Exercise 5.1

Question 1:

Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$.

Answer 1:

Given function $f(x) = 5x - 3$

At $x = 0, f(0) = 5(0) - 3 = -3$

LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(5x - 3) = -3$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(5x - 3) = -3$

Here, at $x = 0$, LHL = RHL = $f(0) = -3$

Hence, the function f is continuous at $x = 0$.

At $x = -3, f(-3) = 5(-3) - 3 = -18$

LHL = $\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-}(5x - 3) = -18$

RHL = $\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+}(5x - 3) = -18$

Here, at $x = -3$, LHL = RHL = $f(-3) = -18$

Hence, the function f is continuous at $x = -3$.

At $x = 5, f(5) = 5(5) - 3 = 22$

LHL = $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-}(5x - 3) = 22$

RHL = $\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+}(5x - 3) = 22$

Here, at $x = 5$, LHL = RHL = $f(5) = 22$

Hence, the function f is continuous at $x = 5$.

Question 2:

Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

Answer 2:

Given function $f(x) = 2x^2 - 1$. At $x = 3, f(3) = 2(3)^2 - 1 = 17$

LHL = $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(2x^2 - 1) = 17$

RHL = $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(2x^2 - 1) = 17$

Here, at $x = 3$, LHL = RHL = $f(3) = 17$

Hence, the function f is continuous at $x = 3$.

Question 3:

Examine the following functions for continuity:

(a) $f(x) = x - 5$ **(b)** $f(x) = \frac{1}{x-5}, x \neq 5$

(c) $f(x) = \frac{x^2-25}{x+5}, x \neq -5$ **(d)** $f(x) = |x - 5|$

Answer 3:

(a) Given function $f(x) = x - 5$

Let, k be any real number. At $x = k, f(k) = k - 5$

LHL = $\lim_{x\to k^-} f(x) = \lim_{x\to k^-}(x - 5) = k - 5$

RHL = $\lim_{x\to k^+} f(x) = \lim_{x\to k^+}(x - 5) = k - 5$

At, $x = k$, LHL = RHL = $f(k) = k - 5$

Hence, the function f is continuous for all real numbers.

(b) Given function $f(x) = \frac{1}{x-5}, x \neq 5$

Let, k $(k \neq 5)$ be any real number. At $x = k, f(k) = \frac{1}{k-5}$

LHL = $\lim_{x\to k^-} f(x) = \lim_{x\to k^-} \left(\frac{1}{x-5}\right) = \frac{1}{k-5}$

RHL = $\lim_{x\to k^+} f(x) = \lim_{x\to k^+} \left(\frac{1}{x-5}\right) = \frac{1}{k-5}$

At, $x = k$, LHL = RHL = $f(k) = \frac{1}{k-5}$

Hence, the function f is continuous for all real numbers (except 5).

(c) Given function $f(x) = \frac{x^2-25}{x+5}, x \neq -5$

Let, k $(k \neq -5)$ be any real number.

At $x = k, f(k) = \frac{k^2-25}{k+5} = \frac{(k+5)(k-5)}{(k+5)} = (k+5)$

LHL = $\lim_{x\to k^-} f(x) = \lim_{x\to k^-} \left(\frac{x^2-25}{x+5}\right) = \lim_{x\to k^-} \left(\frac{(k+5)(k-5)}{(k+5)}\right) = k+5$

RHL = $\lim_{x\to k^+} f(x) = \lim_{x\to k^+} \left(\frac{x^2-25}{x+5}\right) = \lim_{x\to k^+} \left(\frac{(k+5)(k-5)}{(k+5)}\right) = k+5$

At, $x = k$, LHL = RHL = $f(k) = k+5$

Hence, the function f is continuous for all real numbers (except – 5).

(d) Given function $f(x) = |x-5| = \begin{cases} 5-x, & x < 5 \\ x-5, & x \geq 5 \end{cases}$

Let, k be any real number. According to question, $k < 5$ or $k = 5$ or $k > 5$.

First case: If, $k < 5$,

$f(k) = 5-k$ and $\lim_{x\to k} f(x) = \lim_{x\to k}(5-x) = 5-k$, Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers less than 5.

Second case: If, $k = 5$,

$f(k) = k-5$ and $\lim_{x\to k} f(x) = \lim_{x\to k}(x-5) = k-5$, Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous at x = 5.

Third case: If, $k > 5$,

$f(k) = k-5$ and $\lim_{x\to k} f(x) = \lim_{x\to k}(x-5) = k-5$, Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers greater than 5.

Hence, the function f is continuous for all real numbers.

Question 4:

Prove that the function $f(x) = x^n$, is continuous at $x = n$, where n is a positive integer.

Answer 4:

Given function $f(x) = x^n$.

At $x = n, f(n) = n^n$

$$\lim_{x\to n} f(x) = \lim_{x\to n}(x^n) = n^n$$

Here, at $x = n, \lim_{x\to n} f(x) = f(n) = n^n$

Hence, the function f is continuous at $x = n$, where n is positive integer.

Question 5:

Is the function f defined by $f(x) = \begin{cases} x, & x \leq 1 \\ 5, & x > 1 \end{cases}$ continuous at x = 0? At x = 1? At x = 2?

