Mathematics

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(Chapter – 2) (Relations and Functions)

(Class – XI)

Exercise 2.3

Question 1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
(iii) {(1, 3), (1, 5), (2, 5)}

Answer 1:

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}
Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function. Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}
(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}
Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.
Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}
(iii) {(1, 3), (1, 5), (2, 5)}
Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:

Find the domain and range of the following real function:
(i) $f(x) = -|x|$
(ii) $f(x) = \sqrt{9 - x^2}$

Answer 2:

(i) $f(x) = -|x|$, $x \in$ R

We know that $|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$

$$\therefore\ f(x) = -|x| = \begin{cases} -x, & if\ x \geq 0 \\ x, & if\ x < 0 \end{cases}$$

Since $f(x)$ is defined for $x \in$ **R**, the domain of f is **R.**

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.
∴ The range of f is $(-\infty, 0]$.

(ii) $f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9 - x^2}$ is defined for all real numbers that are greater than or equal to −3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \le x \le 3\}$ or $[-3, 3]$.
For any value of x such that $-3 \le x \le 3$, the value of $f(x)$ will lie between 0 and 3. ∴The range of $f(x)$ is $\{x: 0 \le x \le 3\}$ or $[0, 3]$.

Question 3:
A function f is defined by $f(x) = 2x - 5$. Write down the values of
(i) $f(0)$,
(ii) $f(7)$,
(iii) $f(-3)$

Answer 3:
The given function is $f(x) = 2x - 5$.
Therefore,
(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$
(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$
(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Question 4:
The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $f(C) = \frac{9C}{5} + 32$. Find

(i) t (0) **(ii)** t (28) **(iii)** t (−10)
(iv) The value of C, when t(C) = 212

Answer 4:
The given function is $f(C) = \frac{9C}{5} + 32$.
Therefore,

(i) $t(0)=\frac{9\times 0}{5}+32=0+32=32$

(ii) $t(28)=\frac{9\times 28}{5}+32=\frac{252+160}{5}=\frac{412}{5}$

(iii) $t(-10)=\frac{9\times(-10)}{5}+32=9\times(-2)+32=-18+32=14$

(iv) It is given that $t(C) = 212$

$$\therefore 212=\frac{9C}{5}+32$$
$$\Rightarrow \frac{9C}{5}=212-32$$
$$\Rightarrow \frac{9C}{5}=180$$
$$\Rightarrow 9C=180\times 5$$
$$\Rightarrow C=\frac{180\times 5}{9}=100$$

Thus, the value of t, when $t(C) = 212$, is 100.

Question 5:

Find the range of each of the following functions.

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$.

(ii) $f(x) = x^2 + 2$, x, is a real number.

(iii) $f(x) = x$, x is a real number

Answer 5:

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	…
$f(x)$	1.97	1.7	–0.7	–1	–4	–5.5	–10	–13	…

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2.
i.e., range of $f = (-\infty, 2)$

Alter:
Let $x > 0$
$\Rightarrow 3x > 0$
$\Rightarrow 2 - 3x < 2$
$\Rightarrow f(x) < 2$
$\therefore$Range of $f = (-\infty, 2)$
(ii) $f(x) = x^2 + 2$, x, is a real number
The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	**0**	**±0.3**	**±0.8**	**±1**	**±2**	**±3**		**...**
***f*(*x*)**	2	2.09	2.64	3	6	11		.....

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2.
i.e., range of $f = [2, \infty)$

Alter:
Let x be any real
number. Accordingly,
$x^2 \geq 0$
$\Rightarrow x^2 + 2 \geq 0 + 2$
$\Rightarrow x^2 + 2 \geq 2$
$\Rightarrow f(x) \geq 2$
$\therefore$ Range of $f = [2, \infty)$
(iii) $f(x) = x$, x is a real number
It is clear that the range of f is the set of all real
numbers. $\therefore$ Range of f = **R**