Mathematics

(www.tiwariacademy.com : Focus on free Education)

(Chapter – 13) (Limits and Derivatives)

(Class – XI)

Exercise 13.1

Question 1:

Evaluate the Given $\lim_{x\to 3} x+3$ limit:

Answer 1:

$$\lim_{x\to 3} x+3 = 3+3 = 6$$

Question 2:

Evaluate the Given limit: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$

Answer 2:

$$\lim_{x\to \pi}\left(x-\frac{22}{7}\right) = \left(\pi-\frac{22}{7}\right)$$

Question 3:

Evaluate the Given limit: $\lim_{r\to 1} \pi r^2$

Answer 3:

$$\lim_{r\to 1} \pi r^2 = \pi(1)^2 = \pi$$

Question 4:

Evaluate the Given limit: $\lim_{x\to 4}\frac{4x+3}{x-2}$

Answer 4:

$$\lim_{x\to 4}\frac{4x+3}{x-2} = \frac{4(4)+3}{4-2} = \frac{16+3}{2} = \frac{19}{2}$$

Question 5:

Evaluate the Given limit: $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

Answer 5:

$$\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1} = \frac{(-1)^{10}+(-1)^5+1}{-1-1} = \frac{1-1+1}{-2} = -\frac{1}{2}$$

Question 6:

Evaluate the Given limit: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Answer 6:

$$\lim_{x\to 0}\frac{(x+1)^5-1}{x}$$

Put $x + 1 = y$ so that $y \to 1$ as $x \to 0$.

$$\text{Accordingly, } \lim_{x\to 0}\frac{(x+1)^5-1}{x}=\lim_{y\to 1}\frac{y^5-1}{y-1}$$

$$=\lim_{y\to 1}\frac{y^5-1^5}{y-1}$$

$$=5.1^{5-1} \qquad \left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=5$$

$$\therefore \lim_{x\to 0}\frac{(x+5)^5-1}{x}=5$$

Question 7:

Evaluate the Given limit: $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}$

Answer 7:

At $x = 2$, the value of the given rational function takes the form. $\frac{0}{0}$

$$\therefore \lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(3x+5)}{(x-2)(x+2)}$$

$$=\lim_{x\to 2}\frac{3x+5}{x+2}$$

$$=\frac{3(2)+5}{2+2}$$

$$=\frac{11}{4}$$

Question 8:

Evaluate the Given limit: $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

Answer 8:

At x = 2, the value of the given rational function takes the form. $\frac{0}{0}$

$$\therefore \lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3} = \lim_{x \to 3} \frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)}$$

$$= \lim_{x \to 3} \frac{(x+3)(x^2+9)}{2x+1}$$

$$= \frac{(3+3)(3^2+9)}{2(3)+1}$$

$$= \frac{6 \times 18}{7}$$

$$= \frac{108}{7}$$

Question 9:

Evaluate the Given limit: $\lim_{x \to 0} \frac{ax+b}{cx+1}$

Answer 9:

$$\lim_{x \to 0} \frac{ax+b}{cx+1} = \frac{a(0)+b}{c(0)+1} = b$$

Question 10:

Evaluate the Given limit: $\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$

Answer 10:

$$\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

At z = 1, the value of the given function takes the form. $\frac{0}{0}$

Put $z^{\frac{1}{6}} = x$ so that $z \to 1$ as $x \to 1$.

Accordingly, $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=\lim_{x\to 1}\frac{x^2-1}{x-1}$

$$=\lim_{x\to 1}\frac{x^2-1^2}{x-1}$$

$$=2.1^{2-1} \qquad \left[\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=2$$

$$\therefore \lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=2$$

Question 11:

Evaluate the Given limit: $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$

Answer 11:

$$\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}=\frac{a(1)^2+b(1)+c}{c(1)^2+b(1)+a}$$

$$=\frac{a+b+c}{a+b+c}$$

$$=1 \qquad [a+b+c\neq 0]$$

Question 12:

Evaluate the Given limit: $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

Answer 12:

$$\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$$

At $x = -2$, the value of the given function takes the form. $\frac{0}{0}$

$$\text{Now, } \lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} = \lim_{x \to -2} \frac{\left(\frac{2+x}{2x}\right)}{x+2}$$

$$= \lim_{x \to -2} \frac{1}{2x}$$

$$= \frac{1}{2(-2)} = \frac{-1}{4}$$

Question 13:

