

Mathematics

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(Chapter – 11) (Conic Sections)

(Class – XI)

Exercise 11.1

Question 1:

Find the equation of the circle with centre (0, 2) and radius 2

Answer 1:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (0, 2)$ and radius $(r) = 2$.

Therefore, the equation of the circle is

$$(x - 0)^2 + (y - 2)^2 = 2^2$$

$$\Rightarrow x^2 + y^2 + 4 - 4y = 4$$

$$\Rightarrow x^2 + y^2 - 4y = 0$$

Question 2:

Find the equation of the circle with centre $(-2, 3)$ and radius 4

Answer 2:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (-2, 3)$ and radius $(r) = 4$.

Therefore, the equation of the circle is

$$(x + 2)^2 + (y - 3)^2 = (4)^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$\Rightarrow x^2 + y^2 + 4x - 6y - 3 = 0$$

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Question 3:

Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

Answer 3:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = \left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $(r) = \frac{1}{12}$.

Therefore, the equation of the circle is

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{1}{12}\right)^2$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} = \frac{1}{144}$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} - \frac{1}{144} = 0$$

$$144x^2 - 144x + 36 + 144y^2 - 72y + 9 - 1 = 0$$

$$144x^2 - 144x + 144y^2 - 72y + 44 = 0$$

$$36x^2 - 36x + 36y^2 - 18y + 11 = 0$$

$$36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

Question 4:

Find the equation of the circle with centre $(1, 1)$ and radius $\sqrt{2}$.

Answer 4:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (1, 1)$ and radius $(r) = \sqrt{2}$.

Therefore, the equation of the circle is

$$(x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$

$$x^2 + y^2 - 2x - 2y = 0$$

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Question 5:

Find the equation of the circle with centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$.

Answer 5:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (-a, -b)$ and radius $(r) = \sqrt{a^2 - b^2}$.

Therefore, the equation of the circle is

$$(x + a)^2 + (y + b)^2 = (\sqrt{a^2 - b^2})^2$$

$$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$$

$$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

Question 6:

Find the centre and radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$

Answer 6:

The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$.

$$(x + 5)^2 + (y - 3)^2 = 36$$

$$\Rightarrow \{x - (-5)\}^2 + (y - 3)^2 = 6^2,$$

which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = -5$, $k = 3$, and $r = 6$.

Thus, the centre of the given circle is $(-5, 3)$, while its radius is 6.

Question 7:

Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

Answer 7:

The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$.

$$x^2 + y^2 - 4x - 8y - 45 = 0$$

$$\Rightarrow (x^2 - 4x) + (y^2 - 8y) = 45$$

$$\Rightarrow \{x^2 - 2(x)(2) + 2^2\} + \{y^2 - 2(y)(4) + 4^2\} - 4 - 16 = 45$$

$$\Rightarrow (x - 2)^2 + (y - 4)^2 = 65$$

$$\Rightarrow (x - 2)^2 + (y - 4)^2 = \sqrt{65},$$

Which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 2$, $k = 4$ and $r = \sqrt{65}$,

Thus, the centre of the given circle is $(2, 4)$, while its radius is $\sqrt{65}$.

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Question 8:

Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$

Answer 8:

The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$.

$$x^2 + y^2 - 8x + 10y - 12 = 0$$

$$\Rightarrow (x^2 - 8x) + (y^2 + 10y) = 12$$

$$\Rightarrow \{x^2 - 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\} - 16 - 25 = 12$$

$$\Rightarrow (x - 4)^2 + (y + 5)^2 = 53$$

$$\Rightarrow (x - 4)^2 + \{y - (-5)\}^2 = (\sqrt{53})^2$$

which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 4$, $k = -5$ and $r = \sqrt{53}$

Thus, the centre of the given circle is $(4, -5)$, while its radius is $\sqrt{53}$.