Answer 5:

Given function $f(x) = \begin{cases} x, & x \leq 1 \\ 5, & x > 1 \end{cases}$

At $x = 0, f(0) = 0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}(x) = 0$$

Here, $x = 0, \lim_{x\to 0} f(x) = f(0) = 0$

Hence, the function f is discontinuous at $x = 0$.

At $x = 1, f(1) = 1$

LHL = $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x) = 1$

RHL = $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(5) = 5$

Here, at $x = 1$, LHL ≠ RHL. Hence, the function f is discontinuous at x = 1.

At $x = 2, f(2) = 5$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}(5) = 5$$

Here, at $x = 2, \lim_{x\to 2} f(x) = f(2) = 5$

Hence, the function f is continuous at $x = 2$.

Find all points of discontinuity of f, where f is defined by

Question 6:

$$f(x) = \begin{cases} 2x + 3, & \text{If } x \leq 2 \\ 2x - 3, & \text{If } x > 2 \end{cases}$$

Answer 6:

Let, k be any real number. According to question, $k < 2$ or $k = 2$ or $k > 2$

First case: यदि, $k < 2$,

$f(k) = 2k + 3$ and $\lim_{x\to k} f(x) = \lim_{x\to k}(2x + 3) = 2k + 3$, Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers smaller than 2.

Second case: If, $k = 2, f(2) = 2k + 3$

LHL = $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2x + 3) = 7$

RHL = $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(2x - 3) = 1$

Here, at $x = 2$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 2$.

Third case: If, $k > 2$,

$f(k) = 2k - 3$ and $\lim_{x\to k} f(x) = \lim_{x\to k}(2x - 3) = 2k - 3$, Here, $\lim_{x\to k} f(x) = f(k)$

Therefore, the function f is continuous for all real numbers greater than 2.

Hence, the function f is discontinuous only at $x = 2$.

Question 7:

$$f(x) = \begin{cases} |x| + 3, & \text{If } x \leq -3 \\ -2x, & \text{If } -3 < x < 3 \\ 6x + 2, & \text{If } x \geq 3 \end{cases}$$

Answer 7:

Let, k be any real number. According to question,

$k < -3$ or $k = -3$ or $-3 < k < 3$ or $k = 3$ or $k > 3$

First case: If, $k < -3$,
$f(k) = -k + 3$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(-x + 3) = -k + 3$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers less than -3.

Second case: If, $k = -3$, $f(-3) = -(-3) + 3 = 6$
LHL = $\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-}(-x + 3) = -(-3) + 3 = 6$
RHL = $\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+}(-2x) = -2(-3) = 6$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous at $x = -3$.

Third case: If, $-3 < k < 3$,
$f(k) = -2k$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(-2x) = -2k$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous at $-3 < x < 3$.

Fourth case: If $k = 3$,
LHL = $\lim_{x \to k^-} f(x) = \lim_{x \to k^-}(-2x) = -2k$
RHL = $\lim_{x \to k^+} f(x) = \lim_{x \to k^+}(6x + 2) = 6k + 2$,
Here, at $x = 3$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 3$.

Fifth case: If, $k > 3$,
$f(k) = 6k + 2$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(6x + 2) = 6k + 2$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for all numbers greater than 3.
Hence, the function f is discontinuous only at $x = 3$.

Question 8:

$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{If } x \neq 0 \\ 0, & \text{If } x = 0 \end{cases}$$

Answer 8:

After redefining the function f, we get

$$f(x) = \begin{cases} -\frac{x}{x} = -1, & \text{If } x < 0 \\ 0, & \text{If } x = 0 \\ \frac{x}{x} = 1, & \text{If } x > 0 \end{cases}$$

Let, k be any real number. According to question, $k < 0$ or $k = 0$ or $k > 0$.
First case: If, $k < 0$,
$f(k) = -\frac{k}{k} = -1$ and $\lim_{x \to k} f(x) = \lim_{x \to k}\left(-\frac{x}{x}\right) = -1$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers smaller than 0.

Second case: If, $k = 0$, $f(0) = 0$
LHL = $\lim_{x \to k^-} f(x) = \lim_{x \to k^-}\left(-\frac{x}{x}\right) = -1$ and RHL = $\lim_{x \to k^+} f(x) = \lim_{x \to k^+}\left(\frac{x}{x}\right) = 1$,
Here, at $x = 0$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 0$.

Third case: If, $k > 0$,
$f(k) = \frac{k}{k} = 1$ and $\lim_{x \to k} f(x) = \lim_{x \to k}\left(\frac{x}{x}\right) = 1$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers greater than 0.
Therefore, the function f is discontinuous only at $x = 0$.

Question 9:

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{If } x < 0 \\ -1, & \text{If } x \geq 0 \end{cases}$$

Answer 9:

Redefining the function, we get

$$f(x) = \begin{cases} \frac{x}{|x|} = \frac{x}{-x} = -1, & \text{If } x < 0 \\ -1, & \text{If } x \geq 0 \end{cases}$$

Here, $\lim_{x \to k} f(x) = f(k) = -1$, where k is a real number.

Hence, the function f is continuous for all real numbers.

Question 10:

$$f(x) = \begin{cases} x + 1, & \text{If } \geq 1 \\ x^2 + 1, & \text{If } x < 1 \end{cases}$$

Answer 10:

Let, k be any real number. According to question, $k < 1$ or $k = 1$ or $k > 1$

First case: If, $k < 1$,

$f(k) = k^2 + 1$ and $\lim_{x \to k} f(x) = \lim_{x \to k} (x^2 + 1) = k^2 + 1$. Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers smaller than 1.

