

Mathematics

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(Chapter - 1) (Real Numbers)

(Class X)

Exercise 1.1

Question 1:

Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 255

Answer 1:

(i) 135 and 225

Since $225 > 135$, we apply the division lemma to 225 and 135 to obtain

$$225 = 135 \times 1 + 90$$

Since remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to obtain

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain $90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45, therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Since $38220 > 196$, we apply the division lemma to 38220 and 196 to obtain

$$38220 = 196 \times 195 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196, therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Since $867 > 255$, we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$$255 = 102 \times 2 + 51$$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$$102 = 51 \times 2 + 0$$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51, Therefore, HCF of 867 and 255 is 51.

Question 2:

Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Answer 2:

Let a be any positive integer and $b = 6$.

Then, by Euclid's algorithm, $a = 6q + r$ for some integer $q \geq 0$ and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$ Also,

$$6q + 1 = 2 \times 3q + 1 = 2k_1 + 1, \text{ where } k_1 \text{ is a positive integer}$$

$$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1, \text{ where } k_2 \text{ is an integer}$$

$$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1, \text{ where } k_3 \text{ is an integer}$$

Clearly, $6q + 1, 6q + 3, 6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1, 6q + 3, 6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers and therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$.

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Question 3:

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer 3:

HCF (616, 32) will give the maximum number of columns in which they can march. We can use Euclid's algorithm to find the HCF.

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

The HCF (616, 32) is 8. Therefore, they can march in 8 columns each.

Question 4:

Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m .

[Hint: Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Answer 4:

Let a be any positive integer and $b = 3$. Using Euclid's Division Lemma, $a = 3q + r$ for some integer $q \geq 0$ where $r = 0, 1, 2$ because $0 \leq r < 3$.

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

$$\begin{aligned} a^2 &= (3q)^2 \text{ or } (3q + 1)^2 \text{ or } (3q + 2)^2 \\ &= (3q)^2 \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4 \\ &= 3 \times (3q^2) \text{ or } 3 \times (3q^2 + 2q) + 1 \text{ or } 3 \times (3q^2 + 4q + 1) + 1 \\ &= 3k_1 \text{ or } 3k_2 + 1 \text{ or } 3k_3 + 1 \end{aligned}$$

Where k_1, k_2 and k_3 are some positive integers.

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Question 5:

Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Answer 5:

Let a be any positive integer and $b = 3$, using Euclid's Division Lemma, $a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$. Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$.

Therefore, every number can be represented as these three forms.

There are three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$$a^3 = (3q + 1)^3$$

$$a^3 = 27q^3 + 27q^2 + 9q + 1$$

$$a^3 = 9(3q^3 + 3q^2 + q) + 1 = 9m + 1$$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

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Case 3: When $a = 3q + 2$,

$$a^3 = (3q + 2)^3$$

$$a^3 = 27q^3 + 54q^2 + 36q + 8$$

$$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$$

$$a^3 = 9m + 8$$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

