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(Chapter – 12) (Introduction to Three Dimensional Geometry)

(Class - XI)

Exercise 12.3

Question 1:

Find the coordinates of the point which divides the line segment joining the points (-2, 3, 5) and (1, -4, 6) in the ratio (i) 2:3 internally, (ii) 2:3 externally.

Answer 1:

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m: n are

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}, \frac{mz_2 + nz_1}{m + n}\right)$$

Let R (x, y, z) be the point that divides the line segment joining points (-2, 3, 5) and (1, -4, 6) internally in the ratio 2:3

$$x = \frac{2(1) + 3(-2)}{2 + 3}, y = \frac{2(-4) + 3(3)}{2 + 3}, \text{ and } z = \frac{2(6) + 3(5)}{2 + 3}$$

i.e., $x = \frac{-4}{5}, y = \frac{1}{5}, \text{ and } z = \frac{27}{5}$

Thus, the coordinates of the required point are $\left(-\frac{4}{5},\frac{1}{5},\frac{27}{5}\right)$.

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio m: n are

$$\left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}, \frac{mz_2 - nz_1}{m - n}\right)$$

Let R (x, y, z) be the point that divides the line segment joining points (-2, 3, 5) and (1, -4, 6) externally in the ratio 2:3

$$x = \frac{2(1)-3(-2)}{2-3}$$
, $y = \frac{2(-4)-3(3)}{2-3}$, and $z = \frac{2(6)-3(5)}{2-3}$
i.e., $x = -8$, $y = 17$, and $z = 3$

Thus, the coordinates of the required point are (-8, 17, 3).

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Question 2:

Given that P (3, 2, -4), Q (5, 4, -6) and R (9, 8, -10) are collinear. Find the ratio in which Q divides PR.

Answer 2:

Let point Q (5, 4, -6) divide the line segment joining points P (3, 2, -4) and R (9, 8, -10) in the ratio *k*:1.

Therefore, by section formula,

$$(5,4,-6) = \left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1}\right)$$

$$\Rightarrow \frac{9k+3}{k+1} = 5$$

$$\Rightarrow$$
 9k + 3 = 5k + 5

$$\Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$$

Thus, point Q divides PR in the ratio 1:2.

Question 3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points (-2, 4, 7) and (3, -5, 8).

Answer 3:

Let the YZ plane divide the line segment joining points (-2, 4, 7) and (3, -5, 8) in the ratio k:1.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the *x*-coordinate of any point is zero.

$$\frac{3k-2}{k+1} = 0$$
$$\Rightarrow 3k-2 = 0$$

$$\Rightarrow 3k - 2 = 0$$

$$\Rightarrow k = \frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

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Question 4:

Using section formula, show that the points A (2, -3, 4), B (-1, 2, 1) and $C(0, \frac{1}{3}, 2)$ are collinear.

Answer 4:

The given points are A (2, -3, 4), B (-1, 2, 1), and $C(0, \frac{1}{3}, 2)$.

Let P be a point that divides AB in the ratio k:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+1} = 0$, we obtain k = 2.

For k = 2, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$.

i.e., $C\left(0,\frac{1}{3},2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.

Hence, points A, B, and C are collinear.

Ouestion 5:

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, -6) and Q (10, -16, 6).

Answer 5:

Let A and B be the points that trisect the line segment joining points P (4, 2, -6) and Q (10, -16, 6)

$$\begin{array}{cccccc} P & & A & B \\ \hline & & & & \\ (4, 2, -6) & & & & & \\ \end{array} Q$$

Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-6)}{1+2}\right) = (6, -4, -2)$$

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Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1}, \frac{2(-16)+1(2)}{2+1}, \frac{2(6)-1(6)}{2+1}\right) = (8,-10,2)$$

Thus, (6, -4, -2) and (8, -10, 2) are the points that trisect the line segment joining points P (4, 2, -6) and Q (10, -16, 6).