Second case: If, $k = 1$, $f(1) = 1 + 1 = 2$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2 + 1) = 1 + 1 = 2$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x + 1) = 1 + 1 = 2$,

Here, at $x = 1$, LHL = RHL = $f(1)$. Hence, the function f is continuous at $x = 1$.

Third case: If, $k > 1$,

$f(k) = k + 1$ and $\lim_{x \to k} f(x) = \lim_{x \to k} (x + 1) = k + 1$. Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers greater than 1.

Therefore, the function f is continuous for all real numbers.

Question 11:

$$f(x) = \begin{cases} x^3 - 3, & \text{If } x \leq 2 \\ x^2 + 1, & \text{If } x > 2 \end{cases}$$

Answer 11:

Let, k be any real number. According to question, $k < 2$ or $k = 2$ or $k > 2$

First case: If, $k < 2$,

$f(k) = k^3 - 3$ and $\lim_{x \to k} f(x) = \lim_{x \to k} (x^3 - 3) = k^3 - 3$. Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers less than 2.

Second case: If, $k = 2$, $f(2) = 2^3 - 3 = 5$

LHL = $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^3 - 3) = 2^3 - 3 = 5$

RHL = $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 + 1) = 2^2 + 1 = 5$,

Here, at $x = 2$, LHL = RHL = $f(2)$

Hence, the function f is continuous at $x = 2$.

Third case: If, $k > 2$,
$f(k) = k^2 + 1$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(x^2 + 1) = k^2 + 1$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for real numbers greater than 2.
Hence, the function f is continuous for all real numbers.

Question 12:

$f(x) = \begin{cases} x^{10} - 1, & \text{If } x \leq 1 \\ x^2, & \text{If } x > 1 \end{cases}$

Answer 12:

Let, k be any real number. According to question, $k < 1$ or $k = 1$ or $k > 1$
First case: If, $k < 1$,
$f(k) = k^{10} - 1$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(x^{10} - 1) = k^{10} - 1$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers less than 1.

Second case: If, $k = 1$, $f(1) = 1^{10} - 1 = 0$
LHL = $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-}(x^{10} - 1) = 0$
RHL = $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+}(x^2) = 1$,
Here, at $x = 1$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 1$.

Third case: If, $k > 1$,
$f(k) = k^2$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(x^2) = k^2$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for all real values greater than 1.
Hence, the function f is discontinuous only at $x = 1$.

Question 13:

Is the function defined by $f(x) = \begin{cases} x + 5, & \text{If } x \leq 1 \\ x - 5, & \text{If } x > 1 \end{cases}$ a continuous function?

Answer 13:

Let, k be any real number. According to question, $k < 1$ or $k = 1$ or $k > 1$
First case: If, $k < 1$,
$f(k) = k + 5$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(x + 5) = k + 5$. Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers less than 1.

Second case: If, $k = 1$, $f(1) = 1 + 5 = 6$
LHL = $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-}(x + 5) = 6$
RHL = $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+}(x - 5) = -4$,
Here, at $x = 1$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 1$.

Third case: If, $k > 1$,
$f(k) = k - 5$ and $\lim_{x \to k} f(x) = \lim_{x \to k}(x - 5) = k - 5$.
Here, $\lim_{x \to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers greater than 1.
Hence, the function f is discontinuous only at $x = 1$.

Discuss the continuity of the function f, where f is defined by:

Question 14:

$$f(x) = \begin{cases} 3, & \text{If } 0 \le x \le 1 \\ 4, & \text{If } 1 < x < 3 \\ 5, & \text{If } 3 \le x \le 10 \end{cases}$$

Answer 14:

Let, k be any real number. According to question,
$0 \le k \le 1$ or $k = 1$ or $1 < k < 3$ or $k = 3$ or $3 \le k \le 10$

First case: If, $0 \le k \le 1$,
$f(k) = 3$ and $\lim\limits_{x\to k} f(x) = \lim\limits_{x\to k}(3) = 3$. Here, $\lim\limits_{x\to k} f(x) = f(k)$
Hence, the function f is continuous for $0 \le x \le 1$.

Second case: If, $k = 1$, $f(1) = 3$
LHL = $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-}(3) = 3$
RHL = $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+}(4) = 4$,
Here, at $x = 1$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 1$.

Third case: If, $1 < k < 3$,
$f(k) = 4$ and $\lim\limits_{x\to k} f(x) = \lim\limits_{x\to k}(4) = 4$. Here, $\lim\limits_{x\to k} f(x) = f(k)$
Hence, the function f is continuous for $1 < x < 3$.

Fourth case: If $k = 3$,
LHL = $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-}(4) = 4$ and RHL = $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+}(5) = 5$,
Here, at $x = 3$, LHL ≠ RHL. Hence, the function f is discontinuous at $x = 3$.

Fifth case: If, $3 \le k \le 10$,
$f(k) = 5$ and $\lim\limits_{x\to k} f(x) = \lim\limits_{x\to k}(5) = 5$. Here, $\lim\limits_{x\to k} f(x) = f(k)$
Hence, the function f is continuous for $3 \le x \le 10$.
Hence, the function f is discontinuous only at $x = 1$ and $x = 3$.

