Exercise 2.3

Question 16:

Show that:

(i) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$.

(ii) $2x - 3$ is a factor of $x + 2x^3 - 9x^2 + 12$.

Answer 16:

(i). We have $p(x) = 69 + 11x - x^2 + x^3$, $g(x) = x + 3$

Put $g(x) = 0$

$\Rightarrow x + 3 = 0$

$\Rightarrow x = -3$

According to factor theorem if $g(x)$ is a factor of $p(x)$, the remainder $p(-3)$ should be zero.

Remainder $= p(-3)$

$= 69 + 11(-3) - (-3)^2 + (-3)^3$

$= 69 - 33 - 9 - 27$

$= 69 - 69 = 0$

$\Rightarrow g(x)$ is a factor of $p(x)$.

(ii). We have $p(x) = x + 2x^3 - 9x^2 + 12$, $g(x) = 2x - 3$

Put $g(x) = 0$

$\Rightarrow 2x - 3 = 0$

$\Rightarrow x = \frac{3}{2}$

According to factor theorem if $g(x)$ is a factor of $p(x)$, the remainder $p\left(\frac{3}{2}\right)$ should be zero.

Remainder $= p\left(\frac{3}{2}\right)$

$= \left(\frac{3}{2}\right) + 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + 12$

$= \frac{3}{2} + \frac{27}{4} - \frac{81}{4} + 12$

$$= \frac{6 + 27 - 81 + 48}{4}$$

$$= \frac{81 - 81}{4} = 0$$

$\Longrightarrow g(x)$ is a factor of $p(x)$.