

Mathematics

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(Chapter – 8) (Introduction to Trigonometry)

(Class X)

Exercise 8.3

Question 1:

Evaluate

(I) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(III) $\cos 48^\circ - \sin 42^\circ$

(IV) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

Answer 1:

(I) $\frac{\sin 18^\circ}{\cos 72^\circ} = \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ} = \frac{\cos 72^\circ}{\cos 72^\circ} = 1$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ} = \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ} = \frac{\cot 64^\circ}{\cot 64^\circ} = 1$

(III) $\cos 48^\circ - \sin 42^\circ = \cos(90^\circ - 42^\circ) - \sin 42^\circ$

$= \sin 42^\circ - \sin 42^\circ$

$= 0$

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ$

$= \sec 59^\circ - \sec 59^\circ$

$= 0$

Question 2:

Show that

(I) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(II) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Answer 2:

(I) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

$= \tan(90^\circ - 42^\circ) \tan(90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ$

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$$\begin{aligned} &= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ \\ &= (\cot 42^\circ \tan 42^\circ) (\cot 67^\circ \tan 67^\circ) \\ &= (1) (1) \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{(II)} \quad &\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ \\ &= \cos (90^\circ - 52^\circ) \cos (90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ \\ &= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ \\ &= 0 \end{aligned}$$

Question 3:

If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle, find the value of A .

Answer 3:

$$\begin{aligned} \text{Given that, } &\tan 2A = \cot (A - 18^\circ) \\ \cot (90^\circ - 2A) &= \cot (A - 18^\circ) \\ 90^\circ - 2A &= A - 18^\circ \\ 108^\circ &= 3A \\ A &= 36^\circ \end{aligned}$$

Question 4:

If $\tan A = \cot B$, prove that $A + B = 90^\circ$

Answer 4:

$$\begin{aligned} \text{Given that, } &\tan A = \cot B \\ \tan A &= \tan (90^\circ - B) \\ A &= 90^\circ - B \\ A + B &= 90^\circ \end{aligned}$$

Question 5:

If $\sec 4A = \operatorname{cosec} (A - 20^\circ)$, where $4A$ is an acute angle, find the value of A .

Answer 5:

$$\begin{aligned} \text{Given that, } &\sec 4A = \operatorname{cosec} (A - 20^\circ) \\ \operatorname{cosec} (90^\circ - 4A) &= \operatorname{cosec} (A - 20^\circ) \\ 90^\circ - 4A &= A - 20^\circ \\ 110^\circ &= 5A \\ A &= 22^\circ \end{aligned}$$

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Question 6:

If A, B and C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Answer 6:

We know that for a triangle ABC,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A$$

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\begin{aligned}\sin\left(\frac{B+C}{2}\right) &= \sin\left(90^\circ - \frac{A}{2}\right) \\ &= \cos\left(\frac{A}{2}\right)\end{aligned}$$

Question 7:

Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

Answer 7:

$$\sin 67^\circ + \cos 75^\circ$$

$$= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$$

$$= \cos 23^\circ + \sin 15^\circ$$