Exercise 5.5

Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

Question 1:

$\cos x\ .\cos 2x\ .\cos 3x$

Answer 1:

Let $y = \cos x\ .\cos 2x\ .\cos 3x$, taking log on both the sides

$$\log y = \log\cos x + \log\cos 2x + \log\cos 3x$$

Therefore,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}.\frac{d}{dx}\cos x + \frac{1}{\cos 2x}.\frac{d}{dx}\cos 2x + \frac{1}{\cos 3x}.\frac{d}{dx}\cos 3x$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{\cos x}.(-\sin x) + \frac{1}{\cos 2x}.(-\sin 2x).2 + \frac{1}{\cos 3x}.(-\sin 3x).3\right]$$

$$\Rightarrow \frac{dy}{dx} = \cos x\ .\cos 2x\ .\cos 3x\,[-\tan x - 2\tan 2x - 3\tan 3x]$$

Question 2:

$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Answer 2:

Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$, taking log on both the sides

$$\log y = \frac{1}{2}[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)]$$

Therefore,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{(x-1)} + \frac{1}{(x-2)} - \frac{1}{(x-3)} - \frac{1}{(x-4)} - \frac{1}{(x-5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{(x-1)} + \frac{1}{(x-2)} - \frac{1}{(x-3)} - \frac{1}{(x-4)} - \frac{1}{(x-5)}\right]$$

Question 3:

$(\log x)^{\cos x}$

Answer 3:

Let $y = (\log x)^{\cos x}$, taking log on both the sides

$$\log y = \log(\log x)^{\cos x} = \cos x\,.\log\log x$$

Therefore,

$$\frac{1}{y}\frac{dy}{dx} = \cos x\,.\frac{d}{dx}\log\log x + \log\log x\,.\frac{d}{dx}\cos x$$

$$\Rightarrow \frac{dy}{dx} = y\left[\cos x\,.\frac{1}{\log x}.\frac{1}{x} + \log\log x\,.(-\sin x)\right]$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x - \sin x\log\log x}{x\log x}\right]$$

Question 4:
$x^x - 2^{\sin x}$

Answer 4:
Let $u = x^x$ and $v = 2^{\sin x}$ therefore, $y = u - v$
Differentiating with respect to x on both sides
$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ ... (1)
Here, $u = x^x$, taking log on both the sides
$\log u = x\log x$, therefore,
$$\frac{1}{u}\frac{du}{dx} = x.\frac{d}{dx}\log x + \log x . \frac{d}{dx}x = x.\frac{1}{x} + \log x . 1 = 1 + \log x$$
$$\frac{du}{dx} = u[1 + \log x] = x^x[1 + \log x] \quad ... (2)$$
and , $v = 2^{\sin x}$, taking log on both the sides
$\log v = \sin x \log 2$, therefore,
$$\frac{1}{v}\frac{dv}{dx} = \log 2 . \frac{d}{dx}\sin x = \log 2 . \cos x$$
$$\frac{dv}{dx} = v[\cos x \log 2] = 2^{\sin x}[\cos x \log 2] \quad ... (3)$$
Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get
$$\frac{dy}{dx} = x^x[1 + \log x] - 2^{\sin x}[\cos x \log 2]$$

Question 5:
$(x+3)^2 . (x+4)^3 . (x+5)^4$

Answer 5:
Let $y = (x+3)^2 . (x+4)^3 . (x+5)^4$, taking log on both the sides
$$\log y = 2\log(x+3) + 3\log(x+4) + 4\log(x+5)$$
Therefore,
$$\frac{1}{y}\frac{dy}{dx} = 2.\frac{1}{(x+3)} + 3.\frac{1}{(x+4)} + 4.\frac{1}{(x+5)}$$
$$\Rightarrow \frac{dy}{dx} = y\left[\frac{2(x+4)(x+5) + 3(x+3)(x+5) + 4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$$
$$\Rightarrow \frac{dy}{dx} = y\left[\frac{2(x^2+9x+20) + 3(x^2+8x+15) + 4(x^2+7x+12)}{(x+3)(x+4)(x+5)}\right]$$
$$\Rightarrow \frac{dy}{dx} = (x+3)^2 . (x+4)^3 . (x+5)^4\left[\frac{9x^2+70x+133}{(x+3)(x+4)(x+5)}\right]$$
$$\Rightarrow \frac{dy}{dx} = (x+3) . (x+4)^2 . (x+5)^3(9x^2+70x+133)$$

