Mathematics

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(Chapter 11)(Conic Sections)

XI

Exercise 11.3

Question 1:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

Answer 1:

The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$.

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$$

we obtain *a* = 6 and *b* = 4.

Therefore,

The coordinates of the foci are $\left(2\sqrt{5},0\right)$ and $\left(-2\sqrt{5},0\right)$.

The coordinates of the vertices are (6, 0) and (–6, 0).

Length of major axis = 2*a* = 12

Length of minor axis = 2*b* = 8

Eccentricity, $e=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times16}{6}=\frac{16}{3}$

Question 2:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{4}+\frac{y^2}{25}=1$

Answer 2:

The given equation is $\frac{x^2}{4}+\frac{y^2}{25}=1$ or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$.

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.
On comparing the given equation with

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

, we obtain *b* = 2 and *a* = 5.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{25-4}=\sqrt{21}$

Therefore,

The coordinates of the foci are

$\left(0,\sqrt{21}\right)$ and $\left(0,-\sqrt{21}\right)$.

The coordinates of the vertices are (0, 5) and (0, −5)

Length of major axis = 2*a* = 10

Length of minor axis = 2*b* = 4

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{21}}{5}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{5}=\frac{8}{5}$

Question 3:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Answer 3:

The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$.

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.
On comparing the given equation with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

, we obtain a = 4 and b = 3.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{7},0\right)$

The coordinates of the vertices are $\left(\pm 4,0\right)$

Length of major axis = 2a = 8

Length of minor axis = 2b = 6

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

Question 4:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{25}+\frac{y^2}{100}=1$

Answer 4:

The given equation is $\frac{x^2}{25}+\frac{y^2}{100}=1$ or $\frac{x^2}{5^2}+\frac{y^2}{10^2}=1$.

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$$

we obtain *b* = 5 and *a* = 10.

Therefore,

The coordinates of the foci are $\left(0,\pm 5\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±10).

Length of major axis = 2*a* = 20 Length

of minor axis = 2*b* = 10

Eccentricity, $e=\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 25}{10}=5$

Question 5:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{49}+\frac{y^2}{36}=1$

Answer 5:

The given equation is $\frac{x^2}{49}+\frac{y^2}{36}=1$ or $\frac{x^2}{7^2}+\frac{y^2}{6^2}=1$.

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

we obtain $a = 7$ and $b = 6$.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{49-36}=\sqrt{13}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{13},0\right)$.

The coordinates of the vertices are (± 7, 0).

Length of major axis = 2a = 14

Length of minor axis = 2b = 12

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{13}}{7}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 36}{7}=\frac{72}{7}$

Question 6:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100}+\frac{y^2}{400}=1$

Answer 6:

The given equation is $\frac{x^2}{100}+\frac{y^2}{400}=1$ or $\frac{x^2}{10^2}+\frac{y^2}{20^2}=1$.

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$$

we obtain *b* = 10 and *a* = 20.

Therefore,

The coordinates of the foci are $\left(0,\pm 10\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±20)

Length of major axis = 2*a* = 40

Length of minor axis = 2*b* = 20

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{10\sqrt{3}}{20}=\frac{\sqrt{3}}{2}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 100}{20}=10$

Question 7:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $36x^2 + 4y^2 = 144$

Answer 7:

The given equation is $36x^2 + 4y^2 = 144$.

It can be written as

$$36x^2 + 4y^2 = 144$$

$$\text{Or, } \frac{x^2}{4} + \frac{y^2}{36} = 1$$

$$\text{Or, } \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1 \quad \ldots(1)$$

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$$

we obtain $b = 2$ and $a = 6$.

Therefore,

The coordinates of the foci are $\left(0, \pm 4\sqrt{2}\right)$.

The coordinates of the vertices are $(0, \pm 6)$.

Length of major axis = $2a = 12$

Length of minor axis = $2b = 4$

Eccentricity, $e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$

Question 8:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $16x^2 + y^2 = 16$

Answer 8:

The given equation is $16x^2 + y^2 = 16$.

It can be written as

$$16x^2 + y^2 = 16$$

$$\text{Or, } \frac{x^2}{1} + \frac{y^2}{16} = 1$$

$$\text{Or, } \frac{x^2}{1^2} + \frac{y^2}{4^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16-1} = \sqrt{15}$$

we obtain $b = 1$ and $a = 4$.

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{15}\right)$.

The coordinates of the vertices are (0, ±4).

Length of major axis = 2a = 8

Length of minor axis = 2b = 2

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{15}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 1}{4}=\frac{1}{2}$

Question 9:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Answer 9:

The given equation is $4x^2 + 9y^2 = 36$.

It can be written as

$$4x^2+9y^2=36$$

$$\text{Or, } \frac{x^2}{9}+\frac{y^2}{4}=1$$

$$\text{Or, } \frac{x^2}{3^2}+\frac{y^2}{2^2}=1 \qquad ...(1)$$

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

we obtain a = 3 and b = 2.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{9-4}=\sqrt{5}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{5},0\right)$.

The coordinates of the vertices are (±3, 0).

Length of major axis = 2a = 6

Length of minor axis = 2b = 4

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 4}{3} = \frac{8}{3}$

Question 10:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±5, 0), foci (±4, 0)

Answer 10:

Vertices (±5, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where *a* is the semi-major axis.

Accordingly, *a* = 5 and *c* = 4.

It is known that $a^2 = b^2 + c^2$

$\therefore 5^2 = b^2 + 4^2$

$\Rightarrow 25 = b^2 + 16$

$\Rightarrow b^2 = 25 - 16$

$\Rightarrow b = \sqrt{9} = 3$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

Question 11:

Find the equation for the ellipse that satisfies the given conditions: Vertices (0, ±13), foci (0, ±5)

Answer 11:

Vertices (0, ±13), foci (0, ±5)

Here, the vertices are on the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where *a* is the semi-major axis.

