

Exercise 1.4

Question 1:

Find the union of each of the following pairs of sets:

- (i) $X = \{1, 3, 5\}; \quad Y = \{1, 2, 3\}$
- (ii) $A = \{a, e, i, o, u\}; \quad B = \{a, b, c\}$
- (iii) $A = \{x: x \text{ is a natural number and multiple of } 3\}$
 $B = \{x: x \text{ is a natural number less than } 6\}$
- (iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$
 $B = \{x: x \text{ is a natural number and } 6 < x < 10\}$
- (v) $A = \{1, 2, 3\}; \quad B = \Phi$

Answer 1:

- (i) $X = \{1, 3, 5\} \quad Y = \{1, 2, 3\}$
 $X \cup Y = \{1, 2, 3, 5\}$
- (ii) $A = \{a, e, i, o, u\} \quad B = \{a, b, c\}$
 $A \cup B = \{a, b, c, e, i, o, u\}$
- (iii) $A = \{x: x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9 \dots\}$
 $B = \{x: x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5, 6\}$
 $A \cup B = \{1, 2, 4, 5, 3, 6, 9, 12 \dots\}$
 $\therefore A \cup B = \{x: x = 1, 2, 4, 5 \text{ or a multiple of } 3\}$
- (iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$
 $B = \{x: x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$
 $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$
 $\therefore A \cup B = \{x: x \in \mathbb{N} \text{ and } 1 < x < 10\}$
- (v) $A = \{1, 2, 3\}, \quad B = \Phi$
 $A \cup B = \{1, 2, 3\}$

Question 2:

Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Answer 2:

Here, $A = \{a, b\}$ and $B = \{a, b, c\}$

Yes, $A \subset B$.

$A \cup B = \{a, b, c\} = B$

Question 3:

If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Answer 3:

If A and B are two sets such that $A \subset B$, then $A \cup B = B$.

Question 4:

If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cup B$

(ii) $A \cup C$

(iii) $B \cup C$

(iv) $B \cup D$

(v) $A \cup B \cup C$

(vi) $A \cup B \cup D$

(vii) $B \cup C \cup D$

Answer 5:

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$

(i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

- (iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$
- (iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$
- (v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- (vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- (vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

Question 5:

Find the intersection of each pair of sets:

- (i) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$
- (ii) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$
- (iii) $A = \{x: x \text{ is a natural number and multiple of } 3\}$
 $B = \{x: x \text{ is a natural number less than } 6\}$
- (iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\}$
 $B = \{x: x \text{ is a natural number and } 6 < x < 10\}$
- (v) $A = \{1, 2, 3\}$, $B = \Phi$

Answer 5:

- (i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$
 $X \cap Y = \{1, 3\}$
- (ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$
 $A \cap B = \{a\}$
- (iii) $A = \{x: x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9 \dots\}$
 $B = \{x: x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5\}$
 $\therefore A \cap B = \{3\}$
- (iv) $A = \{x: x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$
 $B = \{x: x \text{ is a natural number and } 6 < x < 10\}$
 $= \{7, 8, 9\}$
 $A \cap B = \Phi$
- (v) $A = \{1, 2, 3\}$, $B = \Phi$. So, $A \cap B = \Phi$

Question 6:

If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

- (i) $A \cap B$
- (ii) $B \cap C$
- (iii) $A \cap C \cap D$
- (iv) $A \cap C$
- (v) $B \cap D$
- (vi) $A \cap (B \cup C)$
- (vii) $A \cap D$
- (viii) $A \cap (B \cup D)$
- (ix) $(A \cap B) \cap (B \cup C)$
- (x) $(A \cup D) \cap (B \cup C)$

Answer 6:

- (i) $A \cap B = \{7, 9, 11\}$
- (ii) $B \cap C = \{11, 13\}$
- (iii) $A \cap C \cap D = \{A \cap C\} \cap D = \{11\} \cap \{15, 17\} = \Phi$
- (iv) $A \cap C = \{11\}$
- (v) $B \cap D = \Phi$
- (vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
 $= \{7, 9, 11\} \cup \{11\} = \{7, 9, 11\}$
- (vii) $A \cap D = \Phi$
- (viii) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$
 $= \{7, 9, 11\} \cup \Phi = \{7, 9, 11\}$
- (ix) $(A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$
- (x) $(A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$
 $= \{7, 9, 11, 15\}$

Question 7:

If $A = \{x: x \text{ is a natural number}\}$, $B = \{x: x \text{ is an even natural number}\}$
 $C = \{x: x \text{ is an odd natural number}\}$ and $D = \{x: x \text{ is a prime number}\}$,
find

- (i)** $A \cap B$
- (ii)** $A \cap C$
- (iii)** $A \cap D$
- (iv)** $B \cap C$
- (v)** $B \cap D$
- (vi)** $C \cap D$

Answer 7:

$$A = \{x: x \text{ is a natural number}\} = \{1, 2, 3, 4, 5 \dots\}$$

$$B = \{x: x \text{ is an even natural number}\} = \{2, 4, 6, 8 \dots\}$$

$$C = \{x: x \text{ is an odd natural number}\} = \{1, 3, 5, 7, 9 \dots\}$$

$$D = \{x: x \text{ is a prime number}\} = \{2, 3, 5, 7 \dots\}$$

- (i)** $A \cap B = \{x: x \text{ is an even natural number}\} = B$
- (ii)** $A \cap C = \{x: x \text{ is an odd natural number}\} = C$
- (iii)** $A \cap D = \{x: x \text{ is a prime number}\} = D$
- (iv)** $B \cap C = \Phi$
- (v)** $B \cap D = \{2\}$
- (vi)** $C \cap D = \{x: x \text{ is odd prime number}\}$

Question 8:

Which of the following pairs of sets are disjoint

- (i)** $\{1, 2, 3, 4\}$ and $\{x: x \text{ is a natural number and } 4 \leq x \leq 6\}$
- (ii)** $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$
- (iii)** $\{x: x \text{ is an even integer}\}$ and $\{x: x \text{ is an odd integer}\}$

Answer 8:

(i) $\{1, 2, 3, 4\}$

$\{x: x \text{ is a natural number and } 4 \leq x \leq 6\} = \{4, 5, 6\}$

Now, $\{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\}$

Therefore, this pair of sets is not disjoint.

(ii) $\{a, e, i, o, u\} \cap \{c, d, e, f\} = \{e\}$

Therefore, $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ are not disjoint.

(iii) $\{x: x \text{ is an even integer}\} \cap \{x: x \text{ is an odd integer}\} = \Phi$

Therefore, this pair of sets is disjoint.

Question 9:

If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$,

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$; find

(i) $A - B$

(ii) $A - C$

(iii) $A - D$

(iv) $B - A$

(v) $C - A$

(vi) $D - A$

(vii) $B - C$

(viii) $B - D$

(ix) $C - B$

(x) $D - B$

(xi) $C - D$

(xii) $D - C$

Answer 9:

- (i) $A - B = \{3, 6, 9, 15, 18, 21\}$
- (ii) $A - C = \{3, 9, 15, 18, 21\}$
- (iii) $A - D = \{3, 6, 9, 12, 18, 21\}$
- (iv) $B - A = \{4, 8, 16, 20\}$
- (v) $C - A = \{2, 4, 8, 10, 14, 16\}$
- (vi) $D - A = \{5, 10, 20\}$
- (vii) $B - C = \{20\}$
- (viii) $B - D = \{4, 8, 12, 16\}$
- (ix) $C - B = \{2, 6, 10, 14\}$
- (x) $D - B = \{5, 10, 15\}$
- (xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$
- (xii) $D - C = \{5, 15, 20\}$

Question 10:

If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

- (i) $X - Y$
- (ii) $Y - X$
- (iii) $X \cap Y$

Answer 10:

- (i) $X - Y = \{a, c\}$
- (ii) $Y - X = \{f, g\}$
- (iii) $X \cap Y = \{b, d\}$

Question 11:

If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?

Answer 11:

R: set of real numbers

Q: set of rational numbers

Therefore, $R - Q$ is a set of irrational numbers.

Question 12:

State whether each of the following statement is true or false. Justify your answer.

- (i)** $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.
- (ii)** $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.
- (iii)** $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.
- (iv)** $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

Answer 12:

(i) False

As $3 \in \{2, 3, 4, 5\}$, $3 \in \{3, 6\}$

$\Rightarrow \{2, 3, 4, 5\} \cap \{3, 6\} = \{3\}$

(ii) False

As $a \in \{a, e, i, o, u\}$, $a \in \{a, b, c, d\}$

$\Rightarrow \{a, e, i, o, u\} \cap \{a, b, c, d\} = \{a\}$

(iii) True

As $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \Phi$

(iv) True

As $\{2, 6, 10\} \cap \{3, 7, 11\} = \Phi$