Question 9:

Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

Answer 9:

The equation of the given circle is $2x^2 + 2y^2 - x = 0$.

$$2x^2 + 2y^2 - x = 0$$

$$\Rightarrow (2x^2 - x) + 2y^2 = 0$$

$$\Rightarrow 2 \left[\left(x^2 - \frac{x}{2} \right) + y^2 \right] = 0$$

$$\Rightarrow \left\{ x^2 - 2 \cdot x \left(\frac{1}{4} \right) + \left(\frac{1}{4} \right)^2 \right\} + y^2 - \left(\frac{1}{4} \right)^2 = 0$$

$$\Rightarrow \left(x - \frac{1}{4} \right)^2 + (y - 0)^2 = \left(\frac{1}{4} \right)^2$$

which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 1/4$, $k = 0$ and $r = 1/4$.

Thus, the centre of the given circle is $(1/4, 0)$, while its radius is $1/4$.

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Question 10:

Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line $4x + y = 16$.

Answer 10:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (4, 1) and (6, 5),

$$(4 - h)^2 + (1 - k)^2 = r^2 \dots\dots\dots (1)$$

$$(6 - h)^2 + (5 - k)^2 = r^2 \dots\dots\dots (2)$$

Since the centre (h, k) of the circle lies on line $4x + y = 16$,

$$4h + k = 16 \dots\dots\dots (3)$$

From equations (1) and (2), we obtain

$$(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$$

$$\Rightarrow 16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 25 - 10k + k^2$$

$$\Rightarrow 16 - 8h + 1 - 2k = 36 - 12h + 25 - 10k$$

$$\Rightarrow 4h + 8k = 44$$

$$\Rightarrow h + 2k = 11 \dots\dots\dots (4)$$

On solving equations (3) and (4), we obtain $h = 3$ and $k = 4$.

On substituting the values of h and k in equation (1), we obtain

$$(4 - 3)^2 + (1 - 4)^2 = r^2$$

$$\Rightarrow (1)^2 + (-3)^2 = r^2$$

$$\Rightarrow 1 + 9 = r^2$$

$$\Rightarrow r^2 = 10$$

$$\Rightarrow r = \sqrt{10}$$

Thus, the equation of the required circle is

$$(x - 3)^2 + (y - 4)^2 = (\sqrt{10})^2$$

$$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$$

$$x^2 + y^2 - 6x - 8y + 15 = 0$$

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Question 11:

Find the equation of the circle passing through the points (2, 3) and (-1, 1) and whose centre is on the line $x - 3y - 11 = 0$.

Answer 11:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (2, 3) and (-1, 1),

$$(2 - h)^2 + (3 - k)^2 = r^2 \dots\dots\dots (1)$$

$$(-1 - h)^2 + (1 - k)^2 = r^2 \dots\dots\dots (2)$$

Since the centre (h, k) of the circle lies on line $x - 3y - 11 = 0$,

$$h - 3k = 11 \dots\dots\dots (3)$$

From equations (1) and (2), we obtain

$$(2 - h)^2 + (3 - k)^2 = (-1 - h)^2 + (1 - k)^2$$

$$\Rightarrow 4 - 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$$

$$\Rightarrow 4 - 4h + 9 - 6k = 1 + 2h + 1 - 2k$$

$$\Rightarrow 6h + 4k = 11 \dots (4)$$

On solving equations (3) and (4), we obtain $h = \frac{7}{2}$ and $k = \frac{-5}{2}$.

On substituting the values of h and k in equation (1), we obtain

$$\left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{4-7}{2}\right)^2 + \left(\frac{6+5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{-3}{2}\right)^2 + \left(\frac{11}{2}\right)^2 = r^2$$

$$\Rightarrow \frac{9}{4} + \frac{121}{4} = r^2$$

$$\Rightarrow \frac{130}{4} = r^2$$

Thus, the equation of the required circle is

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$$\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2 + \left(\frac{2y+5}{2}\right)^2 = \frac{130}{4}$$

$$4x^2 - 28x + 49 + 4y^2 + 20y + 25 = 130$$

$$4x^2 + 4y^2 - 28x + 20y - 56 = 0$$

$$4(x^2 + y^2 - 7x + 5y - 14) = 0$$

$$x^2 + y^2 - 7x + 5y - 14 = 0$$

Question 12:

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Answer 12:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the radius of the circle is 5 and its centre lies on the x-axis, $k = 0$ and $r = 5$.

Now, the equation of the circle becomes $(x - h)^2 + y^2 = 25$.

