# Exercise 2.4

## Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

**(i).** $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$ **(ii).** $x^3 - 4x^2 + 5x - 2;\quad 2, 1, 1$

## Answer 1:

**(i)** $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$

Let $p(x) = 2x^3 + x^2 - 5x + 2$

Therefore , $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$

$= 2 \times \frac{1}{8} + \frac{1}{4} - 5 \times \frac{1}{2} + 2$

$= \frac{2}{4} - \frac{5}{2} + 2$

$= \frac{5}{2} - \frac{5}{2} = 0$

So, $\frac{1}{2}$ is one of the zeroes of $p(x)$.

Now, $p(1) = 2(1)^3 + (1)^2 - 5(1) + 2$

$= 2 \times 1 + 1 - 5 \times 1 + 2$

$= 3 - 5 + 2$

$= 5 - 5 = 0$

So, 1 is also the zero of $p(x)$.

Now, $p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

$= 2 \times (-8) + 4 + 10 + 2$

$= -16 + 4 + 10 + 2$

$= -16 + 16 = 0$

So, $-2$ is also the zero of $p(x)$.

Therefore, $\frac{1}{2}$, 1 and $-2$ are the zeroes of $p(x)$.

Now, let $\alpha = \frac{1}{2}, \beta = 1$ and $\gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = \frac{3-4}{2} = \frac{-1}{2} = \frac{-(1)}{2} = \frac{-(\text{Cofficient of } x^2)}{\text{Cofficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2} = \frac{-5}{2} = \frac{-5}{2} = \frac{\text{Cofficient of } x}{\text{Cofficient of } x^3}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = -1 = \frac{-2}{2} = \frac{-(\text{Constant term})}{\text{Cofficient of } x^3}$$

Hence, the relation between zeroes and coefficients is verified.

**(ii)** $x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$

Let, $p(x) = x^3 - 4x^2 + 5x - 2$

Therefore , $p(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

$= 8 - 16 + 10 - 2$

$= 18 - 18 = 0$

So, 2 is one of the zeroes of $p(x)$.

Now, $p(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2$

$= 6 - 6 = 0$

So, 1 is also the zero of $p(x)$.

Therefore, 2, 1 and 1 are the zeroes of $p(x)$.

Now, let $\alpha = 2, \beta = 1$ and $\gamma = 1$

$$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = \frac{-(-1)}{1} = \frac{-(\text{Cofficient of } x^2)}{\text{Cofficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 2 \times 1 + 1 \times 1 + 1 \times 2 = 2 + 1 + 2 = 5 = \frac{5}{1} = \frac{\text{Cofficient of } x}{\text{Cofficient of } x^3}$$

$$\alpha\beta\gamma = 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Constant term})}{\text{Cofficient of } x^3}$$

Hence, the relation between zeroes and coefficients is verified.

## Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

## Answer 2:

Let, $p(x) = ax^3 + bx^2 + cx + d$ be a cubic polynomial whose zeroes are $\alpha, \beta$ and $\gamma$.
Given that:
$\alpha + \beta + \gamma = 2$
$\alpha\beta + \beta\gamma + \gamma\alpha = -7$
$\alpha\beta\gamma = -14$

We know that,

$$\alpha + \beta + \gamma = \frac{-(\text{Cofficient of } x^2)}{\text{Cofficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Cofficient of } x}{\text{Cofficient of } x^3}$$

$$\alpha\beta\gamma = \frac{-(\text{Constant term})}{\text{Cofficient of } x^3}$$

Therefore,

$$\alpha + \beta + \gamma = \frac{-(\text{Cofficient of } x^2)}{\text{Cofficient of } x^3} = \frac{-b}{a} = \frac{2}{1}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Cofficient of } x}{\text{Cofficient of } x^3} = \frac{c}{a} = \frac{-7}{1}$$

$$\alpha\beta\gamma = \frac{-(\text{Constant term})}{\text{Cofficient of } x^3} = \frac{-d}{a} = \frac{-14}{1}$$

On comparing, $a = 1, b = -2, c = -7$ and $d = 14$

Hence, the required cubic polynomial is $p(x) = x^3 - 2x^2 - 7x + 14$.