Evaluate the Given limit: $\lim_{x \to 0} \frac{\sin ax}{bx}$

Answer 13:

$$\lim_{x \to 0} \frac{\sin ax}{bx}$$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$

$$\text{Now, } \lim_{x \to 0} \frac{\sin ax}{bx} = \lim_{x \to 0} \frac{\sin ax}{ax} \times \frac{ax}{bx}$$

$$= \lim_{x \to 0} \left(\frac{\sin ax}{ax}\right) \times \left(\frac{a}{b}\right)$$

$$= \frac{a}{b} \lim_{ax \to 0} \left(\frac{\sin ax}{ax}\right) \qquad [x \to 0 \Rightarrow ax \to 0]$$

$$= \frac{a}{b} \times 1 \qquad \left[\lim_{y \to 0} \frac{\sin y}{y} = 1\right]$$

$$= \frac{a}{b}$$

Question 14:

Evaluate the Given limit: $\lim_{x\to 0}\frac{\sin ax}{\sin bx},\ a,\ b\neq 0$

Answer 14:

$$\lim_{x\to 0}\frac{\sin ax}{\sin bx},\ a,\ b\neq 0$$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$

$$\text{Now, } \lim_{x\to 0}\frac{\sin ax}{\sin bx}=\lim_{x\to 0}\frac{\left(\frac{\sin ax}{ax}\right)\times ax}{\left(\frac{\sin bx}{bx}\right)\times bx}$$

$$=\left(\frac{a}{b}\right)\times\frac{\lim_{ax\to 0}\left(\frac{\sin ax}{ax}\right)}{\lim_{bx\to 0}\left(\frac{\sin bx}{bx}\right)} \qquad \left[\begin{array}{l} x\to 0\Rightarrow ax\to 0 \\ \text{and } x\to 0\Rightarrow bx\to 0\end{array}\right]$$

$$=\left(\frac{a}{b}\right)\times\frac{1}{1} \qquad \left[\lim_{y\to 0}\frac{\sin y}{y}=1\right]$$

$$=\frac{a}{b}$$

Question 15:

Evaluate the Given limit: $\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$

Answer 16:

$$\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$$

It is seen that $x\to\pi\Rightarrow(\pi-x)\to 0$

$$\therefore \lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}=\frac{1}{\pi}\lim_{(\pi-x)\to 0}\frac{\sin(\pi-x)}{(\pi-x)}$$

$$=\frac{1}{\pi}\times 1 \qquad \left[\lim_{y\to 0}\frac{\sin y}{y}=1\right]$$

$$=\frac{1}{\pi}$$

Question 16:

Evaluate the given limit: $\lim_{x\to 0}\frac{\cos x}{\pi-x}$

Answer 16:

$$\lim_{x\to 0}\frac{\cos x}{\pi-x}=\frac{\cos 0}{\pi-0}=\frac{1}{\pi}$$

Question 17:

Evaluate the Given limit: $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$

Answer 17:

$$\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$$

At x = 0, the value of the given function takes the form. $\frac{0}{0}$

Now,

$$\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}=\lim_{x\to 0}\frac{1-2\sin^2 x-1}{1-2\sin^2\frac{x}{2}-1}\qquad\left[\cos x=1-2\sin^2\frac{x}{2}\right]$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{\sin^2\frac{x}{2}}=\lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\times x^2}{\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)\times\frac{x^2}{4}}$$

$$=4\frac{\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)}{\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)}$$

$$=4\frac{\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2}{\left(\lim_{\frac{x}{2}\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2}\qquad\left[x\to 0\Rightarrow\frac{x}{2}\to 0\right]$$

$$=4\frac{1^2}{1^2}\qquad\left[\lim_{y\to 0}\frac{\sin y}{y}=1\right]$$

$$=4$$

Question 18:

Evaluate the Given limit: $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$

Answer 18:

$$\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$$

At $x = 0$, the value of the given function takes the form. $\frac{0}{0}$

Now,

$$\begin{aligned}\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x} &= \frac{1}{b}\lim_{x\to 0}\frac{x(a+\cos x)}{\sin x}\\ &= \frac{1}{b}\lim_{x\to 0}\left(\frac{x}{\sin x}\right)\times\lim_{x\to 0}(a+\cos x)\\ &= \frac{1}{b}\times\frac{1}{\left(\lim\limits_{x\to 0}\frac{\sin x}{x}\right)}\times\lim_{x\to 0}(a+\cos x)\\ &= \frac{1}{b}\times(a+\cos 0) \qquad \left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]\\ &= \frac{a+1}{b}\end{aligned}$$

Question 19:

Evaluate the Given limit: $\lim\limits_{x\to 0} x\sec x$

Answer 19:

$$\lim_{x\to 0} x\sec x = \lim_{x\to 0}\frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = 0$$