Question 15:

$$f(x) = \begin{cases} 2x, & \text{If } x < 0 \\ 0, & \text{If } 0 \le x \le 1 \\ 4x, & \text{If } x > 1 \end{cases}$$

Answer 15:

Let, k be any real number. According to question,
$k < 0$ or $k = 0$ or $0 \le k \le 1$ or $k = 1$ or $k > 1$

First case: If, $k < 0$,
$f(k) = 2k$ and $\lim\limits_{x\to k} f(x) = \lim\limits_{x\to k}(2x) = 2k$. Here, $\lim\limits_{x\to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers less than 0.

Second case: If, $k = 0$, $f(0) = 0$
LHL = $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-}(2x) = 0$
RHL = $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+}(0) = 0$. Here, $\lim\limits_{x\to k} f(x) = f(k)$
Hence, the function f is continuous at x = 0.

Third case: If, $0 \le k \le 1$,
$f(k) = 0$ and $\lim\limits_{x\to k} f(x) = \lim\limits_{x\to k}(0) = 0$. Here, $\lim\limits_{x\to k} f(x) = f(k)$

Hence, the function f is continuous at $0 \leq x \leq 1$.

Fourth case: If $k = 1$,
LHL = $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (0) = 0$
RHL = $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (4x) = 4$,
Here, at $x = 1$, LHL $\neq$ RHL.
Hence, the function f is discontinuous at x = 1.

Fifth case: If, $k > 1$,
$f(k) = 4k$ and $\lim_{x\to k} f(x) = \lim_{x\to k} (4x) = 4k$.
Here, $\lim_{x\to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers greater than 1.
Therefore, the function f is discontinuous only at x = 1.

Question 16:

$$f(x) = \begin{cases} -2, & \text{If } x \leq -1 \\ 2x, & \text{If } -1 < x \leq 1 \\ 2, & \text{If } x > 1 \end{cases}$$

Answer 16:

Let, k be any real number.
According to question, $k < -1$ or $k = -1$ or $-1 < x \leq 1$ or $k = 1$ or $k > 1$
First case: If, $k < -1$,
$f(k) = -2$ and $\lim_{x\to k} f(x) = \lim_{x\to k} (-2) = -2$. Here, $\lim_{x\to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers less than -1.

Second case: If, $k = -1$ पर, $f(-1) = -2$
LHL = $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (-2) = -2$
RHL = $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (2x) = -2$. Here, $\lim_{x\to k} f(x) = f(k)$
Hence, the function f is continuous at x = -1.

Third case: If, $-1 < x \leq 1$,
$f(k) = 2k$ and $\lim_{x\to k} f(x) = \lim_{x\to k} (2x) = 2k$. Here, $\lim_{x\to k} f(x) = f(k)$
Hence, the function f is continuous at $-1 < x \leq 1$.

Fourth case: If, $k = 1$,
LHL = $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x) = 2$
RHL = $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2) = 2$. Here, $\lim_{x\to k} f(x) = f(k)$
Hence, the function f is continuous at x = 1.

Fifth case: If, $k > 1$,
$f(k) = 2$ and $\lim_{x\to k} f(x) = \lim_{x\to k} (2) = 2$.
Here, $\lim_{x\to k} f(x) = f(k)$
Hence, the function f is continuous for all real numbers greater than 1.
Therefore, the function f is continuous for all real numbers.

Question 17:
Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax+1, & \text{If } x \le 3 \\ bx+3, & \text{If } x > 3 \end{cases}$$

is continuous at $x = 3$.

Answer 17:
Given that the function is continuous at $x = 3$. Therefore, LHL = RHL = $f(3)$

$\Rightarrow \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$\Rightarrow \lim_{x\to 3^-} ax+1 = \lim_{x\to 3^+} bx+3 = 3a+1$

$\Rightarrow 3a+1 = 3b+3 = 3a+1$

$\Rightarrow 3a = 3b+2 \quad \Rightarrow a = b + \frac{2}{3}$

Question 18:
For what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2-2x), \text{यदि } x \le 0 \\ 4x+1, \text{यदि } x > 0 \end{cases}$$

continuous at $x = 0$? What about continuity at $x = 1$?

Answer 18:
Given that the function is continuous at $x = 0$. Therefore, LHL = RHL = $f(0)$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0^-} \lambda(x^2-2x) = \lim_{x\to 0^+} 4x+1 = \lambda[(0)^2 - 2(0)]$

$\Rightarrow \lambda[(0)^2 - 2(0)] = 4(0)+1 = \lambda(0)$

$\Rightarrow 0.\lambda = 1 \quad \Rightarrow \lambda = \frac{1}{0}$

Hence, there is no real value of λ for which the given function be continuous.

If, $x = 1$,

$f(1) = 4(1)+1 = 5$ and $\lim_{x\to 1} f(x) = \lim_{x\to 1} 4(1)+1 = 5$, Here, $\lim_{x\to 1} f(x) = f(1)$

Hence, the function f is continuous for all real values of λ.

Question 19:
Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points. Here $[x]$ denotes the greatest integer less than or equal to x.

Answer 19:
Let, k be any integer.

LHL = $\lim_{x\to k^-} f(x) = \lim_{x\to k^-} x - [x] = k - (\mathrm{k}-1) = 1$

RHL = $\lim_{x\to k^+} f(x) = \lim_{x\to k^+} x - [x] = k - (\mathrm{k}) = 0$,

Here, at $x = k$, LHL ≠ RHL. Hence, the function f is discontinuous for all integers.