Question 6:

$\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Answer 6:

Let $u = \left(x+\frac{1}{x}\right)^x$ and $v = x^{\left(1+\frac{1}{x}\right)}$, therefore, $y = u + v$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ... (1)

Here, $u = \left(x+\frac{1}{x}\right)^x$, taking log on both the sides

$\log u = x\log\left(x+\frac{1}{x}\right)$, therefore,

$$\frac{1}{u}\frac{du}{dx} = x.\frac{d}{dx}\log\left(x+\frac{1}{x}\right) + \log\left(x+\frac{1}{x}\right).\frac{d}{dx}x$$

$$= x.\frac{1}{\left(x+\frac{1}{x}\right)}.\left(1-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right).1 = \frac{x^2}{x^2+1}.\frac{x^2-1}{x^2} + \log\left(x+\frac{1}{x}\right)$$

$$\frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] \quad ...(2)$$

and , $v = x^{\left(1+\frac{1}{x}\right)}$, taking log on both the sides

$\log v = \left(1+\frac{1}{x}\right)\log x$, therefore,

$$\frac{1}{v}\frac{dv}{dx} = \left(1+\frac{1}{x}\right).\frac{d}{dx}\log x + \log x.\frac{d}{dx}\left(1+\frac{1}{x}\right) = \left(1+\frac{1}{x}\right).\frac{1}{x} + \log x.\left(-\frac{1}{x^2}\right)$$

$$\frac{dv}{dx} = v\left[\left(\frac{x^2+1}{x}\right).\frac{1}{x} - \frac{\log x}{x^2}\right] = x^{\left(1+\frac{1}{x}\right)}\left[\frac{x^2+1-\log x}{x^2}\right] \quad ...(3)$$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left[\frac{x^2+1-\log x}{x^2}\right]$$

Question 7:

$(\log x)^x + x^{\log x}$

Answer 7:

Let $u = (\log x)^x$ and $v = x^{\log x}$, therefore, $y = u + v$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ... (1)

Here, $u = (\log x)^x$, taking log on both the sides

$\log u = x\log\log x$, therefore,

$$\frac{1}{u}\frac{du}{dx} = x.\frac{d}{dx}\log\log x + \log\log x.\frac{d}{dx}x$$

$$= x.\frac{1}{\log x}.\frac{1}{x} + \log\log x.1 = \frac{1}{\log x} + \log\log x$$

$\frac{du}{dx} = (\log x)^x \left[\frac{1 + \log x \,.\log\log x}{\log x}\right]$
$= (\log x)^{x-1}(1 + \log x \,.\log\log x) \quad ...(2)$

and , $v = x^{\log x}$, taking log on both the sides
$\log v = \log x \log x$, therefore,

$\frac{1}{v}\frac{dv}{dx} = \log x \,.\frac{d}{dx}\log x + \log x \,.\frac{d}{dx}\log x$

$= \log x \,.\frac{1}{x} + \log x \,.\frac{1}{x}$

$\frac{dv}{dx} = v\left[\frac{2\log x}{x}\right] = x^{\log x}\left[\frac{2\log x}{x}\right] = x^{\log x - 1}(2\log x) \quad ...(3)$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get
$\frac{dy}{dx} = (\log x)^{x-1}(1 + \log x \,.\log\log x) + x^{\log x - 1}(2\log x)$

Question 8:
$(\sin x)^x + \sin^{-1}\sqrt{x}$

Answer 8:
Let $u = (\sin x)^x$ and $v = \sin^{-1}\sqrt{x}$, therefore, $y = u + v$
Differentiating with respect to x, we get
$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad ...(1)$
Here, $u = (\sin x)^x$, taking log on both the sides
$\log u = x \log \sin x$, therefore,

