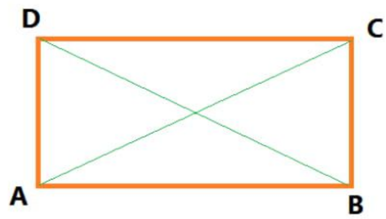
Exercise 8.1

Question 1:
The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Answer 1:
Let the first angle = $3x$
Therefore, the second angle = $5x$,
Third angle = $9x$ and
Fourth angle = $13x$
Sum of all angles of a quadrilateral is 360°. Therefore, $3x + 5x + 9x + 13x = 360°$
$\Rightarrow 30x = 360° \quad \Rightarrow x = \frac{360°}{30} = 12°$
Hence, the first angle = $3 \times 12° = 36°$,
The second angle = $5 \times 12° = 60°$,
Third angle = $9 \times 12° = 108°$
The forth angle = $13 \times 12° = 156°$

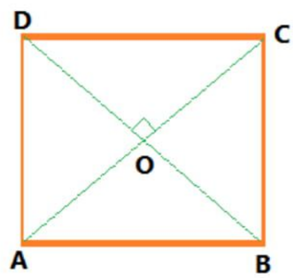
Question 2:
If the diagonals of a parallelogram are equal, then show that it is a rectangle.



Answer 2:
Given: ABCD is a parallelogram with AC = BD.
To Prove: ABCD is a rectangle.
Solution: In ΔABC and ΔBAD,

BC = AD [∵ Opposite sides of a parallelogram are equal]
AC = BD [∵ Given]
AB = AB [∵ Common]
Hence, ΔABC ≅ ΔBAD [∵ SSS Congruency rule]
∠ABC = ∠BAD [∵ CPCT]
But, ∠ABC + ∠BAD = 180° [∵ Co-interior angles]
⇒ 2∠BAD = 180° [∵ ∠ABC = ∠BAD]
$\Rightarrow \angle BAD = \frac{180°}{2} = 90°$
A parallelogram with one of its angle is 90° is a rectangle. Hence, ABCD is a rectangle.

Question 3:
Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.



Answer 3:
Given: ABCD is a quadrilateral in which AO = CO, BO = DO and ∠COD = 90°.
To prove: ABCD is a rhombus.
Solution: In ΔAOB and ΔAOD,

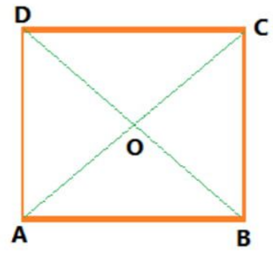
BO = DO [∵ Given]
∠AOB = ∠AOD [∵ Each 90°]
AO = AO [∵ Common]
Hence, ΔAOB ≅ ΔAOD [∵ SAS Congruency rule]
AB = AD [∵ CPCT]
Similarly, AB = BC and BC = CD
Now, all the four sides of quadrilateral ABCD are equal.
Hence, ABCD is a rhombus.

Question 4:

Show that the diagonals of a square are equal and bisect each other at right angles.

Answer 4:

Given: ABCD is a square.
To prove: $AC = BD$, $AO = CO$, $BO = DO$ and $\angle COD = 90°$.
Solution: ΔBAD and ΔABC,



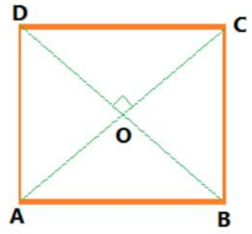
$AD = BC$ [∵ Opposite sides of a square]
$\angle BAD = \angle ABC$ [∵ Each 90°]
$AB = AB$ [∵ Common]
Hence, $\Delta BAD \cong \Delta ABC$ [∵ SAS Congruency rule]
$BD = AC$ [∵ CPCT]
In ΔAOB and ΔCOD,
$\angle OAB = \angle OCD$ [∵ Alternate angles]
$AB = CD$ [∵ Opposite sides of a square]
$\angle OBA = \angle ODC$ [∵ Alternate angles]
Hence, $\Delta BAD \cong \Delta ABC$ [∵ ASA Congruency rule]
$AO = OC$, $BO = OD$ [∵ CPCT]
In ΔAOB and ΔAOD,
$OB = OD$ [∵ Proved above]
$AB = AD$ [∵ Sides of a square]
$OA = OA$ [∵ Common]
Hence, $\Delta BAD \cong \Delta ABC$ [∵ SSS Congruency rule]
$\angle AOB = \angle AOD$ [∵ CPCT]
But, $\angle AOB + \angle AOD = 180°$ [∵ Linear Pair]
$\Rightarrow 2\angle AOB = 180°$ [∵ $\angle AOD = \angle AOB$]
$\Rightarrow \angle AOB = \frac{180°}{2} = 90°$
Hence, the diagonals of a square are equal and bisect each other at right angles.