Question 20:
Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$.

Answer 20:
Given function: $f(x) = x^2 - \sin x + 5$,

At $x = \pi$, $f(\pi) = \pi^2 - \sin \pi + 5 = \pi^2 - 0 + 5 = \pi^2 + 5$

$\lim_{x\to n} f(x) = \lim_{x\to n} x^2 - \sin x + 5 = \pi^2 - \sin \pi + 5 = \pi^2 - 0 + 5 = \pi^2 + 5$

Here, at $x = \pi$, $\lim_{x\to n} f(x) = f(\pi) = \pi^2 + 5$

Hence, the function f is continuous at $x = \pi$.

Question 21:

Discuss the continuity of the following functions:

(a) $f(x) = \sin x + \cos x$ **(b)** $f(x) = \sin x - \cos x$

(c) $f(x) = \sin x\ .\cos x$

Answer 21:

Let, $g(x) = \sin x$

Let, k be any real number. At $x = k$, $g(k) = \sin k$

LHL = $\lim_{x\to k^-} g(x) = \lim_{x\to k^-} \sin x = \lim_{h\to 0} \sin(k-h) = \lim_{h\to 0} \sin k \cos h - \cos k \sin h = \sin k$

RHL = $\lim_{x\to k^+} g(x) = \lim_{x\to k^+} \sin x = \lim_{h\to 0} \sin(k+h) = \lim_{h\to 0} \sin k \cos h + \cos k \sin h = \sin k$

Here, at $x = k$, LHL = RHL = $g(k)$.

Hence, the function g is continuous for all real numbers.

Let, $h(x) = \cos x$

Let, k be any real number. $x = k$ पर, $h(k) = \cos k$

LHL = $\lim_{x\to k^-} h(x) = \lim_{x\to k^-} \cos x = \lim_{h\to 0} \cos(k-h) = \lim_{h\to 0} \cos k \cos h + \sin k \sin h = \cos k$

RHL = $\lim_{x\to k^+} h(x) = \lim_{x\to k^+} \cos x = \lim_{h\to 0} \cos(k+h) = \lim_{h\to 0} \cos k \cos h - \sin k \sin h = \cos k$

Here, at $x = k$, LHL = RHL = $h(k)$.

Hence, the function h is continuous for all real numbers.

We know that if g and h are two continuous functions, then the functions $g + h, g - h$ and gh also be a continuous functions.

Hence, **(a)** $f(x) = \sin x + \cos x$ **(b)** $f(x) = \sin x - \cos x$ and **(c)** $f(x) = \sin x\ .\cos x$ are continuous functions.

Question 22:

Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

Answer 22:

Let $g(x) = \sin x$

Let, k be any real number. At $x = k$, $g(k) = \sin k$

LHL = $\lim_{x\to k^-} g(x) = \lim_{x\to k^-} \sin x = \lim_{h\to 0} \sin(k-h) = \lim_{h\to 0} \sin k \cos h - \cos k \sin h = \sin k$

RHL = $\lim_{x\to k^+} g(x) = \lim_{x\to k^+} \sin x = \lim_{h\to 0} \sin(k+h) = \lim_{h\to 0} \sin k \cos h + \cos k \sin h = \sin k$

Here, at $x = k$, LHL = RHL = $g(k)$.

Hence, the function g is continuous for all real numbers.

Let $h(x) = \cos x$

Let, k be any real number. At $x = k$, $h(k) = \cos k$

LHL = $\lim_{x\to k^-} h(x) = \lim_{x\to k^-} \cos x = \lim_{h\to 0} \cos(k-h) = \lim_{h\to 0} \cos k \cos h + \sin k \sin h = \cos k$

RHL = $\lim_{x\to k^+} h(x) = \lim_{x\to k^+} \cos x = \lim_{h\to 0} \cos(k+h) = \lim_{h\to 0} \cos k \cos h - \sin k \sin h = \cos k$

Here, at $x = k$, LHL = RHL = $h(k)$.

Hence, the function h is continuous for all real numbers.

We know that if g and h are two continuous functions, then the functions $\frac{g}{h}$, $h \neq 0$, $\frac{1}{h}$, $h \neq 0$ and $\frac{1}{g}$, $g \neq 0$ be continuous functions.

Therefore, $cosec\ x = \frac{1}{\sin x}, \sin x \neq 0$ is continuous $\Rightarrow x \neq n\pi\ (n \in Z)$ is continuous.
Hence, $cosec\ x$ is continuous except $x = n\pi\ (n \in Z)$.

$sec\ x = \frac{1}{\cos x}, \cos x \neq 0$ is continuous. $\Rightarrow x \neq \frac{(2n+1)\pi}{2}\ (n \in Z)$ is continuous.
Hence, $sec\ x$ is continuous except $x = \frac{(2n+1)\pi}{2}\ (n \in Z)$.

$cot\ x = \frac{\cos x}{\sin x}, \sin x \neq 0$ is continuous. $\Rightarrow x \neq n\pi\ (n \in Z)$ is continuous.
Hence, $cot\ x$ is continuous except $x = n\pi\ (n \in Z)$.

