Mathematics

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(Chapter – 6) (Triangles)

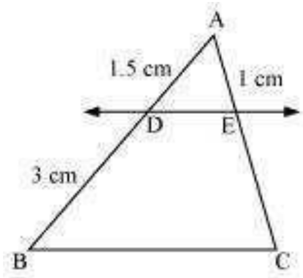
(Class – X)

Exercise 6.2

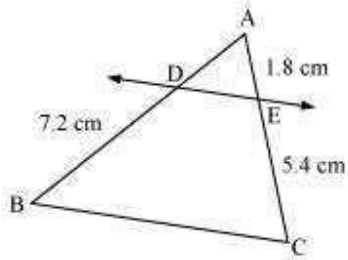
Question 1:

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i)

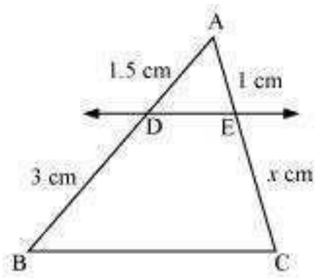


(ii)



Answer 1:

(i)



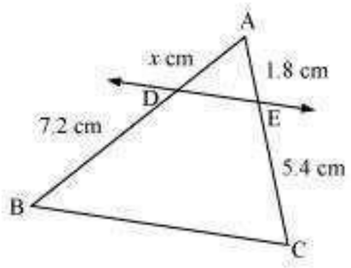
Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB}=\frac{AE}{EC}$$
$$\frac{1.5}{3}=\frac{1}{x}$$
$$x=\frac{3\times 1}{1.5}$$
$$x=2$$
$$\therefore EC=2 \text{ cm}$$

(ii)



Let AD = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB}=\frac{AE}{EC}$$
$$\frac{x}{7.2}=\frac{1.8}{5.4}$$
$$x=\frac{1.8\times 7.2}{5.4}$$
$$x=2.4$$
$$\therefore AD=2.4 \text{ cm}$$

Question 2:

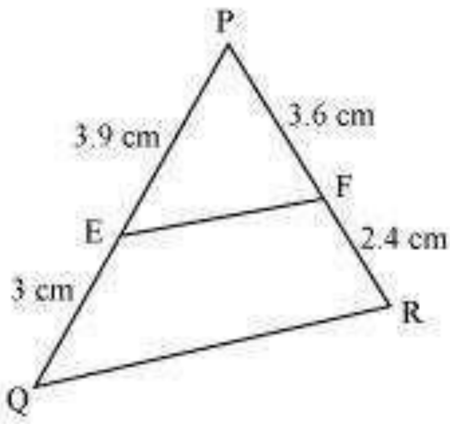
E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm (iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer 2:

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

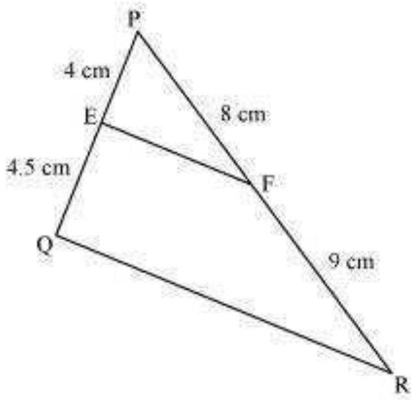
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

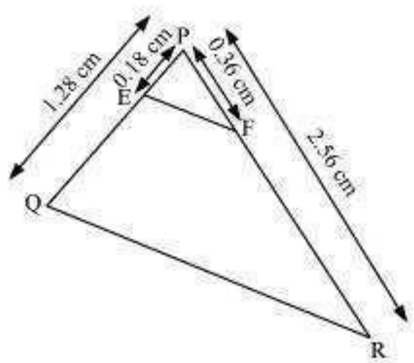
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

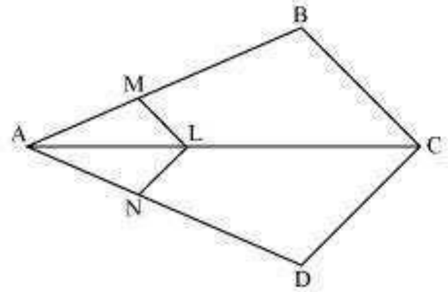
$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

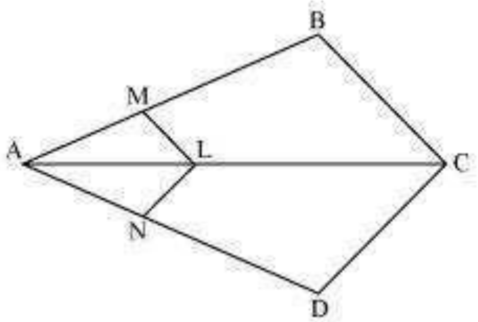
Therefore, EF is parallel to QR.

