Exercise 3.3

Question 1:

Solve the following pair of linear equations by the substitution method.

(i) $x+y=14$

$x-y=4$

(ii) $s-t=3$

$\frac{s}{3}+\frac{t}{2}=6$

(iii) $3x-y=3$

$9x-3y=9$

(iv) $0.2x+0.3y=1.3$

$0.4x+0.5y=2.3$

(v) $\sqrt{2}x+\sqrt{3}y=0$

$\sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

Answer 1:

(i) $x + y = 14$ .....(1)

$x - y = 4$ .....(2)

From (1), we obtain $x = 14 - y$ (3)

Substituting this value in equation (2), we obtain

$(14-y)-y=4$

$14-2y=4$

$10=2y$

$y=5$ (4)

Substituting this in equation (3), we obtain

$x=9$

$\therefore x=9, y=5$

(ii) $s-t=3$ (1)

$\frac{s}{3}+\frac{t}{2}=6$ (2)

From (1), we obtain

$s=t+3$ (3)

Substituting this value in equation (2), we obtain

$$\frac{t+3}{3}+\frac{t}{2}=6$$
$$2t+6+3t=36$$
$$5t=30$$
$$t=6 \qquad (4)$$

Substituting in equation (3), we obtain s

$= 9$

$\therefore s = 9, t = 6$

(iii)$3x - y = 3$ (1)

$9x - 3y = 9$ (2) From (1), we obtain $y = 3x - 3$ (3)

Substituting this value in equation (2), we obtain

$$9x-3(3x-3)=9$$
$$9x-9x+9=9$$

$9 = 9$

This is always true.

Hence, the given pair of equations has infinite possible solutions and the relation between these variables can be given by $y = 3x - 3$

Therefore, one of its possible solutions is $x = 1, y = 0$.

(iv) $0.2x+0.3y=1.3 \qquad (1)$
$0.4x+0.5y=2.3 \qquad (2)$

From equation (1), we obtain

$$x=\frac{1.3-0.3y}{0.2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$0.4\left(\frac{1.3-0.3y}{0.2}\right)+0.5y=2.3$$
$$2.6-0.6y+0.5y=2.3$$
$$2.6-2.3=0.1y$$
$$0.3=0.1y$$
$$y=3 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x = \frac{1.3 - 0.3 \times 3}{0.2}$$
$$= \frac{1.3 - 0.9}{0.2} = \frac{0.4}{0.2} = 2$$
$$\therefore x = 2,\ y = 3$$

(v) $\sqrt{2}x + \sqrt{3}y = 0 \qquad (1)$
$\sqrt{3}x - \sqrt{8}y = 0 \qquad (2)$

From equation (1), we obtain

$$x = \frac{-\sqrt{3}y}{\sqrt{2}} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\sqrt{3}\left(-\frac{\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$$
$$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$
$$y\left(-\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$$
$$y = 0 \qquad (4)$$

Substituting this value in equation (3), we obtain

$x = 0$

$\therefore x = 0,\ y = 0$

(vi) $\frac{3}{2}x - \frac{5}{3}y = -2 \qquad (1)$
$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \qquad (2)$

From equation (1), we obtain

$$9x - 10y = -12$$
$$x = \frac{-12 + 10y}{9} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\frac{\frac{-12+10y}{9}}{3}+\frac{y}{2}=\frac{13}{6}$$

$$\frac{-12+10y}{27}+\frac{y}{2}=\frac{13}{6}$$

$$\frac{-24+20y+27y}{54}=\frac{13}{6}$$

$$47y=117+24$$

$$47y=141$$

$$y=3 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x=\frac{-12+10\times 3}{9}=\frac{18}{9}=2$$

Hence, $x = 2$, $y = 3$

Question 2:

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Answer 2:

$$2x+3y=11 \qquad (1)$$

$$2x-4y=-24 \qquad (2)$$

From equation (1), we obtain

$$x=\frac{11-3y}{2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$2\left(\frac{11-3y}{2}\right)-4y=-24$$

$$11-3y-4y=-24$$

$$-7y=-35$$

$$y=5 \qquad (4)$$

Putting this value in equation (3), we obtain

$$x=\frac{11-3\times 5}{2}=-\frac{4}{2}=-2$$

Hence, $x = -2$, $y = 5$

Also,

$y = mx + 3$

$5 = -2m + 3$

$-2m = 2$

$m = -1$

Question 3:

Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km.