Accordingly, *a* = 13 and *c* = 5.

It is known that $a^2 = b^2 + c^2$

$\therefore 13^2 = b^2 + 5^2$

$\Rightarrow 169 = b^2 + 25$

$\Rightarrow b^2 = 169 - 25$

$\Rightarrow b = \sqrt{144} = 12$

Thus, the equation of the ellipse is $\frac{x^2}{12^2} + \frac{y^2}{13^2} = 1$ or $\frac{x^2}{144} + \frac{y^2}{169} = 1$.

Question 12:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±6, 0), foci (±4, 0)

Answer 12:

Vertices (±6, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where *a* is the semi-major axis.

Accordingly, $a = 6$, $c = 4$.

It is known that $a^2 = b^2 + c^2$

$\therefore 6^2 = b^2 + 4^2$

$\Rightarrow 36 = b^2 + 16$

$\Rightarrow b^2 = 36 - 16$

$\Rightarrow b = \sqrt{20}$

Thus, the equation of the ellipse is $\frac{x^2}{6^2} + \frac{y^2}{\left(\sqrt{20}\right)^2} = 1$ or $\frac{x^2}{36} + \frac{y^2}{20} = 1$

Question 13:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Answer 13:

Ends of major axis (±3, 0), ends of minor axis (0, ±2) Here, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = 3 and *b* = 2.

Thus, the equation of the ellipse is $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ i.e., $\frac{x^2}{9}+\frac{y^2}{4}=1$.

Question 14:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Answer 14:

Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Here, the major axis is along the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where *a* is the semi-major axis.

Accordingly, *a* = $\sqrt{5}$ and *b* = 1.

Thus, the equation of the ellipse is $\frac{x^2}{1^2}+\frac{y^2}{\left(\sqrt{5}\right)^2}=1$ or $\frac{x^2}{1}+\frac{y^2}{5}=1$.

Question 15:

Find the equation for the ellipse that satisfies the given conditions: Length of major axis 26, foci (±5, 0)

Answer 15:

Length of major axis = 26; foci = (±5, 0).

Since the foci are on the *x*-axis, the major axis is along the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where *a* is the semi-major axis.

Accordingly, 2*a* = 26 ⇒ *a* = 13 and *c* = 5.

It is known that $a^2=b^2+c^2$

$\therefore 13^2=b^2+5^2$

$\Rightarrow 169=b^2+25$

$\Rightarrow b^2=169-25$

$\Rightarrow b=\sqrt{144}=12$

Thus, the equation of the ellipse is $\frac{x^2}{13^2}+\frac{y^2}{12^2}=1$ or $\frac{x^2}{169}+\frac{y^2}{144}=1$

Question 16:

Find the equation for the ellipse that satisfies the given conditions: Length of minor axis 16, foci (0, ±6)

Answer 16:

Length of minor axis = 16; foci = (0, ±6).

Since the foci are on the *y*-axis, the major axis is along the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where *a* is the semi-major axis.

Accordingly, 2*b* = 16 ⇒ *b* = 8 and *c* = 6.

It is known that $a^2=b^2+c^2$

$\therefore a^2=8^2+6^2=64+36=100$

$\Rightarrow a=\sqrt{100}=10$

Thus, the equation of the ellipse is $\frac{x^2}{8^2}+\frac{y^2}{10^2}=1$ or $\frac{x^2}{64}+\frac{y^2}{100}=1$.

Question 17:

Find the equation for the ellipse that satisfies the given conditions: Foci (±3, 0), *a* = 4

Answer 17:

Foci (±3, 0), a = 4

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, c = 3 and a = 4.

It is known that $a^2 = b^2 + c^2$

$\therefore 4^2 = b^2 + 3^2$

$\Rightarrow 16 = b^2 + 9$

$\Rightarrow b^2 = 16 - 9 = 7$

Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{7}=1$

Question 18:

Find the equation for the ellipse that satisfies the given conditions: b = 3, c = 4, centre at the origin; foci on the x axis.

Answer 18:

It is given that b = 3, c = 4, centre at the origin; foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, b = 3, c = 4.

It is known that $a^2 = b^2 + c^2$

$\therefore a^2 = 3^2 + 4^2 = 9 + 16 = 25$

$\Rightarrow a = 5$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Question 19:

Find the equation for the ellipse that satisfies the given conditions: Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Answer 19:

Since the centre is at (0, 0) and the major axis is on the *y*-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (3, 2) and (1, 6). Hence,

$$\frac{9}{b^2}+\frac{4}{a^2}=1 \qquad ...(2)$$

$$\frac{1}{b^2}+\frac{36}{a^2}=1 \qquad ...(3)$$

On solving equations (2) and (3), we obtain $b^2 = 10$ and $a^2 = 40$.

Thus, the equation of the ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$ or $4x^2+y^2=40$

Question 20:

Find the equation for the ellipse that satisfies the given conditions: Major axis on the *x* - axis and passes through the points (4, 3) and (6, 2).

Answer 20:

Since the major axis is on the *x*-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (4, 3) and (6, 2). Hence,

$$\frac{16}{a^2}+\frac{9}{b^2}=1 \qquad ...(2)$$

$$\frac{36}{a^2}+\frac{4}{b^2}=1 \qquad ...(3)$$

On solving equations (2) and (3), we obtain $a^2 = 52$ and $b^2 = 13$.

Thus, the equation of the ellipse is $\frac{x^2}{52}+\frac{y^2}{13}=1$ or $x^2+4y^2=52$