Mathematics

(Chapter – 2) (Relations and Functions)

(Class – XI)

Exercise 2.1

Question 1:

If

$\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$, find the values of *x* and *y*.

Answer 1:

It is given that

$$\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right).$$

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\frac{x}{3}+1=\frac{5}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$ $\quad y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{x}{3}=\frac{2}{3}$ $\quad \Rightarrow y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow x=2$ $\quad \Rightarrow y=1$

∴ $x = 2$ and $y = 1$

Question 2:

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

Answer 2:

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

⇒ Number of elements in set B = 3

Number of elements in (A × B)
= (Number of elements in A) × (Number of elements in B)
= 3 × 3 = 9

Thus, the number of elements in (A × B) is 9.

Question 3:
If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.

Answer 3:
G = {7, 8} and H = {5, 4, 2}
We know that the Cartesian product P × Q of two non-empty sets P and Q is defined as P × Q = {(p, q): $p \in$ P, $q \in$ Q}
∴ G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}
H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

Question 4:
State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.
(i) If P = {m, n} and Q = {n, m}, then P × Q = {(m, n), (n, m)}.

(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (x, y) such that $x \in$ A and $y \in$ B.

(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.

Answer 4:
(i) False
If P = {m, n} and Q = {n, m}, then
P × Q = {(m, m), (m, n), (n, m), (n, n)}
(ii) True
(iii) True

Question 5:
If A = {−1, 1}, find A × A × A.

Answer 5:
It is known that for any non-empty set A, A × A × A is defined as
A × A × A = {(a, b, c): a, b, c ∈ A}

It is given that A = {−1, 1}

∴ A × A × A = {(−1, −1, −1), (−1, −1, 1), (−1, 1, −1), (−1, 1, 1), (1, −1, −1), (1, −1, 1), (1, 1, −1), (1, 1, 1)}

Question 6:
If A × B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.

Answer 6:
It is given that A × B = {(a, x), (a, y), (b, x), (b, y)}
We know that the Cartesian product of two non-empty sets P and Q is defined as P × Q = {(p, q): p ∈ P, q ∈ Q}

∴ A is the set of all first elements and B is the set of all second elements.

Thus, A = {a, b} and B = {x, y}

Question 7:
Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that
(i) A × (B ∩ C) = (A × B) ∩ (A × C)
(ii) A × C is a subset of B × D

Answer 7:
(i) To verify: A × (B ∩ C) = (A × B) ∩ (A × C)

We have B ∩ C = {1, 2, 3, 4} ∩ {5, 6} = Φ

∴ L.H.S. = A × (B ∩ C) = A × Φ = Φ
A × B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

$\therefore$ R.H.S. = (A × B) ∩ (A × C) = Φ

$\therefore$ L.H.S. = R.H.S

Hence, A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) To verify: A × C is a subset of B × D

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
A × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

We can observe that all the elements of set A × C are the elements of set B × D. Therefore, A × C is a subset of B × D.

Question 8:
Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.

Answer 8:
A = {1, 2} and B = {3, 4}

$\therefore$ A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}
⇒ n(A × B) = 4

We know that if C is a set with n(C) = m, then n[P(C)] = 2^m.

Therefore, the set A × B has 2^4 = 16 subsets. These are

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

Question 9:

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1)$, $(y, 2)$, $(z, 1)$ are in $A \times B$, find A and B, where x, y and z are distinct elements.

Answer 9:

It is given that $n(A) = 3$ and $n(B) = 2$; and $(x, 1)$, $(y, 2)$, $(z, 1)$ are in $A \times B$.

We know that
A = Set of first elements of the ordered pair elements of $A \times B$
B = Set of second elements of the ordered pair elements of $A \times B$.

$\therefore$ x, y, and z are the elements of A; and 1 and 2 are the elements of B.

Since $n(A) = 3$ and $n(B) = 2$,

it is clear that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

Question 10:

The Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.

Answer 10:

We know that if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

$\therefore n(A \times A) = n(A) \times n(A)$

It is given that $n(A \times A) = 9$

$\therefore n(A) \times n(A) = 9$
$\Rightarrow n(A) = 3$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of the nine elements of $A \times A$.

We know that $A \times A = \{(a, a): a \in A\}$. Therefore, -1, 0, and 1 are elements of A.
Since $n(A) = 3$, it is clear that $A = \{-1, 0, 1\}$.
The remaining elements of set $A \times A$ are $(-1, -1)$, $(-1, 1)$, $(0, -1)$, $(0, 0)$, $(1, -1)$, $(1, 0)$, and $(1, 1)$.