(v) A fraction becomes 8/11, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Answer 3:

(i) Let the first number be x and the other number be y such that $y > x$.

According to the given information,

$y = 3x \qquad (1)$

$y - x = 26 \qquad (2)$

On substituting the value of y from equation (1) into equation (2), we obtain

$3x - x = 26$

$x = 13 \qquad (3)$

Substituting this in equation (1), we obtain $y = 39$

Hence, the numbers are 13 and 39.

(ii) Let the larger angle be x and smaller angle be y.

We know that the sum of the measures of angles of a supplementary pair is always 180º.

According to the given information,

$$x+y=180^\circ \quad (1)$$

$$x-y=18^\circ \quad (2)$$

From (1), we obtain $x = 180^\circ - y$ (3)

Substituting this in equation (2), we obtain

$$180^\circ - y - y = 18^\circ$$

$$162^\circ = 2y$$

$$81^\circ = y \quad (4)$$

Putting this in equation (3), we obtain x

= 180º – 81º

= 99º

Hence, the angles are 99º and 81º.

(iii) Let the cost of a bat and a ball be x and y respectively.

According to the given information,

$$7x+6y=3800 \quad (1)$$

$$3x+5y=1750 \quad (2)$$

From (1), we obtain

$$y=\frac{3800-7x}{6} \quad (3)$$

Substituting this value in equation (2), we obtain

$$3x+5\left(\frac{3800-7x}{6}\right)=1750$$

$$3x+\frac{9500}{3}-\frac{35x}{6}=1750$$

$$3x-\frac{35x}{6}=1750-\frac{9500}{3}$$

$$\frac{18x-35x}{6}=\frac{5250-9500}{3}$$
$$-\frac{17x}{6}=\frac{-4250}{3}$$
$$-17x=-8500$$
$$x=500 \qquad (4)$$

Substituting this in equation (3), we obtain

$$y=\frac{3800-7\times 500}{6}$$
$$=\frac{300}{6}=50$$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

(iv)Let the fixed charge be Rs x and per km charge be Rs y.

According to the given information,

$$x+10y=105 \qquad (1)$$
$$x+15y=155 \qquad (2)$$

From (3), we obtain

$$x=105-10y \qquad (3)$$

Substituting this in equation (2), we obtain

$$105-10y+15y=155$$
$$5y=50$$
$$y=10 \qquad (4)$$

Putting this in equation (3), we obtain

$$x=105-10\times 10$$
$$x=5$$

Hence, fixed charge = Rs 5

And per km charge = Rs 10

Charge for 25 km = x + 25y

= 5 + 250 = Rs 255

(v) Let the fraction be x/y.

According to the given information,

$$\frac{x+2}{y+2}=\frac{9}{11}$$
$$11x+22=9y+18$$
$$11x-9y=-4 \qquad (1)$$
$$\frac{x+3}{y+3}=\frac{5}{6}$$
$$6x+18=5y+15$$
$$6x-5y=-3 \qquad (2)$$

From equation (1), we obtain $x=\frac{-4+9y}{11}$ (3)

Substituting this in equation (2), we obtain

$$6\left(\frac{-4+9y}{11}\right)-5y=-3$$
$$-24+54y-55y=-33$$
$$-y=-9$$
$$y=9 \qquad (4)$$

Substituting this in equation (3), we obtain

$$x=\frac{-4+81}{11}=7$$

Hence, the fraction is 7/9.

(vi) Let the age of Jacob be *x* and the age of his son be *y*.

According to the given information,

$$(x+5)=3(y+5)$$
$$x-3y=10 \qquad (1)$$
$$(x-5)=7(y-5)$$
$$x-7y=-30 \qquad (2)$$

From (1), we obtain

$$x=3y+10 \qquad (3)$$

Substituting this value in equation (2), we obtain

$$3y+10-7y=-30$$
$$-4y=-40$$
$$y=10 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x=3\times10+10$$
$$=40$$

Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years.