$\frac{1}{u}\frac{du}{dx} = x.\frac{d}{dx}\log\sin x + \log\sin x \,.\frac{d}{dx}x$

$= x.\frac{1}{\sin x}.\cos x + \log\sin x \,.1 = x\cot x + \log\sin x$
$\frac{du}{dx} = (\sin x)^x(x\cot x + \log\sin x) \quad ...(2)$

and , $v = \sin^{-1}\sqrt{x}$, therefore,

$\frac{1}{v}\frac{dv}{dx} = \log x \,.\frac{d}{dx}\log x + \log x \,.\frac{d}{dx}\log x$

$= \log x \,.\frac{1}{x} + \log x \,.\frac{1}{x}$

$\frac{dv}{dx} = \frac{1}{\sqrt{1-x}}.\frac{d}{dx}\sqrt{x} = \frac{1}{\sqrt{1-x}}.\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x - x^2}} \quad ...(3)$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get
$\frac{dy}{dx} = (\sin x)^x(x\cot x + \log\sin x) + \frac{1}{2\sqrt{x - x^2}}$

Question 9:
$x^{\sin x} + (\sin x)^{\cos x}$

Answer 9:
Let $u = x^{\sin x}$ and $v = (\sin x)^{\cos x}$ therefore, $y = u + v$
Differentiating with respect to x, we get
$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ... (1)
Here, $u = x^{\sin x}$, taking log on both the sides
$\log u = \sin x \log x$, therefore,

$$\frac{1}{u}\frac{du}{dx} = \sin x \,.\frac{d}{dx}\log x + \log x \,.\frac{d}{dx}\sin x = \sin x \,.\frac{1}{x} + \log x \,.\cos x = \frac{\sin x}{x} + \log x \cos x$$

$$\frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x\right] = x^{\sin x - 1}(\sin x + x\log x \cos x) \quad ... (2)$$

and , $v = (\sin x)^{\cos x}$, taking log on both the sides
$\log v = \cos x \log \sin x$, therefore,

$$\frac{1}{v}\frac{dv}{dx} = \cos x \,.\frac{d}{dx}\log \sin x + \log \sin x \,.\frac{d}{dx}\cos x = \cos x \,.\frac{1}{\sin x}\cos x + \log \sin x\,(-\sin x)$$

$$\frac{dv}{dx} = v[\cos x \cot x - \sin x \log \sin x] = (\sin x)^{\cos x}(\cos x \cot x - \sin x \log \sin x) \quad ... (3)$$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get

$$\frac{dy}{dx} = x^{\sin x - 1}(\sin x + x\log x \cos x) + (\sin x)^{\cos x}(\cos x \cot x - \sin x \log \sin x)$$

Question 10:
$x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Answer 10:
Let $u = x^{x\cos x}$ and $v = \frac{x^2+1}{x^2-1}$ therefore, $y = u + v$
Differentiating with respect to x, we get
$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ... (1)
Here, $u = x^{x\cos x}$, taking log on both the sides
$\log u = x\log x$, therefore,

$$\frac{1}{u}\frac{du}{dx} = x\cos x \,.\frac{d}{dx}\log x + \log x \,.\frac{d}{dx}x\cos x = x\cos x \,.\frac{1}{x} + \log x \,.(-x.\sin x + \cos x)$$

$$= \cos x - x\sin x \log x + \cos x \log x$$

$$\frac{du}{dx} = u[\cos x - x\sin x \log x + \cos x \log x]$$

$$= x^{x\cos x}[\cos x - x\sin x \log x + \cos x \log x] \quad ... (2)$$

and , $v = \frac{x^2+1}{x^2-1}$, Taking log on both the sides
$\log v = \log(x^2+1) - \log(x^2-1)$, therefore,