Question 23:

Find all points of discontinuity of f, where

$$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{If } x < 0 \\ x + 1, & \text{If } x \geq 0 \end{cases}$$

Answer 23:

Let, k be any real number. According to question, $k < 0$ or $k = 0$ or $k > 0$

First case: If, $k < 0$,

$f(k) = \frac{\sin k}{k}$ and $\lim_{x \to k} f(x) = \lim_{x \to k} \left(\frac{\sin x}{x}\right) = \frac{\sin k}{k}$, Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers less than 0.

Second case: If, $k = 0$, $f(0) = 0 + 1 = 1$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x + 1) = 0 + 1 = 1$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x + 1) = 0 + 1 = 1$,

Here, at $x = 0$, LHL = RHL = $f(0)$. Hence, the function f is continuous at x = 0.

Third case: If, $k > 0$,

$f(k) = k + 1$ and $\lim_{x \to k} f(x) = \lim_{x \to k} (x + 1) = k + 1$, Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function f is continuous for all real numbers greater than 0.
Therefore, the function f is continuous for all real numbers.

Question 24:

Determine if f defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{If } x \neq 0 \\ 0, & \text{If } x = 0 \end{cases}$$

is a continuous function?

Answer 24:

Let, k be any real number. According to question, $k \neq 0$ or $k = 0$

First case: If, $k \neq 0$,

$f(k) = k^2 \sin\frac{1}{k}$ and $\lim_{x \to k} f(x) = \lim_{x \to k} \left(x^2 \sin\frac{1}{x}\right) = k^2 \sin\frac{1}{k}$, Here, $\lim_{x \to k} f(x) = f(k)$

Hence, the function f is continuous for $k \neq 0$.

Second case: If, $k = 0$, $f(0) = 0$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(x^2 \sin\frac{1}{x}\right) = \lim_{x \to 0} \left(x^2 \sin\frac{1}{x}\right)$

We know that, $-1 \le \sin\frac{1}{x} \le 1, \quad x \ne 0 \qquad \Rightarrow -x^2 \le \sin\frac{1}{x} \le x^2$

$\Rightarrow \lim_{x\to 0}(-x^2) \le \lim_{x\to 0} \sin\frac{1}{x} \le \lim_{x\to 0} x^2$

$\Rightarrow 0 \le \lim_{x\to 0} \sin\frac{1}{x} \le 0 \qquad \Rightarrow \lim_{x\to 0} \sin\frac{1}{x} = 0 \qquad \Rightarrow \lim_{x\to 0^-} x^2 \sin\frac{1}{x} = 0 \qquad \Rightarrow \lim_{x\to 0^-} f(x) = 0$

Similarly, RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(x^2 \sin\frac{1}{x}\right) = \lim_{x\to 0} \left(x^2 \sin\frac{1}{x}\right) = 0,$

Here, at $x = 0$, LHL = RHL = $f(0)$

Hence, at$x = 0$, f is continuous.

Hence, the function$f$ is continuous for all real numbers.

Question 25:

Examine the continuity of *f*, where *f* is defined by

$$f(x) = \begin{cases} \sin x - \cos x, & \text{If } x \ne 0 \\ -1, & \text{If } x = 0 \end{cases}$$

Answer 25:

Let, k be any real number. According to question, $k \ne 0$ or $k = 0$

First case: If, $k \ne 0$, $f(0) = 0 - 1 = -1$

LHL = $\lim_{k\to 0^-} f(x) = \lim_{k\to 0^-} (\sin x - \cos x) = 0 - 1 = -1$

RHL = $\lim_{k\to 0^+} f(x) = \lim_{k\to 0^+} (\sin x - \cos x) = 0 - 1 = -1,$

Hence, at $x \ne 0$, LHL = RHL = $f(x)$

Hence, the function f is continuous at $x \ne 0$.

Second case: If, $k = 0$, $f(k) = -1$

and $\lim_{x\to k} f(x) = \lim_{x\to k}(-1) = -1$, Here, $\lim_{x\to k} f(x) = f(k)$

Hence, the function f is continuous at x = 0.

Therefore, the function f is continuous for all real numbers.

Find the values of k so that the function f is continuous at the indicated point in exercises 26 to 29.

Question 26:

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{If } x \ne \dfrac{\pi}{2} \\ 3, & \text{If } x = \dfrac{\pi}{2} \end{cases} \quad \text{at } x = \frac{\pi}{2}$$

Answer 26:

Given that the function is continuous at $x = \frac{\pi}{2}$. Therefore, LHL = RHL = $f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{x\to \frac{\pi}{2}^-} f(x) = \lim_{x\to \frac{\pi}{2}^+} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{x\to \frac{\pi}{2}^-} \frac{k\cos x}{\pi - 2x} = \lim_{x\to \frac{\pi}{2}^+} \frac{k\cos x}{\pi - 2x} = 3$

$\Rightarrow \lim_{h\to 0} \frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = \lim_{h\to 0} \frac{k\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)} = 3$

$\Rightarrow \lim_{h\to 0} \frac{k \sin h}{2h} = \lim_{h\to 0} \frac{-k \sin h}{-2h} = 3$

$\Rightarrow \frac{k}{2} = \frac{k}{2} = 3$ $\left[\because \lim_{h\to 0} \frac{\sin h}{h} = 1\right]$

$\Rightarrow k = 6$

Question 27:

$$f(x) = \begin{cases} kx^2, & \text{If } x \le 2 \\ 3, & \text{If } x > 2 \end{cases} \text{ at } x = 2$$

Answer 27:

Given that the function is continuous at $x = 2$.