Question 20:

Evaluate the Given limit: $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}\ a,b,a+b\neq 0$

Answer 20:

At x = 0, the value of the given function takes the form. $\frac{0}{0}$

Now,

$$\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}$$

$$=\lim_{x\to 0}\frac{\left(\frac{\sin ax}{ax}\right)ax+bx}{ax+bx\left(\frac{\sin bx}{bx}\right)}$$

$$=\frac{\left(\lim_{ax\to 0}\frac{\sin ax}{ax}\right)\times\lim_{x\to 0}(ax)+\lim_{x\to 0}bx}{\lim_{x\to 0}ax+\lim_{x\to 0}bx\left(\lim_{bx\to 0}\frac{\sin bx}{bx}\right)}\qquad [\text{As } x\to 0\Rightarrow ax\to 0 \text{ and } bx\to 0]$$

$$=\frac{\lim_{x\to 0}(ax)+\lim_{x\to 0}bx}{\lim_{x\to 0}ax+\lim_{x\to 0}bx}\qquad \left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{\lim_{x\to 0}(ax+bx)}{\lim_{x\to 0}(ax+bx)}$$

$$=\lim_{x\to 0}(1)$$

$$=1$$

Question 21:

Evaluate the Given limit: $\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$

Answer 21:

At $x = 0$, the value of the given function takes the form. $\infty - \infty$

Now,

$$\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$$

$$= \lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{1-\cos x}{\sin x}\right)$$

$$= \lim_{x\to 0}\frac{\left(\frac{1-\cos x}{x}\right)}{\left(\frac{\sin x}{x}\right)}$$

$$= \frac{\lim_{x\to 0}\frac{1-\cos x}{x}}{\lim_{x\to 0}\frac{\sin x}{x}}$$

$$= \frac{0}{1} \qquad \left[\lim_{x\to 0}\frac{1-\cos x}{x} = 0 \text{ and } \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= 0$$

Question 22:

$$\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}$$

Answer 22:

$$\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}$$

At $x=\frac{\pi}{2}$, the value of the given function takes the form $\frac{0}{0}$.

Now, put $x-\frac{\pi}{2}=y$ so that $x\to\frac{\pi}{2},\ y\to 0$.

$$\therefore \lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}=\lim_{y\to 0}\frac{\tan 2\left(y+\frac{\pi}{2}\right)}{y}$$

$$=\lim_{y\to 0}\frac{\tan(\pi+2y)}{y}$$

$$=\lim_{y\to 0}\frac{\tan 2y}{y} \qquad \left[\tan(\pi+2y)=\tan 2y\right]$$

$$=\lim_{y\to 0}\frac{\sin 2y}{y\cos 2y}$$

$$=\lim_{y\to 0}\left(\frac{\sin 2y}{2y}\times\frac{2}{\cos 2y}\right)$$

$$=\left(\lim_{2y\to 0}\frac{\sin 2y}{2y}\right)\times\lim_{y\to 0}\left(\frac{2}{\cos 2y}\right) \qquad [y\to 0\Rightarrow 2y\to 0]$$

$$=1\times\frac{2}{\cos 0} \qquad \left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=1\times\frac{2}{1}$$

$$=2$$

Question 23:

Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$

Answer 23:

The given function is $f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}[2x+3] = 2(0)+3 = 3$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} 3(x+1) = 3(0+1) = 3$$

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 3$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} 3(x+1) = 3(1+1) = 6$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} 3(x+1) = 3(1+1) = 6$$

$$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x) = 6$$

Question 24:

Find $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^2-1, & x \le 1 \\ -x^2-1, & x > 1 \end{cases}$

Answer 24:

The given function is

$$f(x)=\begin{cases} x^2-1, & x\le 1 \\ -x^2-1, & x>1 \end{cases}$$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1}\left[x^2-1\right]=1^2-1=1-1=0$$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1}\left[-x^2-1\right]=-1^2-1=-1-1=-2$$

It is observed that $\lim_{x\to 1^-} f(x)\neq \lim_{x\to 1^+} f(x)$.

Hence, $\lim_{x\to 1} f(x)$ does not exist.

