Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i). $x^2 - 2x - 8$ (ii). $4s^2 - 4s + 1$ (iii). $6x^2 - 3 - 7x$

(iv). $4u^2 + 8u$ (v). $t^2 - 15$ (vi). $3x^2 - x - 4$

Answer 1:

(i) $x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x-4) + 2(x-4)$

$= (x+2)(x-4)$

The value of $x^2 - 2x - 8$ is zero if $x + 2 = 0$ or $x - 4 = 0$

$\Rightarrow x = -2$ or $x = 4$

Therefore, the zeroes of $x^2 - 2x - 8$ are -2 and 4. Now

Sum of zeroes $= -2 + 4 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Cofficient of } x)}{\text{Cofficient of } x^2}$

Product of zeroes $= (-2) \times 4 = -8 = \frac{-8}{1} = \frac{\text{Constant term}}{\text{Cofficient of } x^2}$

(ii) $4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1$

$= 2s(2s-1) - 1(2s-1)$

$= (2s-1)(2s-1)$

The value of $4s^2 - 4s + 1$ is zero if $2s - 1 = 0 \Rightarrow s = \frac{1}{2}$.

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$. Now

Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(4)}{4} = \frac{-(\text{Cofficient of } s)}{\text{Cofficient of } s^2}$

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Cofficient of } s^2}$

(iii) $6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x-3) + 1(2x-3)$

$= (3x+1)(2x-3)$

The value of $6x^2 - 7x - 3$ is zero if $3x + 1 = 0$ or $2x - 3 = 0$.

$\Rightarrow x = -\frac{1}{3}$ or $x = \frac{3}{2}$.

Therefore, the zeroes of $6x^2 - 7x - 3$ are $-\frac{1}{3}$ and $\frac{3}{2}$. Now

Sum of zeroes $= -\frac{1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Cofficient of } x)}{\text{Cofficient of } x^2}$

Product of zeroes $= \left(-\frac{1}{3}\right) \times \frac{3}{2} = -\frac{1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Cofficient of } x^2}$

(iv) $4u^2 + 8u$
$= 4u^2 + 8u$
$= 4u(u + 2)$
The value of $4u^2 + 8u$ is zero if $4u = 0$ or $u + 2 = 0$.
$\Rightarrow u = 0$ or $u = -2$.
Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2. Now

$$\text{Sum of zeroes} = 0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Cofficient of } u)}{\text{Cofficient of } u^2}$$

$$\text{Product of zeroes} = 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Cofficient of } u^2}$$

(v) $t^2 - 15$
$= t^2 - \left(\sqrt{15}\right)^2$
$= \left(t + \sqrt{15}\right)\left(t - \sqrt{15}\right)$
The value of $t^2 - 15$ is zero if $t + \sqrt{15} = 0$ or $t - \sqrt{15} = 0$.
$\Rightarrow x = -\sqrt{15}$ or $x = \sqrt{15}$.
Therefore, the zeroes of $t^2 - 15$ are $-\sqrt{15}$ and $\sqrt{15}$. Now

$$\text{Sum of zeroes} = -\sqrt{15} + \sqrt{15} = 0 = \frac{-(0)}{1} = \frac{-(\text{Cofficient of } t)}{\text{Cofficient of } t^2}$$

$$\text{Product of zeroes} = \left(-\sqrt{15}\right) \times \sqrt{15} = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Cofficient of } t^2}$$

(vi) $3x^2 - x - 4$
$= 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4)$
$= (3x - 4)(x + 1)$
The value of $3x^2 - x - 4$ is zero if $3x - 4 = 0$ or $x + 1 = 0$.
$\Rightarrow x = \frac{4}{3}$ or $x = -1$.
Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1. Now

$$\text{Sum of zeroes} = \frac{4}{3} + (-1) = \frac{4-3}{3} = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(\text{Cofficient of } x)}{\text{Cofficient of } x^2}$$

$$\text{Product of zeroes} = \frac{4}{3} \times (-1) = -\frac{4}{3} = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Cofficient of } x^2}$$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i). $\frac{1}{4}, -1$ **(ii).** $\sqrt{2}, \frac{1}{3}$ **(iii).** $0, \sqrt{5}$
(iv). $1, 1$ **(v).** $-\frac{1}{4}, \frac{1}{4}$ **(vi).** $4, 1$

Answer 2

(i) Let α and β are the zeroes of the polynomial $ax^2 + bx + c$, then we have

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

On comparing,

a = 4, b = – 1 and c = – 4

Hence, the required quadratic polynomial is $4x^2 - x - 4$.

(ii) Let α and β are the zeroes of the polynomial $ax^2 + bx + c$, then we have

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

On comparing,

$a = 3,\ b = -3\sqrt{2}$ and $c = 1$

Hence, the required quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) Let α and β are the zeroes of the polynomial $ax^2 + bx + c$, then we have

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha\beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

On comparing,

a = 1, b = 0 and c = $\sqrt{5}$

Hence, the required quadratic polynomial is $x^2 + 0.x + \sqrt{5}$.

(iv) Let α and β are the zeroes of the polynomial $ax^2 + bx + c$, then we have

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$

$$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

On comparing,

a = 1, b = – 1 and c = 1

Hence, the required quadratic polynomial is $x^2 - x + 1$.

(v) Let α and β are the zeroes of the polynomial $ax^2 + bx + c$, then we have

$$\alpha + \beta = \frac{-1}{4} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{4} = \frac{c}{a}$$

On comparing,

a = 4, b = 1 and c = 1

Hence, the required quadratic polynomial is $4x^2 + x + 1$.

(vi) Let α and β are the zeroes of the polynomial $ax^2 + bx + c$, then we have

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$

$$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

On comparing,

a = 1, b = – 4 and c = 1

Hence, the required quadratic polynomial is $x^2 - 4x + 1$.