$$\frac{1}{v}\frac{dv}{dx} = \frac{1}{x^2+1}.2x - \frac{1}{x^2-1}.2x = \frac{2x(x^2-1) - 2x(x^2+1)}{(x^2+1)(x^2-1)} = \frac{-4x}{(x^2+1)(x^2-1)}$$

$$\frac{dv}{dx} = v\left[\frac{-4x}{(x^2+1)(x^2-1)}\right] = \frac{x^2+1}{x^2-1}\left[\frac{-4x}{(x^2+1)(x^2-1)}\right] = -\frac{4x}{(x^2-1)^2} \quad \dots(3)$$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get

$$\frac{dy}{dx} = x^{x\cos x}[\cos x - x\sin x \log x + \cos x \log x] - \frac{4x}{(x^2-1)^2}$$

Question 11:

$(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Answer 11:

Let $u = (x\cos x)^x$ and $v = (x\sin x)^{\frac{1}{x}}$, therefore, $y = u + v$
Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

Here, $u = (x\cos x)^x$, Taking log on both the sides
$\log u = x\log(x\cos x)$, therefore,

$$\frac{1}{u}\frac{du}{dx} = x.\frac{d}{dx}\log(x\cos x) + \log(x\cos x).\frac{d}{dx}x$$

$$= x.\frac{1}{(x\cos x)}(-x\sin x + \cos x) + \log(x\cos x).1 = -x\tan x + 1 + \log(x\cos x)$$

$$\frac{du}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)]$$

$$= (x\cos x)^x[1 - x\tan x + \log(x\cos x)] \quad \dots(2)$$

and, $v = (x\sin x)^{\frac{1}{x}}$, Taking log on both the sides
$\log v = \frac{1}{x}\log(x\sin x)$, therefore,

$$\frac{1}{v}\frac{dv}{dx} = \frac{1}{x}.\frac{d}{dx}\log(x\sin x) + \log(x\sin x).\frac{d}{dx}\frac{1}{x}$$

$$= \frac{1}{x}.\frac{1}{x\sin x}(x\cos x + \sin x) + \log(x\sin x)\left(-\frac{1}{x^2}\right)$$

$$\frac{dv}{dx} = v\left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right]$$

$$= (x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right] \quad \dots(3)$$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get

$$\frac{dy}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)] + (x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right]$$

Find $\frac{dy}{dx}$ of the functions given in Exercises 12 to 15:

Question 12:

$x^y + y^x = 1$

Answer 12:

Let $u = x^y$ and $v = y^x$, therefore, $u + v = 1$

Differentiating with respect to x, we get

$$\frac{du}{dx} + \frac{dv}{dx} = 0 \quad \dots (1)$$

Here, $u = x^y$, Taking log on both the sides, $\log u = y \log x$, therefore,

$$\frac{1}{u}\frac{du}{dx} = y.\frac{d}{dx}\log x + \log x.\frac{d}{dx}y = y.\frac{1}{x} + \log x.\frac{dy}{dx}$$

$$\frac{du}{dx} = x^y\left[\frac{y}{x} + \log x.\frac{dy}{dx}\right] \quad \dots (2)$$

and , $v = y^x$, Taking log on both the sides

$\log v = x\log y$, therefore,

$$\frac{1}{v}\frac{dv}{dx} = x.\frac{d}{dx}\log y + \log y.\frac{d}{dx}x = x.\frac{1}{y}\frac{dy}{dx} + \log y.1$$

$$\frac{dv}{dx} = v\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] \quad \dots (3)$$

Putting the value of $\frac{du}{dx}$ from (2) and $\frac{dv}{dx}$ from (3) in equation (1), we get

$$x^y\left[\frac{y}{x} + \log x.\frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = 0$$

$$\Rightarrow yx^{y-1} + x^y \log x\frac{dy}{dx} + xy^{x-1}\frac{dy}{dx} + y^x \log y = 0$$

$$\Rightarrow \frac{dy}{dx}(x^y \log x + xy^{x-1}) = -(y^x \log y + yx^{y-1})$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^x \log y + yx^{y-1}}{x^y \log x + xy^{x-1}}$$