Therefore, LHL = RHL = $f(2)$

$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2^-} kx^2 = \lim_{x\to 2^+} 3 = k(2)^2$

$\Rightarrow 4k = 3 = 4k$

$\Rightarrow k = \frac{3}{4}$

Question 28:

$$f(x) = \begin{cases} kx + 1, & \text{If } x \le \pi \\ \cos x, & \text{If } x > \pi \end{cases} \text{ at } x = \pi$$

Answer 28:

Given that the function is continuous at $x = \pi$.

Therefore, LHL = RHL = $f(\pi)$

$\Rightarrow \lim_{x\to \pi^-} f(x) = \lim_{x\to \pi^+} f(x) = f(\pi)$

$\Rightarrow \lim_{x\to \pi^-} kx + 1 = \lim_{x\to \pi^+} \cos x = k(\pi) + 1$

$\Rightarrow k(\pi) + 1 = \cos \pi = k\pi + 1$

$\Rightarrow k\pi + 1 = -1 = k\pi + 1$

$\Rightarrow \pi k = -2$

$\Rightarrow k = -\frac{2}{\pi}$

Question 29:

$$f(x) = \begin{cases} kx + 1, & \text{If } x \le 5 \\ 3x - 5, & \text{If } x > 5 \end{cases} \text{ at } x = 5$$

Answer 29:

Given that the function is continuous at $x = 5$.

Therefore, LHL = RHL = $f(5)$

$\Rightarrow \lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x) = f(5)$

$\Rightarrow \lim_{x\to 5^-} kx + 1 = \lim_{x\to 5^+} 3x - 5 = 5k + 1$

$\Rightarrow 5k + 1 = 15 - 5 = 5k + 1$

$\Rightarrow 5k = 9$

$\Rightarrow k = \frac{9}{5}$

Question 30:

Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & \text{If } x \le 2 \\ ax + b, & \text{If } 2 < x < 10 \\ 21, & \text{If } x \ge 10 \end{cases}$$

is a continuous function.

Answer 30:

Given that the function is continuous at $x = 2$. Therefore, LHL = RHL = $f(2)$

$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2^-} 5 = \lim_{x\to 2^+} ax + b = 5$

$\Rightarrow 2a + b = 5$ ... (1)

Given that the function is continuous at $x = 10$. Therefore, LHL = RHL = $f(10)$

$\Rightarrow \lim_{x\to 10^-} f(x) = \lim_{x\to 10^+} f(x) = f(10)$

$\Rightarrow \lim_{x\to 10^-} ax + b = \lim_{x\to 10^+} 21 = 21$

$\Rightarrow 10a + b = 21$ ... (2)

Solving the equation (1) and (2), we get

$a = 2 \quad b = 1$

Question 31:

Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.

Answer 31:

Assuming that the functions are well defined for all real numbers, we can write the given function f in the combination of g and h $(f = goh)$. Where, $g(x) = \cos x$ and $h(x) = x^2$. If g and h both are continuous function then f also be continuous.

$[\because goh(x) = g(h(x)) = g(x^2) = \cos(x^2)]$

Function $g(x) = \cos x$

Let, k be any real number. At $x = k$, $g(k) = \cos k$

$\lim_{x\to k} g(x) = \lim_{x\to k} \cos x = \lim_{h\to 0} \cos(k+h) = \lim_{h\to 0} \cos k \cos h - \sin k \sin h = \cos k$

Here, $\lim_{x\to k} g(x) = g(k)$, Hence, the function g is continuous for all real numbers.

Function $h(x) = x^2$

Let, k be any real number. At $x = k$, $h(k) = k^2$

$\lim_{x\to k} h(x) = \lim_{x\to k} x^2 = k^2$

Here, $\lim_{x\to k} h(x) = h(k)$, Hence, the function h is continuous for all real numbers.

Therefore, g and h both are continuous function. Hence, f is continuous.

Question 32:

Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

Answer 32:

Assuming that the functions are well defined for all real numbers, we can write the given function f in the combination of g and h $(f = goh)$. Where, $g(x) = |x|$ and $h(x) = \cos x$. If g and h both are continuous function then f also be continuous.

$[\because goh(x) = g(h(x)) = g(\cos x) = |\cos x|]$

Function $g(x) = |x|$

Rearranging the function g, we get

$$g(x) = \begin{cases} -x, & \text{If } x < 0 \\ \text{x}, & \text{If } x \geq 0 \end{cases}$$

Let, k be any real number. According to question, $k < 0$ or $k = 0$ or $k > 0$

First case: If, $k < 0$,

$g(k) = 0$ and $\lim_{x \to k} g(x) = \lim_{x \to k}(-\text{x}) = 0$, here, $\lim_{x \to k} g(x) = g(k)$

Hence, the function g is continuous for all real numbers less than 0.

Second case: If, $k = 0$, $g(0) = 0 + 1 = 1$

LHL= $\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-}(-x) = 0$

RHL = $\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+}(x) = 0$,

Here, at $x = 0$, LHL = RHL = $g(0)$

Hence, the function g is continuous at x = 0.

Third case: If, $k > 0$,

$g(k) = 0$ and $\lim_{x \to k} g(x) = \lim_{x \to k}(x\ ) = 0$, Here, $\lim_{x \to k} g(x) = g(k)$

Hence, the function g is continuous for all real numbers greater than 0.