Question 5:

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer 5:

Given: ABCD is a quadrilateral such that $AC = BD$, $AO = CO$, $BO = DO$ and $\angle COD = 90°$.
To prove: ABCD is a square.
Solution: If the diagonals of a quadrilateral bisects each other at right angle, it is a rhombus.
Hence, $AB = BC = CD = DA$
In ΔBAD and ΔABC,

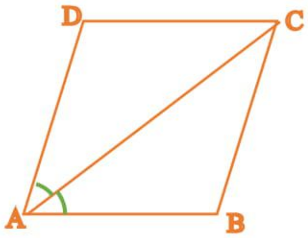


$AD = BC$ [∵ Proved above]
$BD = AC$ [∵ Given]
$AB = AB$ [∵ Common]
Hence, $\Delta BAD \cong \Delta ABC$ [∵ SSS Congruency rule]
$\angle BAD = \angle ABC$ [∵ CPCT]
But, $\angle BAD + \angle ABC = 180°$ [∵ Co-interior angles]
$\Rightarrow 2\angle ABC = 180°$ [∵ $\angle BAD = \angle ABC$]
$\Rightarrow \angle ABC = \frac{180°}{2} = 90°$
Hence, if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Question 6:

Diagonal AC of a parallelogram ABCD bisects ∠A (see Figure). Show that

(i) it bisects ∠C also, **(ii)** ABCD is a rhombus.



Answer 6:

(i) ∠DAC = ∠BAC ... (1) [∵ Given]

∠DAC = ∠BCA ... (2) [∵ Alternate angles]

∠BAC = ∠ACD ... (3) [∵ Alternate angles]

From the equations (1), (2) and (3), we have

∠ACD = ∠BCA ... (4)

Hence, diagonal AC bisects angle C also.

(ii) From the equation (2) and (4), we have

∠ACD = ∠DAC

In ΔADC,

∠ACD = ∠DAC [∵ Proved above]

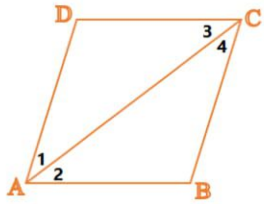
AD = DC [∵ In a triangle, the sides opposite to equal angle are equal]

A parallelogram whose adjacent sides are equal, is a rhombus. Hence, ABCD is a rhombus.

Question 7:

ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Answer 7:



In ΔADC,

AD = DC [∵ ABCD is a rhombus]

∠3 = ∠1 ... (1) [∵ Angles opposite to equal sides are equal]

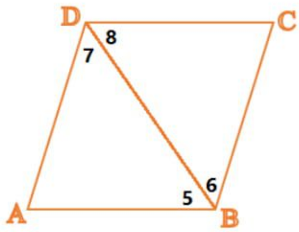
But, ∠3 = ∠2 ... (2) [∵ Alternate angles]

Hence, ∠1 = ∠2 ... (3) [∵ From (1) and (2)]

and ∠1 = ∠4 ... (4) [∵ Alternate angles]

Hence, ∠3 = ∠4 ... (5) [∵ From (1) and (4)]

Hence, from (3) and (5), diagonal AC bisects angle A as well as angle C.



In ΔADB,

AD = AB [∵ ABCD is a rhombus]

∠5 = ∠7 ... (6) [∵ Angles opposite to equal sides are equal]

But, ∠7 = ∠6 ... (7) [∵ Alternate angles]

Hence, ∠5 = ∠6 ... (8) [∵ From (6) and (7)]

and ∠5 = ∠8 ... (9) [∵ Alternate angles]

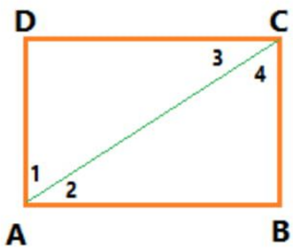
Hence, ∠7 = ∠8 ... (10) [∵ From (6) and (9)]

Hence, from (8) and (10), diagonal BD bisects angle B as well as angle D.

