Exercise 5.2

Differentiate the functions with respect to x in Exercises 1 to 8.

Question 1:

$$\sin(x^2 + 5)$$

Answer 1:

Let $y = \sin(x^2 + 5)$

Therefore,

$$\frac{dy}{dx} = \cos(x^2 + 5).\frac{d}{dx}(x^2 + 5)$$

$$= cos(x^2 + 5).\,2x$$

Question 2:

$$\cos(\sin x)$$

Answer 2:

Let $y = \cos(\sin x)$

Therefore,

$$\frac{dy}{dx} = -\sin(\sin x).\frac{d}{dx}(\sin x)$$

$$= -sin(\sin x).\cos x$$

Question 3:

$\sin(ax + b)$

Answer 3:

Let

$$y = \sin(ax + b)$$

Therefore,

$$\frac{dy}{dx} = \cos(ax + b).\frac{d}{dx}(ax + b)$$

$$= cos(ax + b).\,a$$

Question 4:

$$\sec(\tan(\sqrt{x}))$$

Answer 4:

Let $y = \sec(\tan(\sqrt{x}))$

Therefore,

$$\frac{dy}{dx} = \sec(\tan\sqrt{x})\tan(\tan\sqrt{x}).\frac{d}{dx}(\tan\sqrt{x})$$

$$= \sec(\tan\sqrt{x})\tan(\tan\sqrt{x}).\sec^2\sqrt{x}\frac{d}{dx}(\sqrt{x})$$

$$= \sec(\tan\sqrt{x})\tan(\tan\sqrt{x}).\sec^2\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)$$

Question 5:

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

Answer 5:

Let

$$y = \frac{sin(ax+b)}{cos(cx+d)}$$

Therefore,

$$\frac{dy}{dx} = \frac{cos(cx+d).\frac{d}{dx}sin(ax+b) - sin(ax+b).\frac{d}{dx}cos(cx+d)}{[cos(cx+d)]^2}$$

$$= \frac{cos(cx+d).\sin(ax+b).\frac{d}{dx}(ax+b) - sin(ax+b).[-sin(cx+d).\frac{d}{dx}(cx+d)]}{cos^2(cx+d)}$$

$$= \frac{cos(cx+d).\sin(ax+b).a + sin(ax+b).sin(cx+d)\,c}{cos^2(cx+d)}$$

Question 6:

$$\cos x^3 . \sin^2(x^5)$$

Answer 6:

Let $y = \cos x^3 . \sin^2(x^5)$

Therefore,

$$\frac{dy}{dx} = \cos x^3 .\frac{d}{dx}\sin^2(x^5) + \sin^2(x^5).\frac{d}{dx}\cos x^3$$

$$= \cos x^3 . 2\sin x^5 \cos x^5 .\frac{d}{dx}x^5 + \sin^2(x^5)\,[-\sin x^3].\frac{d}{dx}x^3$$

$$= \cos x^3 . 2\sin x^5 \cos x^5 . 5x^4 - \sin^2(x^5)\sin x^3 . 3x^2$$

Question 7:

$$2\sqrt{\cot(x^2)}$$

Answer 7:

Let $y = 2\sqrt{\cot(x^2)}$

Therefore,

$$\frac{dy}{dx} = 2.\frac{1}{2\sqrt{\cot(x^2)}}.\frac{d}{dx}[\cot(x^2)]$$

$$= \frac{1}{\sqrt{\cot(x^2)}}.[-cossec\ x^2].\frac{d}{dx}x^2$$

$$= \frac{1}{\sqrt{\cot(x^2)}}.[-cossec\ x^2].2x$$

Question 8:

$$\cos(\sqrt{x})$$

Answer 8:

Let $y = \cos(\sqrt{x})$
Therefore,

$$\frac{dy}{dx} = -\sin(\sqrt{x}).\frac{d}{dx}\sqrt{x}$$

$$= -\sin(\sqrt{x}).\frac{1}{2\sqrt{x}}$$

Question 9:

Prove that the function f given by $f(x) = |x-1|, x \in \boldsymbol{R}$, is not differentiable at $x = 1$.

Answer 9:

At $x = 1$,

$$LHD = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{|1-h-1|-|1-1|}{-h} = \lim_{h\to 0}\frac{h}{-h} = -1$$

$$RHD = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{|1+h-1|-|1-1|}{h} = \lim_{h\to 0}\frac{h}{h} = 1$$

Here, $LHD \neq RHD$, therefore, the function $f(x) = |x-1|, x \in \boldsymbol{R}$, is not differentiable at $x = 1$.

Question 10:

Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$, is not differentiable at $x = 1$ and $x = 2$.

Answer 10:

At $x = 1$,

$$LHD = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{[1-h]-|1|}{-h} = \lim_{h\to 0}\frac{0-1}{-h} = \infty$$

$$RHD = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{[1+h]-[1]}{h} = \lim_{h\to 0}\frac{1-1}{h} = 0$$

Here, $LHD \neq RHD$, therefore, the function $f(x) = [x], 0 < x < 3$, is not differentiable at $x = 1$.

At $x = 2$,

$$LHD = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{[2-h]-[2]}{-h} = \lim_{h\to 0}\frac{1-2}{-h} = \infty$$

$$RHD = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{[2+h]-[2]}{h} = \lim_{h\to 0}\frac{2-2}{h} = 0$$

Here, $LHD \neq RHD$, therefore, the function $f(x) = [x], 0 < x < 3$, is not differentiable at $x = 2$.