## Question 3:

If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, $a$, $a + b$, find $a$ and $b$.

## Answer 3:

We know that,

$$\text{Sum of zeroes} = \frac{-(\text{Cofficient of } x^2)}{\text{Cofficient of } x^3}$$

Therefore,

$$(\mathrm{a} - \mathrm{b}) + \mathrm{a} + (\mathrm{a} + \mathrm{b}) = \frac{-(-3)}{1}$$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

$$\text{Product of zeroes} = \frac{-(\text{Constant term})}{\text{Cofficient of } x^3}$$

Therefore,

$$(\mathrm{a} - \mathrm{b})(\mathrm{a})(\mathrm{a} + \mathrm{b}) = \frac{-(1)}{1}$$

$\Rightarrow (1 - \mathrm{b})1(1 + \mathrm{b}) = -1$ [Because $a = 1$]

$\Rightarrow 1 - b^2 = -1$

$\Rightarrow b^2 = 2$

$\Rightarrow b = \pm\sqrt{2}$

Hence, $a = 1$ and $b = \pm\sqrt{2}$

## Question 4:

If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 13x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

## Answer 4:

Let $p(x) = x^4 - 6x^3 - 26x^2 + 13x - 35$

Given that: $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are two of the zeroes of $p(x)$.

So, $(x - 2 - \sqrt{3})$ and $(x - 2 + \sqrt{3})$ are the factors of $p(x)$.

Therefore, $(x - 2)^2 - (\sqrt{3})^2$ is a factor of $p(x)$.

$\Rightarrow x^2 - 4x + 1$ is a factor of $p(x)$. [As $(x - 2)^2 - (\sqrt{3})^2 = x^2 - 4x + 1$]

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Therefore,

$p(x) = x^4 - 6x^3 - 26$:

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

$= (x^2 - 4x + 1)(x^2 - 7x + 5x - 35)$

$= (x^2 - 4x + 1)[x(x - 7) + 5(x - 7)]$

$= (x^2 - 4x + 1)(x + 5)(x - 7)$

To get the other zeroes, putting $x + 5 = 0$ and $x - 7 = 0$, we get

$x = -5$ and $x = 7$

Hence, the other two zeroes of $p(x)$ are $x = -5$ and $x = 7$.

## Question 5:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find $k$ and $a$.

## Answer 5:

Given that:

Divisor $= x^2 - 2x + k$

Dividend $= x^4 - 6x^3 + 16x^2 - 25x + 10$

Remainder $= x + a$

We know that:

Dividend = Divisor × Quotient + Remainder

Therefore,

$x^4 - 6x^3 + 16x^2 - 25x + 10 = (x^2 - 2x + k) \times \text{Quotient} + (x + a)$

$x^4 - 6x^3 + 16x^2 - 25x + 10 - (x + a) = (x^2 - 2x + k) \times \text{Quotient}$

$$\Rightarrow \frac{x^4 - 6x^3 + 16x^2 - 26x + 10 - a}{x^2 - 2x + k} = \text{Quotient}$$

So, if the polynomial $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ is divided by $x^2 - 2x + k$, the remainder will be zero.

$$
\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 26x \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (26-4k)x + 10 - a \\
 & (8-k)x^2 - (16-2k)x + (8k - k^2) \\
 & - \quad + \quad - \\
\hline
 & (-10+2k)x + (10 - a - 8k + k^2)
\end{array}
$$

On comparing,

$-10 + 2k = 0$

$\Rightarrow 2k = 10$

$\Rightarrow k = 5$

and

$10 - a - 8k + k^2 = 0$

$\Rightarrow 10 - a - 8 \times 5 + 5^2 = 0$ [As $k = 5$]

$\Rightarrow 10 - a - 40 + 25 = 0$

$\Rightarrow -a - 5 = 0$

$\Rightarrow a = -5$

Hence, $k = 5$ and $a = -5$