Question 8:

ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:

(i) ABCD is a square **(ii)** diagonal BD bisects ∠B as well as ∠D.



Answer 8:

(i) Given: ABCD is a rectangle ∠1 = ∠2 and ∠3 = ∠4.

To prove: ABCD is a square.

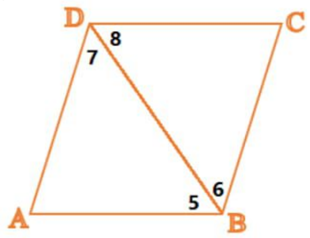
Solution: ∠1 = ∠4 ... (1) [∵ Alternate angles]

∠3 = ∠4 ... (2) [∵ Given]

अतः, ∠1 = ∠3 ... (3) [∵ From (1) and (2)]

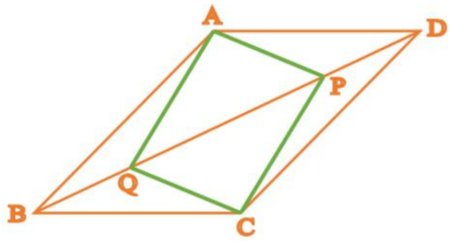
In ΔADC,

∠1 = ∠3 [∵ From (3)]
DC = AD [∵ In a triangle, sides opposite to equal angle are equal]
A rectangle, whose adjacent sides are equal, is a square.
Hence, ABCD is a square.



(ii) To prove: Diagonal BD bisects angle B as well as angle D.
Solution: ∠5 = ∠8 ... (4) [∵ Alternate angles]
In ΔADB,
AB = AD [∵ ABCD is a square]
∠7 = ∠5 ... (5) [∵ Angles opposite to equal sides are equal]
Hence, ∠7 = ∠8 ... (6) [∵ From (4) and (5)]
and ∠7 = ∠6 ... (7) [∵ Alternate angles]
Hence, ∠5 = ∠6 ... (8) [∵ From (5) and (7)]
Hence, from (6) and (8), diagonal BD bisects angle B as well as D.

Question 9:
In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Figure). Show that:
(i) ΔAPD ≅ ΔCQB
(ii) AP = CQ
(iii) ΔAQB ≅ ΔCPD
(iv) AQ = CP
(v) APCQ is a parallelogram



Answer 9:
(i) In ΔAPD and ΔCQB,
DP = BQ [∵ Given]
∠ADP = ∠CBQ [∵ Alternate angles]
AD = BC [∵ Opposite sides of a parallelogram]
Hence, ΔAPD ≅ ΔCQB [∵ SAS Congruency rule]

(ii) ΔAPD ≅ ΔCQB [∵ Proved above]
AP = CQ ... (1) [∵ CPCT]

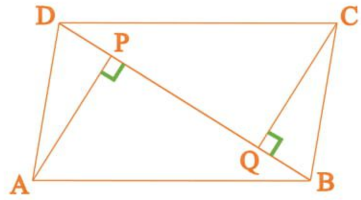
(iii) In ΔAQB and ΔCPD,
QB = DP [∵ Given]
∠ABQ = ∠CDP [∵ Alternate angles]
AB = CD [∵ Opposite sides of a parallelogram]
Hence, ΔAQB ≅ ΔCPD [∵ SAS Congruency rule]

(iv) ΔAQB ≅ ΔCPD [∵ Proved above]
AQ = CP ... (2) [∵ CPCT]

(v) In APCQ,
AP = CQ [∵ From (1)]
AQ = CP [∵ From (2)]
The opposite sides of quadrilateral APCQ are equal.
Hence, APCQ is a parallelogram.

Question 10:

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Figure). Show that: **(i)** ΔAPB ≅ ΔCQD **(ii)** AP = CQ



Answer 10:

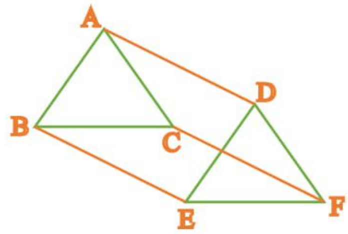
(i) In ΔAPB and ΔCQD,
∠APB = ∠CQD [∵ Each 90°]
∠ABP = ∠CDQ [∵ Alternate angles]
AB = CD [∵ Opposite sides of a parallelogram]
Hence, ΔAPB ≅ ΔCQD [∵ SAS Congruency rule]

(ii) ΔAPB ≅ ΔCQD [∵ Proved above]
AP = CQ [∵ CPCT]

Question 11:

In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Figure). Show that

(i) quadrilateral ABED is a parallelogram **(ii)** quadrilateral BEFC is a parallelogram
(iii) AD || CF and AD = CF **(iv)** quadrilateral ACFD is a parallelogram
(v) AC = DF **(vi)** ΔABC ≅ ΔDEF.



