Exercise 1.4

Question 5:

If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + y^2$.

Answer 5:

Given that: $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$$= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{3-2}$$

$$= (\sqrt{3}+\sqrt{2})^2$$

$$= 3 + 2 + 2\sqrt{6}$$

$$= 5 + 2\sqrt{6}$$

and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$$

$$= (\sqrt{3}-\sqrt{2})^2$$

$$= 3 + 2 - 2\sqrt{6}$$

$$= 5 - 2\sqrt{6}$$

Now $x^2 + y^2 = \left(5 + 2\sqrt{6}\right)^2 + \left(5 - 2\sqrt{6}\right)^2$

$= 25 + 24 + 20\sqrt{6} + 25 + 24 - 20\sqrt{6} = 98$