Question 25:

Evaluate $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$

Answer 25:

The given function is $f(x) = \begin{cases} \frac{|x|}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$

$$\begin{aligned}\lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-}\left[\frac{|x|}{x}\right] \\ &= \lim_{x\to 0}\left(\frac{-x}{x}\right) \qquad \left[\text{When } x \text{ is negaitve, } |x|=-x\right] \\ &= \lim_{x\to 0}(-1) \\ &= -1\end{aligned}$$

$$\begin{aligned}\lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+}\left[\frac{|x|}{x}\right] \\ &= \lim_{x\to 0}\left[\frac{x}{x}\right] \qquad \left[\text{When } x \text{ is positive, } |x|=x\right] \\ &= \lim_{x\to 0}(1) \\ &= 1\end{aligned}$$

It is observed that $\lim_{x\to 0^-} f(x)\neq \lim_{x\to 0^+} f(x)$.

Hence, $\lim_{x\to 0} f(x)$ does not exist.

Question 26:

Find $\lim\limits_{x\to 0} f(x)$, where $f(x) = \begin{cases} \dfrac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Answer 26:

The given function is

$$f(x) = \begin{cases} \dfrac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-}\left[\frac{x}{|x|}\right] \\ &= \lim_{x\to 0}\left[\frac{x}{-x}\right] \qquad [\text{When } x<0,\ |x|=-x] \\ &= \lim_{x\to 0}(-1) \\ &= -1 \end{aligned}$$

$$\begin{aligned} \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+}\left[\frac{x}{|x|}\right] \\ &= \lim_{x\to 0}\left[\frac{x}{x}\right] \qquad [\text{When } x>0,\ |x|=x] \\ &= \lim_{x\to 0}(1) \\ &= 1 \end{aligned}$$

It is observed that $\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$.

Hence, $\lim\limits_{x\to 0} f(x)$ does not exist.

Question 27:

Find $\lim_{x\to 5} f(x)$, where $f(x) = |x| - 5$

Answer 27:

The given function is $f(x) = |x| - 5$.

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-}\left[|x|-5\right]$$
$$= \lim_{x\to 5}(x-5) \qquad \left[\text{When } x>0,\ |x|=x\right]$$
$$= 5-5$$
$$= 0$$
$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+}\left(|x|-5\right)$$
$$= \lim_{x\to 5}(x-5) \qquad \left[\text{When } x>0,\ |x|=x\right]$$
$$= 5-5$$
$$= 0$$
$$\therefore\ \lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x) = 0$$
$$\text{Hence, } \lim_{x\to 5} f(x) = 0$$

Question 28:

Suppose $f(x) = \begin{cases} a+bx, & if x<1 \\ 4, & if\ x=0 \\ b-ax, & if x>1 \end{cases}$,

and $\lim_{x\to 1} f(x) = f(1)$ what are possible values of *a* and *b*?

Answer 28:

The given function is

$$f(x) = \begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax & x>1 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(a+bx) = a+b$$
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(b-ax) = b-a$$
$$f(1) = 4$$

It is given that $\lim_{x\to 1} f(x) = f(1)$.

$$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x) = f(1)$$
$$\Rightarrow a+b=4 \text{ and } b-a=4$$

On solving these two equations, we obtain $a=0$ and $b=4$.

Thus, the respective possible values of *a* and *b* are 0 and 4.

Question 29:

Let $a_1, a_2, \ldots, a_n$ be fixed real numbers and define a function

$f(x) = (x - a_1)(x - a_2)\ldots(x - a_n)$.

What is $\lim_{x\to a_1} f(x)$? For some $a \neq a_1, a_2 \ldots a_n$, compute $\lim_{x\to a} f(x)$.

Answer 29:

The given function is $f(x) = (x-a_1)(x-a_2)\ldots(x-a_n)$

$$\lim_{x\to a_1} f(x) = \lim_{x\to a_1}\left[(x-a_1)(x-a_2)\ldots(x-a_n)\right]$$

$$= \left[\lim_{x\to a_1}(x-a_1)\right]\left[\lim_{x\to a_1}(x-a_2)\right]\ldots\left[\lim_{x\to a_1}(x-a_n)\right]$$

$$= (a_1-a_1)(a_1-a_2)\ldots(a_1-a_n) = 0$$

$$\therefore \lim_{x\to a_1} f(x) = 0$$

$$\text{Now, } \lim_{x\to a} f(x) = \lim_{x\to a}\left[(x-a_1)(x-a_2)\ldots(x-a_n)\right]$$

$$= \left[\lim_{x\to a}(x-a_1)\right]\left[\lim_{x\to a}(x-a_2)\right]\ldots\left[\lim_{x\to a}(x-a_n)\right]$$

$$= (a-a_1)(a-a_2)\ldots\ldots(a-a_n)$$

$$\therefore \lim_{x\to a} f(x) = (a-a_1)(a-a_2)\ldots(a-a_n)$$

Question 30:

If $f(x) = \begin{cases} |x|+1, & x<0 \\ 0, & x=0 \\ |x|-1, & x>0 \end{cases}$.

