Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

Answer 1:

(i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

$$
\begin{array}{r|l}
 & x-3 \\
x^2-2 & x^3-3x^2+5x-3 \\
 & x^3 \qquad -2x \\
 & - \qquad\ \ + \\ \hline
 & -3x^2+7x-3 \\
 & -3x^2 \qquad +6 \\
 & + \qquad\ \ - \\ \hline
 & 7x-9
\end{array}
$$

Therefore,

Quotient = $x - 2$

And remainder = $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ g(x) = x^2 + 1 - x$

$$
\begin{array}{r|l}
 & x^2+x-3 \\
x^2-x+1 & x^4+0.x^3-3x^2+4x+5 \\
 & x^4-\ x^3+x^2 \\
 & -\ \ +\ \ - \\ \hline
 & x^3-4x^2+4x+5 \\
 & x^3-\ x^2+\ x \\
 & -\ \ +\ \ - \\ \hline
 & -3x^2+3x+5 \\
 & -3x^2+3x-3 \\
 & +\ \ -\ \ + \\ \hline
 & 8
\end{array}
$$

Therefore,

Quotient = $x^2 + x - 3$

And remainder = 8

(iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

$$
\begin{array}{r|l}
 & -x^2-2 \\
-x^2+2 & x^4+0.x^2-5x+6 \\
 & x^4-2x^2 \\
 & -\ \ + \\ \hline
 & 2x^2-5x+6 \\
 & 2x^2 \qquad -4 \\
 & - \qquad\ \ + \\ \hline
 & -5x+10
\end{array}
$$

Therefore,

Quotient = $-x^2 - 2$

And remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

Answer 2:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 + 0.t - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 + 0.t^3 - 6t^2 \\
 & -\quad -\quad + \\
\hline
 & 3t^3 + 4t^2 - 9t - 12 \\
 & 3t^3 + 0.t^2 - 9t \\
 & -\quad -\quad + \\
\hline
 & 4t^2 + 0.t - 12 \\
 & 4t^2 + 0.t - 12 \\
 & -\quad -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since, remainder is 0, therefore the polynomial $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
\hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & -\quad -\quad - \\
\hline
 & -4x^3 - 10x^2 + 2x + 2 \\
 & -4x^3 - 12x^2 - 4x \\
 & +\quad +\quad + \\
\hline
 & 2x^2 + 6x + 2 \\
 & 2x^2 + 6x + 2 \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since, remainder is 0, therefore the polynomial $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & -\quad +\quad - \\
\hline
 & -x^3 \qquad + 3x + 1 \\
 & -x^3 \qquad + 3x - 1 \\
 & + \qquad\ -\quad + \\
\hline
 & 2 \\
\hline
\end{array}
$$

Since, remainder is not 0, therefore the polynomial $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Question 3:

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer 3:

Let $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

So, $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are zeroes of $p(x)$.

Therefore, $\left(x - \sqrt{\frac{5}{3}}\right)$ and $\left(x + \sqrt{\frac{5}{3}}\right)$ are the factors of $p(x)$.

or $x^2 - \frac{5}{3}$ is a factor of $p(x)$.

$$\begin{array}{r|l} & 3x^2 + 6x + 3 \\ x^2 + 0.x - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & 3x^4 + 0x^3 - 5x^2 \\ & - \quad - \quad + \\ \hline & 6x^3 + 3x^2 - 10x - 5 \\ & 6x^3 + 0x^2 - 10x \\ & - \quad - \quad + \\ \hline & 3x^2 + 0x - 5 \\ & 3x^2 + 0x - 5 \\ & - \quad - \quad + \\ \hline & 0 \end{array}$$

$$3x^4 + 6x^3 - 2x^2 - 10x - 5 = \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3)$$
$$= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$$

Now,

$p(x) = 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1)$

$= 3\left(x^2 - \frac{5}{3}\right)[x^2 + x + x + 1]$

$= 3\left(x^2 - \frac{5}{3}\right)[x(x + 1) + 1(x + 1)]$

$= 3\left(x^2 - \frac{5}{3}\right)(x + 1)(x + 1)$

Hence, the zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$ are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1.

Question 4:
On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Answer 4:

Given that:

Divisor $= g(x)$

Quotient $= x - 2$

Dividend $= x^3 - 3x^2 + x + 2$

Remainder $= -2x + 4$

We know that:

Dividend = Divisor × Quotient + Remainder

Therefore,

$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) - (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 - (-2x + 4) = g(x) \times (x - 2)$

$$\Rightarrow \frac{x^3 - 3x^2 + x + 2 - (-2x + 4)}{(x - 2)} = g(x)$$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \end{array}$$

Therefore, $g(x) = x^2 - x + 1$

Question 5:
Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$ **(ii)** deg $q(x)$ = deg $r(x)$ **(iii)** deg $r(x)$ = 0

Answer 5:

According to Euclid's division lemma, $(x) = g(x) \times q(x) + r(x)$, where $q(x) \neq 0$, degree of $r(x) = 0$ or degree of $r(x)$ < degree $g(x)$.

(i) degree of $p(x)$ = degree of $q(x)$

The degree of dividend and quotient can be equal, if the divisor is a constant (degree 0) term. Therefore

Let $p(x) = 3x^2 - 6x + 5$

Let $g(x) = 3$

Therefore, $q(x) = x^2 - 2x + 1$ and $r(x) = 2$

(ii) degree of $q(x)$ = degree of $r(x)$

Let $p(x) = 2x^2 - 4x + 3$

Let $g(x) = x^2 - 2x + 1$

Therefore, $q(x) = 2$ and $r(x) = 1$

(iii) degree of $r(x) = 0$

Let $p(x) = 2x^2 - 4x + 3$

Let $g(x) = x^2 - 2x + 1$

Therefore, $q(x) = 2$ and $r(x) = 1$

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