It is given that the circle passes through point (2, 3).

$$\therefore (2 - h)^2 + 3^2 = 25$$

$$\Rightarrow (2 - h)^2 = 25 - 9$$

$$\Rightarrow (2 - h)^2 = 16$$

$$\Rightarrow 2 - h = \pm\sqrt{16} = \pm 4$$

If $2 - h = 4$, then $h = -2$.

If $2 - h = -4$, then $h = 6$.

When $h = -2$, the equation of the circle becomes

$$(x + 2)^2 + y^2 = 25$$

$$x^2 + 4x + 4 + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

When $h = 6$, the equation of the circle becomes

$$(x - 6)^2 + y^2 = 25$$

$$x^2 - 12x + 36 + y^2 = 25$$

$$x^2 + y^2 - 12x + 11 = 0$$

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Question 13:

Find the equation of the circle passing through (0, 0) and making intercepts a and b on the coordinate axes.

Answer 13:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the centre of the circle passes through (0, 0),

$$(0 - h)^2 + (0 - k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 = r^2$$

The equation of the circle now becomes $(x - h)^2 + (y - k)^2 = h^2 + k^2$.

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points $(a, 0)$ and $(0, b)$. Therefore,

$$(a - h)^2 + (0 - k)^2 = h^2 + k^2 \dots\dots\dots (1)$$

$$(0 - h)^2 + (b - k)^2 = h^2 + k^2 \dots\dots\dots (2)$$

From equation (1), we obtain $a^2 - 2ah + h^2 + k^2 = h^2 + k^2$

$$\Rightarrow a^2 - 2ah = 0$$

$$\Rightarrow a(a - 2h) = 0$$

$$\Rightarrow a = 0 \text{ or } (a - 2h) = 0$$

However, $a \neq 0$; hence, $(a - 2h) = 0 \Rightarrow h = a/2$.

From equation (2), we obtain $h^2 + b^2 - 2bk + k^2 = h^2 + k^2$

$$\Rightarrow b^2 - 2bk = 0$$

$$\Rightarrow b(b - 2k) = 0$$

$$\Rightarrow b = 0 \text{ or } (b - 2k) = 0$$

However, $b \neq 0$; hence, $(b - 2k) = 0 \Rightarrow k = b/2$.

Thus, the equation of the required circle is

$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$$

$$\Rightarrow \left(\frac{2x - a}{2}\right)^2 + \left(\frac{2y - b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

$$\Rightarrow 4x^2 - 4ax + a^2 + 4y^2 - 4by + b^2 = a^2 + b^2$$

$$\Rightarrow 4x^2 + 4y^2 - 4ax - 4by = 0$$

$$\Rightarrow x^2 + y^2 - ax - by = 0$$

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Question 14:

Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Answer 14:

The centre of the circle is given as $(h, k) = (2, 2)$.

Since the circle passes through point (4, 5), the radius (r) of the circle is the distance between the points (2, 2) and (4, 5).

$$\therefore r = \sqrt{(2-4)^2 + (2-5)^2} = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-2)^2 = (\sqrt{13})^2$$

$$x^2 - 4x + 4 + y^2 - 4y + 4 = 13$$

$$x^2 + y^2 - 4x - 4y - 5 = 0$$

Question 15:

Does the point $(-2.5, 3.5)$ lie inside, outside or on the circle $x^2 + y^2 = 25$?

Answer 15:

The equation of the given circle is $x^2 + y^2 = 25$.

$$x^2 + y^2 = 25$$

$$\Rightarrow (x-0)^2 + (y-0)^2 = 5^2,$$

which is of the form $(x-h)^2 + (y-k)^2 = r^2$, where $h = 0$, $k = 0$, and $r = 5$.

\therefore Centre = $(0, 0)$ and radius = 5

Distance between point $(-2.5, 3.5)$ and centre $(0, 0)$

$$= \sqrt{(-2.5-0)^2 + (3.5-0)^2}$$

$$= \sqrt{6.25 + 12.25}$$

$$= \sqrt{18.5}$$

$$= 4.3 \text{ (approx.)} < 5$$

Since the distance between point $(-2.5, 3.5)$ and centre $(0, 0)$ of the circle is less than the radius of the circle, point $(-2.5, 3.5)$ lies inside the circle.