Question 3:

In the following figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Answer 3:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$\frac{AM}{AB} = \frac{AL}{AC}$ (i)

Similarly, LN || CD

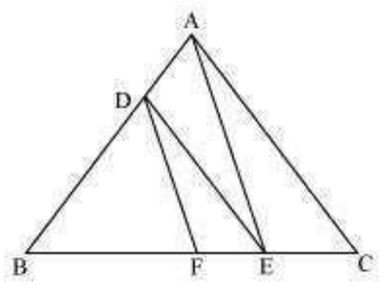
$\therefore \frac{AN}{AD} = \frac{AL}{AC}$ (ii)

From (i) and (ii), we obtain

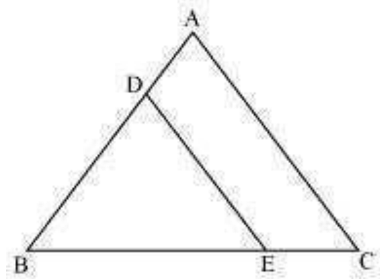
$\frac{AM}{AB} = \frac{AN}{AD}$

Question 4:

In the following figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

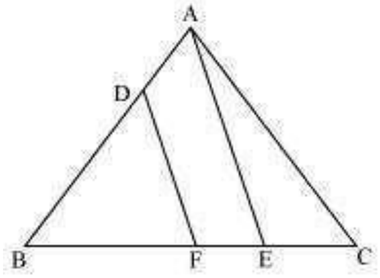


Answer 4:



In ΔABC, DE || AC

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ (Basic Proportionality Theorem) (i)



In ΔBAE, $DF \parallel AE$

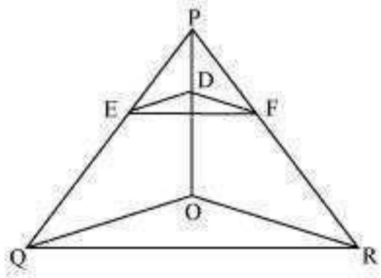
$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ (Basic Proportionality Theorem) (ii)

From (i) and (ii), we obtain

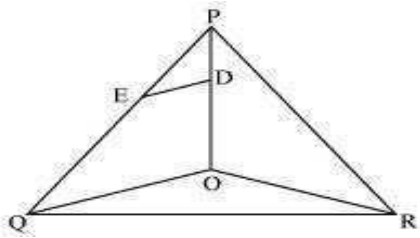
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Question 5:

In the following figure, DE || OQ and DF || OR, show that EF || QR.

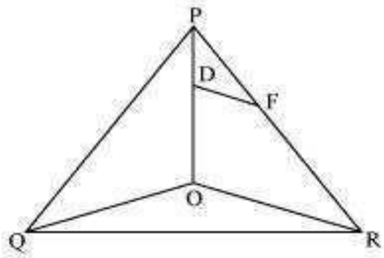


Answer 5:



In Δ POQ, DE || OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ (Basic proportionality theorem) (i)



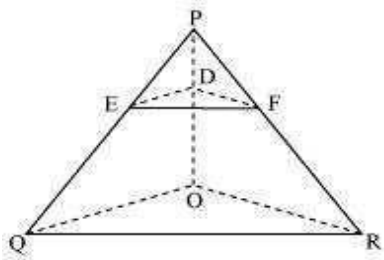
In ΔPOR, DF || OR

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we obtain

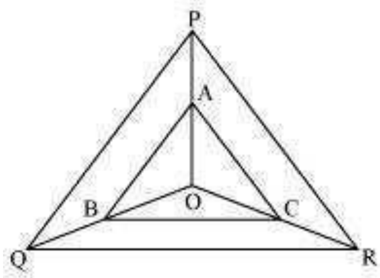
$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore$ EF || QR (Converse of basic proportionality theorem)

