

Mathematics

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(Chapter – 3) (Linear equations in two variables)

(Class – X)

Exercise 3.5

Question 1:

Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

$$(i) \quad \begin{aligned} x - 3y - 3 &= 0 \\ 3x - 9y - 2 &= 0 \end{aligned}$$

$$(ii) \quad \begin{aligned} 2x + y &= 5 \\ 3x + 2y &= 8 \end{aligned}$$

$$(iii) \quad \begin{aligned} 3x - 5y &= 20 \\ 6x - 10y &= 40 \end{aligned}$$

$$(iv) \quad \begin{aligned} x - 3y - 7 &= 0 \\ 3x - 3y - 15 &= 0 \end{aligned}$$

Answer 1:

$$(i) \quad \begin{aligned} x - 3y - 3 &= 0 \\ 3x - 9y - 2 &= 0 \end{aligned}$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

$$(ii) \quad \begin{aligned} 2x + y &= 5 \\ 3x + 2y &= 8 \end{aligned}$$

$$\frac{a_1}{a_2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-5}{-8}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations. By cross-multiplication method,

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$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{-8 - (-10)} = \frac{y}{-15 + 16} = \frac{1}{4 - 3}$$

$$\frac{x}{2} = \frac{y}{1} = 1$$

$$\frac{x}{2} = 1, \quad \frac{y}{1} = 1$$

$$x = 2, \quad y = 1$$

$$\therefore x = 2, y = 1$$

(iii) $3x - 5y = 20$

$$6x - 10y = 40$$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

(iv) $x - 3y - 7 = 0$

$$3x - 3y - 15 = 0$$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-3}{-3} = 1, \quad \frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

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$$\frac{x}{45-(21)} = \frac{y}{-21-(-15)} = \frac{1}{-3-(-9)}$$

$$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$$

$$\frac{x}{24} = \frac{1}{6} \text{ and } \frac{y}{-6} = \frac{1}{6}$$

$$x = 4 \text{ and } y = -1$$

$$\therefore x = 4, y = -1$$

Question 2:

(i) For which values of a and b will the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$

$$(a-b)x + (a+b)y = 3a+b-2$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$

$$(2k-1)x + (k-1)y = 2k+1$$

Answer 2:

$$(i) \quad 2x + 3y - 7 = 0$$

$$(a-b)x + (a+b)y - (3a+b-2) = 0$$

$$\frac{a_1}{a_2} = \frac{2}{a-b}, \quad \frac{b_1}{b_2} = \frac{3}{a+b}, \quad \frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{(3a+b-2)}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{7}{3a+b-2}$$

$$6a + 2b - 4 = 7a - 7b$$

$$a - 9b = -4 \quad (1)$$

$$\frac{2}{a-b} = \frac{3}{a+b}$$

$$2a + 2b = 3a - 3b$$

$$a - 5b = 0 \quad (2)$$

Subtracting (1) from (2), we obtain

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$$4b = 4$$

$$b = 1$$

Substituting this in equation (2), we obtain

$$a - 5 \times 1 = 0$$

$$a = 5$$

Hence, $a = 5$ and $b = 1$ are the values for which the given equations give infinitely many solutions.

$$(ii) \quad 3x + y - 1 = 0$$

$$(2k-1)x + (k-1)y - 2k - 1 = 0$$

$$\frac{a_1}{a_2} = \frac{3}{2k-1}, \quad \frac{b_1}{b_2} = \frac{1}{k-1}, \quad \frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$$3k - 3 = 2k - 1$$

$$k = 2$$

Hence, for $k = 2$, the given equation has no solution.

Question 3:

Solve the following pair of linear equations by the substitution and cross multiplication methods:

$$8x + 5y = 9$$

$$3x + 2y = 4$$

Answer 3:

$$8x + 5y = 9 \quad (i)$$

$$3x + 2y = 4 \quad (ii)$$

From equation (ii), we obtain

$$x = \frac{4-2y}{3} \quad (iii)$$

Substituting this value in equation (i), we obtain

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$$8\left(\frac{4-2y}{3}\right)+5y=9$$

$$32-16y+15y=27$$

$$-y=-5$$

$$y=5 \quad (iv)$$

Substituting this value in equation (ii), we obtain

$$3x+10=4$$

$$x=-2$$

Hence, $x=-2, y=5$

Again, by cross-multiplication method, we obtain

$$8x+5y-9=0$$

$$3x+2y-4=0$$

$$\frac{x}{-20-(-18)} = \frac{y}{-27-(-32)} = \frac{1}{16-15}$$

$$\frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$$\frac{x}{-2} = 1 \text{ and } \frac{y}{5} = 1$$

$$x = -2 \text{ and } y = 5$$

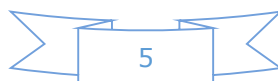
Question 4:

Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i). A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii). A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii). Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?



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(iv). Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v). The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer 4:

(i) Let x be the fixed charge of the food and y be the charge for food per day.

According to the given information,

$$x + 20y = 1000 \quad (1)$$

$$x + 26y = 1180 \quad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$6y = 180$$

$$y = 30$$

Substituting this value in equation (1), we obtain

$$x + 20 \times 30 = 1000$$

$$x = 1000 - 600$$

$$x = 400$$

Hence, fixed charge = Rs 400

And charge per day = Rs 30

(ii) Let the fraction be x/y .

According to the given information,

$$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow \quad 3x - y = 3 \quad (1)$$

$$\frac{x}{y+8} = \frac{1}{4} \quad \Rightarrow \quad 4x - y = 8 \quad (2)$$

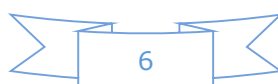
Subtracting equation (1) from equation (2), we obtain

$$x = 5 \quad (3)$$

Putting this value in equation (1), we obtain

$$15 - y = 3$$

$$y = 12$$



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Hence, the fraction is $\frac{5}{12}$.

(iii) Let the number of right answers and wrong answers be x and y respectively.

According to the given information,

$$3x - y = 40 \quad (1)$$

$$4x - 2y = 50$$

$$\Rightarrow 2x - y = 25 \quad (2)$$

Subtracting equation (2) from equation (1), we obtain x
 $= 15$ (3)

Substituting this in equation (2), we obtain

$$30 - y = 25$$

$$y = 5$$

Therefore, number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv) Let the speed of 1st car and 2nd car be u km/h and v km/h.

Respective speed of both cars while they are travelling in same direction = $(u - v)$ km/h

Respective speed of both cars while they are travelling in opposite directions i.e.,
travelling towards each other = $(u + v)$ km/h

According to the given information,

$$5(u - v) = 100$$

$$\Rightarrow u - v = 20 \quad \dots(1)$$

$$1(u + v) = 100 \quad \dots(2)$$

Adding both the equations, we obtain

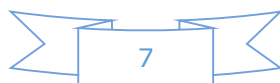
$$2u = 120$$

$$u = 60 \text{ km/h} \quad (3)$$

Substituting this value in equation (2), we obtain v

$$= 40 \text{ km/h}$$

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h



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(v) Let length and breadth of rectangle be x unit and y unit respectively.

Area = xy

According to the question,

$$(x-5)(y+3) = xy-9$$

$$\Rightarrow 3x-5y-6=0 \quad (1)$$

$$(x+3)(y+2) = xy+67$$

$$\Rightarrow 2x+3y-61=0 \quad (2)$$

By cross-multiplication method, we obtain

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$x=17, y=9$$

Hence, the length and breadth of the rectangle are 17 units and 9 units respectively.