Question 13:

$y^x = x^y$

Answer 13:

$$y^x = x^y$$

Taking log on both the sides, $x \log y = y \log x$, therefore,

$$x.\frac{d}{dx}\log y + \log y.\frac{d}{dx}x = y.\frac{d}{dx}\log x + \log x.\frac{d}{dx}y$$

$$\Rightarrow x.\frac{1}{y}\frac{dy}{dx} + \log y.1 = y.\frac{1}{x} + \log x.\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x}{y} - \log x\right) = \frac{y}{x} - \log y$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{x - y\log x}{y}\right) = \frac{y - x\log y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(y - x\log y)}{x(x - y\log x)}$$

Question 14:

$(\cos x)^y = (\cos y)^x$

Answer 14:

$$(\cos x)^y = (\cos y)^x$$

Taking log on both the sides, $y \cos x = x \cos y$, therefore,

$y.\frac{d}{dx}\cos x + \cos x.\frac{d}{dx}y = x.\frac{d}{dx}\cos y + \cos y.\frac{d}{dx}x$

$\Rightarrow y(-\sin x) + \cos x.\frac{dy}{dx} = x.(-\sin y)\frac{dy}{dx} + \cos y.1$

$\Rightarrow \frac{dy}{dx}(\cos x + x\sin y) = \cos y + y\sin x \qquad \Rightarrow \frac{dy}{dx} = \frac{\cos y + y\sin x}{\cos x + x\sin y}$

Question 15:

$xy = e^{(x-y)}$

Answer 15:

$$xy = e^{(x-y)}$$

Taking log on both the sides,

$\log x + \log y = (x-y)\log e \qquad \Rightarrow \log x + \log y = (x-y)$, therefore,

$\frac{1}{x} + \frac{1}{y}.\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} + 1\right) = 1 - \frac{1}{x} \qquad \Rightarrow \frac{dy}{dx}\left(\frac{1+y}{y}\right) = \frac{x-1}{x} \qquad \Rightarrow \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$

Question 16:

Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

Answer 16:

$f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$

Taking log on both the sides,

$\log f(x) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$, therefore,

$\frac{1}{f(x)}.\frac{d}{dx}f(x) = \frac{1}{1+x} + \frac{1}{1+x^2}.\frac{d}{dx}x^2 + \frac{1}{1+x^4}.\frac{d}{dx}x^4 + \frac{1}{1+x^8}.\frac{d}{dx}x^8$

$\Rightarrow \frac{1}{f(x)}.f'(x) = \frac{1}{1+x} + \frac{1}{1+x^2}.2x + \frac{1}{1+x^4}.4x^3 + \frac{1}{1+x^8}.8x^7$

$\Rightarrow f'(x) = f(x)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$

$\Rightarrow f'(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$

$\Rightarrow f'(1) = (1+1)(1+1)(1+1)(1+1)\left[\frac{1}{1+1} + \frac{2}{1+1} + \frac{4}{1+1} + \frac{8}{1+1}\right]$

$\Rightarrow f'(1) = (2)(2)(2)(2)\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right] = 16\left(\frac{15}{2}\right) = 120$

Question 17:

Differentiate $(x^2-5x+8)(x^3+7x+9)$ in three ways mentioned below:

(i) by using product rule

(ii) by expanding the product to obtain a single polynomial.

(iii) by logarithmic differentiation.

Do they all give the same answer?