For what value(s) of a does $\lim_{x\to a} f(x)$ exists?

Answer 30:

The given function is

$$f(x)=\begin{cases}|x|+1, & x<0\\ 0, & x=0\\ |x|-1, & x>0\end{cases}$$

When $a=0$,

$$\begin{aligned}\lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-}(|x|+1)\\ &= \lim_{x\to 0}(-x+1) \qquad [\text{If } x<0,\ |x|=-x]\\ &= -0+1\\ &= 1\end{aligned}$$

$$\begin{aligned}\lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+}(|x|-1)\\ &= \lim_{x\to 0}(x-1) \qquad [\text{If } x>0,\ |x|=x]\\ &= 0-1\\ &= -1\end{aligned}$$

Here, it is observed that $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$.

$\therefore \lim_{x\to 0} f(x)$ does not exist.

When $a < 0$,

$$\begin{aligned}\lim_{x\to a^-} f(x) &= \lim_{x\to a^-}(|x|+1)\\ &= \lim_{x\to a}(-x+1) \qquad [x<a<0 \Rightarrow |x|=-x]\\ &= -a+1\end{aligned}$$

$$\begin{aligned}\lim_{x\to a^+} f(x) &= \lim_{x\to a^+}(|x|+1)\\ &= \lim_{x\to a}(-x+1) \qquad [a<x<0 \Rightarrow |x|=-x]\\ &= -a+1\end{aligned}$$

$\therefore \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = -a+1$

Thus, limit of $f(x)$ exists at $x=a$, where $a<0$.

When $a > 0$

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^-}(|x|-1)$$
$$= \lim_{x\to a}(x-1) \qquad [0 < x < a \Rightarrow |x| = x]$$
$$= a-1$$
$$\lim_{x\to a^+} f(x) = \lim_{x\to a^+}(|x|-1)$$
$$= \lim_{x\to a}(x-1) \qquad [0 < a < x \Rightarrow |x| = x]$$
$$= a-1$$
$$\therefore \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = a-1$$

Thus, limit of $f(x)$ exists at $x = a$, where $a > 0$.

Thus, $\lim_{x\to a} f(x)$ exists for all $a \neq 0$.

Question 31:

If the function *f*(*x*) satisfies, $\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$ evaluate $\lim_{x\to 1} f(x)$.

Answer 31:

$$\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$$
$$\Rightarrow \frac{\lim_{x\to 1}(f(x)-2)}{\lim_{x\to 1}(x^2-1)} = \pi$$
$$\Rightarrow \lim_{x\to 1}(f(x)-2) = \pi\lim_{x\to 1}(x^2-1)$$
$$\Rightarrow \lim_{x\to 1}(f(x)-2) = \pi(1^2-1)$$
$$\Rightarrow \lim_{x\to 1}(f(x)-2) = 0$$
$$\Rightarrow \lim_{x\to 1} f(x) - \lim_{x\to 1} 2 = 0$$
$$\Rightarrow \lim_{x\to 1} f(x) - 2 = 0$$
$$\therefore \lim_{x\to 1} f(x) = 2$$

Question 32:

If. $f(x)=\begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1 \\ nx^3+m, & x>1 \end{cases}$ For what integers *m* and *n* does

$\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exist?

Answer 32: The given function is

$$f(x)=\begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1 \\ nx^3+m, & x>1 \end{cases}$$

$$\begin{aligned}\lim_{x\to 0^-} f(x) &= \lim_{x\to 0}\left(mx^2+n\right) \\ &= m(0)^2+n \\ &= n\end{aligned}$$

$$\begin{aligned}\lim_{x\to 0^+} f(x) &= \lim_{x\to 0}(nx+m) \\ &= n(0)+m \\ &= m.\end{aligned}$$

Thus, $\lim_{x\to 0} f(x)$ exists if $m = n$.

$$\begin{aligned}\lim_{x\to 1^-} f(x) &= \lim_{x\to 1}(nx+m) \\ &= n(1)+m \\ &= m+n\end{aligned}$$

$$\begin{aligned}\lim_{x\to 1^+} f(x) &= \lim_{x\to 1}\left(nx^3+m\right) \\ &= n(1)^3+m \\ &= m+n\end{aligned}$$

$$\therefore \lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=\lim_{x\to 1} f(x).$$

Thus $\lim_{x\to 1} f(x)$ exist for any integral value of m and n.