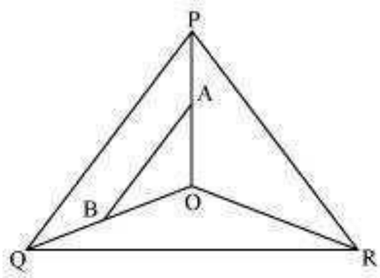


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

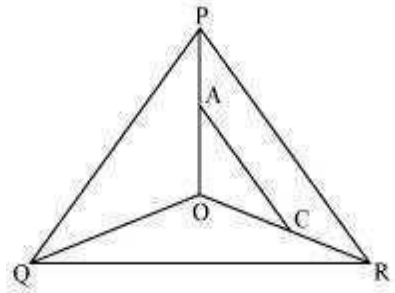


Answer 6:



In Δ POQ, AB || PQ

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



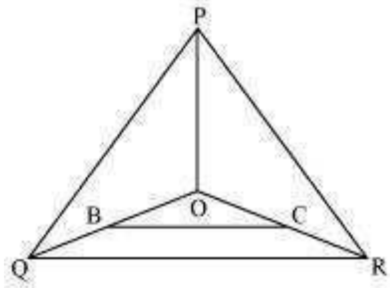
In ΔPOR, $AC \parallel PR$

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ (By basic proportionality theorem) (ii)

From (i) and (ii), we obtain

$\frac{OB}{BQ} = \frac{OC}{CR}$

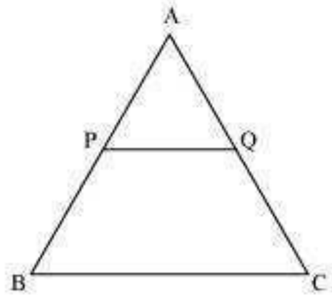
$\therefore BC \parallel QR$ (By the converse of basic proportionality theorem)



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer 7:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad (\text{P is the mid-point of AB. } \therefore \ AP = PB)$$

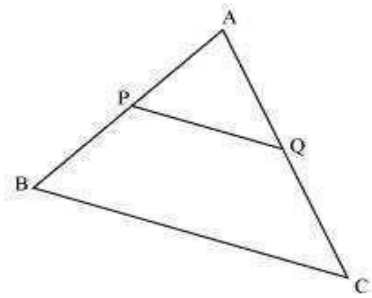
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer 8:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.
i.e., AP = PB and AQ = QC It
can be observed that

$$\frac{AP}{PB}=\frac{1}{1}$$

and $\frac{AQ}{QC}=\frac{1}{1}$

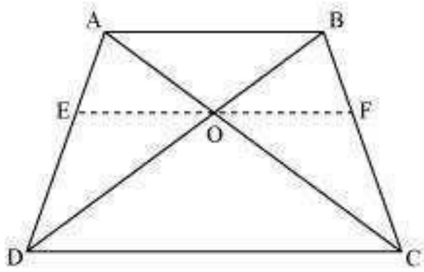
$$\therefore \frac{AP}{PB}=\frac{AQ}{QC}$$

Hence, by using basic proportionality theorem, we obtain
$PQ \| BC$

Question 9:

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO}=\frac{CO}{DO}$.

Answer 9:



Draw a line EF through point O, such that $EF \| CD$
In ΔADC, $EO \| CD$
By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$$

In ΔABD, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

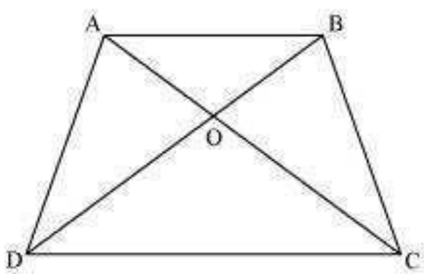
Question 10:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that

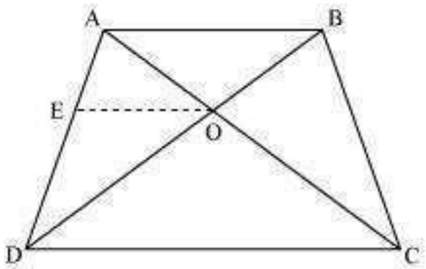
$\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer 10:

Let us consider the following figure for the given question.



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

∴ ABCD is a trapezium.