Answer 11:

(i) In ABED, AB = DE [∵ Given]
AB || DE [∵ Given]
Hence, ABED is a parallelogram.

(ii) In BEFC, BC = EF [∵ Given]
BC || EF [∵ Given]
Hence, BEFC is a parallelogram.

(iii) In ABED,
AD = BE ... (1) [∵ ABED is a parallelogram]
AD || BE ... (2) [∵ ABED is a parallelogram]
In BEFC,
BE = CF ... (3) [∵ ABED is a parallelogram]
BE || CF ... (4) [∵ ABED is a parallelogram]
From (2) and (4), we have
AD || CF ... (5)
From (1) and (3), we have
AD = CF ... (6)

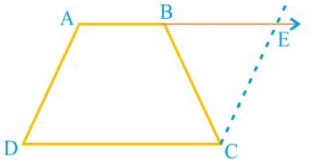
(iv) In ACFD,
AD = CF [∵ From (6)]
AD || CF [∵ From (5)]
Hence, ACFD is a parallelogram.

(v) In ACFD,
AC = DF [∵ ACFD is a parallelogram]

(vi) In ΔABC and ΔDEF,
AB = DE [∵ Given]
AC = DF [∵ Proved above]
BC = EF [∵ Given]
Hence, ΔABC ≅ ΔDEF [∵ SSS Congruency rule]

Question 12:

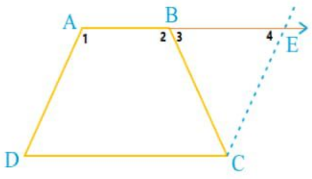
ABCD is a trapezium in which AB || CD and AD = BC (see Figure). Show that



(i) $\angle A = \angle B$
(ii) $\angle C = \angle D$
(iii) $\Delta ABC \cong \Delta BAD$
(iv) diagonal AC = diagonal BD
[***Hint***: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Answer 12:

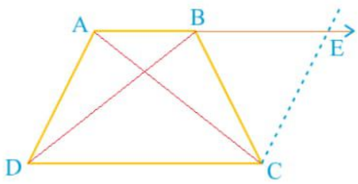
(i) Construction: Produce AB and draw a line through C parallel to AD, which intersects produced AB at E.



In AECD,
AE || DC [∵ Given]
AD || CE [∵ By construction]
Hence, AECD is a parallelogram.
AD = CE ... (1) [∵ Opposite sides of a parallelogram are equal]
AD = BC ... (2) [∵ Given]
Hence, CE = BC [∵ From the equation (1) and (2)]
Therefore, in ΔBCE,
$\angle 3 = \angle 4$ ... (3) [∵ In a triangle, the angles opposite to equal sides are equal]
Here, $\angle 2 + \angle 3 = 180°$ ... (4) [∵ Linear Pair]
$\angle 1 + \angle 4 = 180°$ ... (5) [∵ Co-interior angles]
Therefore, $\angle 2 + \angle 3 = \angle 1 + \angle 4$ [∵ From the equation (4) and (5)]
$\Rightarrow \angle 2 = \angle 1$ $\Rightarrow \angle B = \angle A$ [∵ $\angle 3 = \angle 4$]

(ii) ABCD is a trapezium in which AB || DC, hence,
$\angle 1 + \angle D = 180°$ ... (6) [∵ Co-interior angles]
$\angle 2 + \angle C = 180°$ ... (7) [∵ Co-interior angles]
Therefore, $\angle 1 + \angle D = \angle 2 + \angle C$ [∵ From the equation (6) and (7)]
$\Rightarrow \angle D = \angle C$ [∵ $\angle 2 = \angle 1$]

(iii) In ΔABC and ΔBAD,



BC = AD [∵ Given]
$\angle ABC = \angle BAD$ [∵ Proved above]
AB = AB [∵ Common]
Hence, $\Delta ABC \cong \Delta BAD$ [∵ SAS Congruency rule]

(iv) $\Delta ABC \cong \Delta BAD$ [∵ Proved above]
Diagonal AC = diagonal BD [∵ CPCT]