Hence, the function g is continuous for all real numbers.

Function $h(x) = \cos x$

Let, k be any real number. At $x = k$, $h(k) = \cos k$

$\lim_{x \to k} h(x) = \lim_{x \to k} \cos x = \cos k$

Here, $\lim_{x \to k} h(x) = h(k)$, Hence, the function h is continuous for all real numbers.

Therefore, g and h both are continuous function. Hence, f is continuous.

Question 33:

Examine that $\sin|x|$ is a continuous function.

Answer 33:

Assuming that the functions are well defined for all real numbers, we can write the given function f in the combination of g and h ($f = hog$). Where, $h(x) = \sin x$ and $g(x) = |x|$. If g and h both are continuous function then f also be continuous.

[$\because hog(x) = h(g(x)) = h(|x|) = \sin|x|$]

Function $h(x) = \sin x$

Let, k be any real number. At $x = k$, $h(k) = \sin k$

$\lim_{x \to k} h(x) = \lim_{x \to k} \sin x = \sin k$

Here, $\lim_{x \to k} h(x) = h(k)$, Hence, the function h is continuous for all real numbers.

Function $g(x) = |x|$

Redefining the function g, we get

$$g(x) = \begin{cases} -x, & \text{If } x < 0 \\ \text{x}, & \text{If } x \geq 0 \end{cases}$$

Let, k be any real number. According to question, $k < 0$ or $k = 0$ or $k > 0$

First case: If, $k < 0$,

$g(k) = 0$ and $\lim_{x \to k} g(x) = \lim_{x \to k}(-\text{x}) = 0$, Here, $\lim_{x \to k} g(x) = g(k)$

Hence, the function g is continuous for all real numbers less than 0.

Second case: If, $k = 0$, $g(0) = 0 + 1 = 1$

LHL = $\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$

RHL = $\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0,$

Here, at $x = 0$, LHL = RHL = $g(0)$

Hence, at $x = 0$, the function g is continuous.

Third case: If, $k > 0$,

$g(k) = 0$ and $\lim_{x \to k} g(x) = \lim_{x \to k} (x) = 0$, Here, $\lim_{x \to k} g(x) = g(k)$

Hence, the function g is continuous for all real numbers greater than 0.

Hence, the function g is continuous for all real numbers.

Therefore, g and h both are continuous function. Hence, f is continuous.

Question 34:

Find all the points of discontinuity of f defined by $f(x) = |x| - |x + 1|$.

Answer 34:

Assuming that the functions are well defined for all real numbers, we can write the given function f in the combination of g and h $(f = g - h)$, where, $g(x) = |x|$ and $h(x) = |x + 1|$. If g and h both are continuous function then f also be continuous.

Function $g(x) = |x|$

Redefining the function g, we get,

$$g(x) = \begin{cases} -x, & \text{If } x < 0 \\ \text{x}, & \text{If } x \geq 0 \end{cases}$$

Let, k be any real number. According to question, $k < 0$ or $k = 0$ or $k > 0$

First case: If, $k < 0$,

$g(k) = 0$ and $\lim_{x \to k} g(x) = \lim_{x \to k} (-\text{x}) = 0$, Here, $\lim_{x \to k} g(x) = g(k)$

Hence, the function g is continuous for all real numbers less than 0.

Second case: If, $k = 0$, $g(0) = 0 + 1 = 1$

LHL = $\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$ and RHL = $\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0,$

Here, at $x = 0$, LHL = RHL = $g(0)$

Hence, the function g is continuous at $x = 0$.

Third case: If, $k > 0$,

$g(k) = 0$ and $\lim_{x \to k} g(x) = \lim_{x \to k} (x) = 0$, Here, $\lim_{x \to k} g(x) = g(k)$

Hence, the function g is continuous for all real numbers more than 0.

Hence, the function g is continuous for all real numbers.

Function $h(x) = |x + 1|$

Redefining the function h, we get

$$h(x) = \begin{cases} -(x + 1), & \text{If } x < -1 \\ x + 1, & \text{If } x \geq -1 \end{cases}$$

Let, k be any real number. According to question, $k < -1$ or $k = -1$ or $k > -1$

First case: If, $k < -1$,

$h(k) = -(k + 1)$ and $\lim_{x \to k} h(x) = \lim_{x \to k} -(k + 1) = -(k + 1)$, Here, $\lim_{x \to k} h(x) = h(k)$

Hence, the function g is continuous for all real numbers less than – 1.

Second case: If, $k = -1$, $h(-1) = -1 + 1 = 0$

LHL = $\lim_{x\to -1^-} h(x) = \lim_{x\to -1^-} -(-1+1) = 0$

RHL = $\lim_{x\to -1^+} h(x) = \lim_{x\to -1^+} (x+1) = -1+1 = 0,$

Here, at $x = -1$, LHL = RHL = $h(-1)$

Hence, the function h is continuous at $x = -1$.

Third case: If, $k > -1$,

$h(k) = k+1$ and $\lim_{x\to k} h(x) = \lim_{x\to k}(k+1) = \text{k} + 1$, Here, $\lim_{x\to k} h(x) = h(k)$

Hence, the function g is continuous for all real numbers greater than -1.

Hence, the function h is continuous for all real numbers.

Therefore, g and h both are continuous function. Hence, f is continuous.