Exercise 3.1

Question 1:

Find the radian measures corresponding to the following degree measures:

(i) 25° **(ii)** – 47° 30' **(iii)** 240° **(iv)** 520°

Answer 1:

(i) 25°

We know that 180° = π radian

$$\therefore 25° = \frac{\pi}{180} \times 25 \text{ radian} = \frac{5\pi}{36} \text{ radian}$$

(ii) –47° 30'

–47° 30' $-47\frac{1}{2}$

$= \frac{-95}{2}$ degree

Since 180° = π radian

$$\frac{-95}{2} \text{ degree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian} = \left(\frac{-19}{36 \times 2}\right)\pi \text{ radian} = \frac{-19}{72}\pi \text{ radian}$$

$$\therefore -47° 30' = \frac{-19}{72}\pi \text{ radian}$$

(iii) 240°

We know that 180° = π radian

$$\therefore 240° = \frac{\pi}{180} \times 240 \text{ radian} = \frac{4}{3}\pi \text{ radian}$$

(iv) 520°

We know that 180° = π radian

$$\therefore 520° = \frac{\pi}{180} \times 520 \text{ radian} = \frac{26\pi}{9} \text{ radian}$$

Question 2:

Find the degree measures corresponding to the following radian measures $\left(\text{Use } \pi = \frac{22}{7}\right)$

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(i) $\frac{11}{16}$ **(ii)** – 4 **(iii)** $\frac{5\pi}{3}$ **(iv)** $\frac{7\pi}{6}$

Answer 2:

(i) $\frac{11}{16}$

We know that π radian = 180°

$$\therefore \frac{11}{16}\text{ radain} = \frac{180}{\pi}\times\frac{11}{16}\text{ degree} = \frac{45\times 11}{\pi\times 4}\text{degree}$$

$$= \frac{45\times 11\times 7}{22\times 4}\text{ degree} = \frac{315}{8}\text{ degree}$$

$$= 39\frac{3}{8}\text{ degree}$$

$$= 39^\circ + \frac{3\times 60}{8}\text{ minutes} \qquad [1^\circ = 60']$$

$$= 39^\circ + 22' + \frac{1}{2}\text{ minutes}$$

$$= 39^\circ 22' 30'' \qquad [1' = 60'']$$

(ii) – 4

We know that π radian = 180°

$$-4\text{ radian} = \frac{180}{\pi}\times(-4)\text{ degree} = \frac{180\times 7(-4)}{22}\text{ degree}$$

$$= \frac{-2520}{11}\text{ degree} = -229\frac{1}{11}\text{ degree}$$

$$= -229^\circ + \frac{1\times 60}{11}\text{ minutes} \qquad [1^\circ = 60']$$

$$= -229^\circ + 5' + \frac{5}{11}\text{ minutes}$$

$$= -229^\circ 5' 27'' \qquad [1' = 60'']$$

(iii) $\frac{5\pi}{3}$

We know that π radian = 180°

$$\therefore \frac{5\pi}{3}\text{radian} = \frac{180}{\pi}\times\frac{5\pi}{3}\text{ degree} = 300^\circ$$

(iv) $\frac{7\pi}{6}$

We know that π radian = 180°

$$\therefore \frac{7\pi}{6}\text{ radian} = \frac{180}{\pi} \times \frac{7\pi}{6} = 210°$$

Question 3:

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Answer 3:

Number of revolutions made by the wheel in 1 minute = 360

∴ Number of revolutions made by the wheel in 1 second = $\frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of 6 × 2π radian, i.e., 12 π radian

Thus, in one second, the wheel turns an angle of 12π radian.

Question 4:

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm.

$\left(\text{Use } \pi = \frac{22}{7}\right)$

Answer 4:

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle *θ* radian at the centre, then

$$\theta = \frac{l}{r}$$

Therefore, forr = 100 cm, l = 22 cm, we have

$$\theta = \frac{22}{100}\ \text{radian} = \frac{180}{\pi} \times \frac{22}{100}\ \text{deg ree} = \frac{180 \times 7 \times 22}{22 \times 100}\ \text{deg ree}$$

$$= \frac{126}{10}\ \text{deg ree} = 12\frac{3}{5}\ \text{deg ree} = 12°36' \qquad [1° = 60']$$

Thus, the required angle is 12°36’.

Question 5:

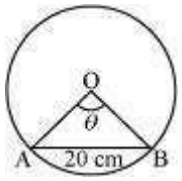
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Answer 5:

Diameter of the circle = 40 cm

∴Radius (*r*) of the circle = $\frac{40}{2}\ \text{cm} = 20\ \text{cm}$

Let AB be a chord (length = 20 cm) of the circle.



In ΔOAB, OA = OB = Radius of circle = 20 cm

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

$\therefore \theta = 60° = \frac{\pi}{3}$ radian

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle θ

$$\theta = \frac{l}{r}$$

$$\frac{\pi}{3} = \frac{\overset{\frown}{AB}}{20} \Rightarrow \overset{\frown}{AB} = \frac{20\pi}{3} \text{ cm}$$

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3}$ cm

Question 6:

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Answer 6:

Let the radii of the two circles be r_1 and r_2. Let an arc of length *l* subtend an angle of 60° at the centre of the circle of radius r_1, while let an arc of length *l* subtend an angle of 75° at the centre of the circle of radius r_2.

Now, 60°= $\frac{\pi}{3}$ radian and 75° = $\frac{5\pi}{12}$ radian

We know that in a circle of radius *r* unit, if an arc of length *l* unit subtends an angle θ

$$\theta = \frac{l}{r} \text{ or } l = r\theta$$

$$\therefore l = \frac{r_1\pi}{3} \text{ and } l = \frac{r_2 5\pi}{12}$$

$$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$$

$$\Rightarrow r_1 = \frac{r_2 5}{4}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$$

Thus, the ratio of the radii is 5:4.

Question 7:

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm **(ii)** 15 cm **(iii)** 21 cm

Answer 7:

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$

It is given that r = 75 cm

(i) Here, l = 10 cm

$$\theta = \frac{10}{75} \text{ radian} = \frac{2}{15} \text{ radian}$$

(ii) Here, l = 15 cm

$$\theta = \frac{15}{75} \text{ radian} = \frac{1}{5} \text{ radian}$$

(iii) Here, l = 21 cm

$$\theta = \frac{21}{75} \text{ radian} = \frac{7}{25} \text{ radian}$$