Answer 17:

Let $y=(x^2-5x+8)(x^3+7x+9$

(i) Differentiating using product rule

$$\frac{dy}{dx}=(x^2-5x+8)\frac{\mathrm{d}}{\mathrm{dx}}(x^3+7x+9)+(x^3+7x+9)\frac{\mathrm{d}}{\mathrm{dx}}(x^2-5x+8)$$

$$=(x^2-5x+8)(3x^2+7)+(x^3+7x+9)(2x-5)$$

$$=(3x^4+7x^2-15x^3-35x+24x^2+56)+2x^4-5x^3+14x^2-35x+18x-45$$

$$=5x^4-20x^3+45x^2-52x+11$$

(ii) Differentiating by expanding the product to obtain a single polynomial

$$y=(x^2-5x+8)(x^3+7x+9)$$

$$=x^5+7x^3+9x^2-5x^4-35x^2-45x+8x^3+56x+72$$

$$=x^5-5x^4+15x^3-26x^2+11x+72$$

$$\frac{dy}{dx}=\frac{d}{dx}x^5-5\frac{d}{dx}x^4+15\frac{d}{dx}x^3-26\frac{d}{dx}x^2+11\frac{d}{dx}x+\frac{d}{dx}72$$

$$=5x^4-20x^3+45x^2-52x+11$$

(iii) Logarithmic differentiation

$$y=(x^2-5x+8)(x^3+7x+9)$$

Taking log on both sides, $\log y=\log(x^2-5x+8)+\log(x^3+7x+9)$

$$\frac{1}{y}.\frac{dy}{dx}=\frac{1}{(x^2-5x+8)}.\frac{d}{dx}(x^2-5x+8)+\frac{1}{(x^3+7x+9)}.\frac{d}{dx}(x^3+7x+9)$$

$$\frac{1}{y}.\frac{dy}{dx}=\frac{1}{x^2-5x+8}.(2x-5)+\frac{1}{x^3+7x+9}.(3x^2+7)$$

$$\frac{dy}{dx}=y\left[\frac{(2x-5)(x^3+7x+9)+(3x^2+7)(x^2-5x+8)}{(x^2-5x+8)(x^3+7x+9)}\right]$$

$$=\mathrm{y}\left[\frac{2x^4+14x^2+18x-5x^3-35x-45+3x^4-15x^3+24x^2+7x^2-35x+56}{(x^2-5x+8)(x^3+7x+9)}\right]$$

$$\Rightarrow\frac{dy}{dx}=(x^2-5x+8)(x^3+7x+9)\left[\frac{5x^5-20x^3+45x^2-52x+11}{(x^2-5x+8)(x^3+7x+9)}\right]$$

$$\Rightarrow\frac{dy}{dx}==5x^4-20x^3+45x^2-52x+11$$

Hence, all the three answers are same.

Question 18:

If u, v and w are functions of x, then show that

$$\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w + u.\frac{dv}{dx}.w + u.v\frac{dw}{dx}$$

in two ways - first by repeated application of product rule, second by logarithmic differentiation.

Answer 18:

Let $y = u.v.w = u.(v.w)$

Differentiation by repeated application of product rule

$$\frac{dy}{dx} = u.\frac{d}{dx}(v.w) + (v.w).\frac{d}{dx}u$$

$$= u\left[v\frac{d}{dx}w + w\frac{d}{dx}v\right] + v.w.\frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = u.v.\frac{dw}{dx} + u.w.\frac{dv}{dx} + v.w.\frac{du}{dx}$$

Differentiation using logarithmic

Let $y = u.v.w$

Taking log on both the sides, $\log y = \log u + \log v + \log w$

$$\frac{1}{y}.\frac{dy}{dx} = \frac{1}{u}.\frac{du}{dx} + \frac{1}{v}.\frac{dv}{dx} + \frac{1}{w}.\frac{dw}{dx}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{u}.\frac{du}{dx} + \frac{1}{v}.\frac{dv}{dx} + \frac{1}{w}.\frac{dw}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx} = u.v.w\left[\frac{1}{u}.\frac{du}{dx} + \frac{1}{v}.\frac{dv}{dx} + \frac{1}{w}.\frac{dw}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{u.v.w}{u}.\frac{du}{dx} + \frac{u.v.w}{v}.\frac{dv}{dx} + \frac{u.v.w}{w}.\frac{dw}{dx}$$

$$\Rightarrow \frac{dy}{dx} = v.w.\frac{du}{dx} + u.w.\frac{dv}{dx} + u.v.\frac{dw}{dx}$$