Exercise 3.4

Question 1:

Find the principal and general solutions of the equation $\tan x = \sqrt{3}$

Answer 1:

$\tan x = \sqrt{3}$

It is known that $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now, $\tan x = \tan\frac{\pi}{3}$

$\Rightarrow x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{\pi}{3}$, where $n \in Z$

Question 2:

Find the principal and general solutions of the equation $\sec x = 2$

Answer 2:

$\sec x = 2$

It is known that $\sec\frac{\pi}{3} = 2$ and $\sec\frac{5\pi}{3} = \sec\left(2\pi - \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$

Therefore, the principal solutions are $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Now, $\sec x = \sec\frac{\pi}{3}$

$\Rightarrow \cos x = \cos\frac{\pi}{3}$ $\qquad \left[\sec x = \frac{1}{\cos x}\right]$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

Therefore, the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in \mathbf{Z}$

Question 3:

Find the principal and general solutions of the equation $\cot x = -\sqrt{3}$

Answer 3:

$\cot x = -\sqrt{3}$

It is known that $\cot \frac{\pi}{6} = \sqrt{3}$

$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$ and $\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6} = -\sqrt{3}$

i.e., $\cot\frac{5\pi}{6} = -\sqrt{3}$ and $\cot\frac{11\pi}{6} = -\sqrt{3}$

Therefore, the principal solutions are $x = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now, $\cot x = \cot\frac{5\pi}{6}$

$\Rightarrow \tan x = \tan\frac{5\pi}{6}$ $\qquad \left[\cot x = \frac{1}{\tan x}\right]$

$\Rightarrow x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

Therefore, the general solution is $x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

Question 4:

Find the general solution of cosec x = −2

Answer 4:

cosec x = −2

It is known that

$$\operatorname{cosec}\frac{\pi}{6} = 2$$

$$\therefore \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2 \text{ and } \operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

$$\text{i.e., } \operatorname{cosec}\frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec}\frac{11\pi}{6} = -2$$

Therefore, the principal solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

$$\text{Now, } \operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$

$$\Rightarrow \sin x = \sin\frac{7\pi}{6} \qquad \left[\operatorname{cosec} x = \frac{1}{\sin x}\right]$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

Question 5:

Find the general solution of the equation $\cos 4x = \cos 2x$

Answer 5:

$$\cos 4x = \cos 2x$$
$$\Rightarrow \cos 4x - \cos 2x = 0$$
$$\Rightarrow -2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$$
$$\left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$
$$\Rightarrow \sin 3x \sin x = 0$$
$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad \sin x = 0$$
$$\therefore 3x = n\pi \quad \text{or} \quad x = n\pi, \text{ where } n \in Z$$
$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = n\pi, \text{ where } n \in Z$$

Question 6:

Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$

Answer 6:

$$\cos 3x + \cos x - \cos 2x = 0$$
$$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$
$$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$$
$$\Rightarrow \cos 2x(2\cos x - 1) = 0$$
$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$
$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$
$$\therefore 2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos\frac{\pi}{3}, \text{ where } n \in Z$$
$$\Rightarrow x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Question 7:

Find the general solution of the equation $\sin 2x + \cos x = 0$

Answer 7:

$$\sin 2x + \cos x = 0$$
$$\Rightarrow 2\sin x \cos x + \cos x = 0$$
$$\Rightarrow \cos x(2\sin x + 1) = 0$$
$$\Rightarrow \cos x = 0 \quad \text{or} \quad 2\sin x + 1 = 0$$
$$\text{Now, } \cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}, \text{ where } n \in Z$$
$$2\sin x + 1 = 0$$
$$\Rightarrow \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{7\pi}{6}$$
$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in Z$$

Therefore, the general solution is $(2n+1)\frac{\pi}{2}$ or $n\pi + (-1)^n \frac{7\pi}{6}, n \in Z$

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Question 8:

Find the general solution of the equation $\sec^2 2x = 1 - \tan 2x$

Answer 8:

$$\sec^2 2x = 1 - \tan 2x$$
$$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$$
$$\Rightarrow \tan^2 2x + \tan 2x = 0$$
$$\Rightarrow \tan 2x(\tan 2x + 1) = 0$$
$$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$$

$$\text{Now, } \tan 2x = 0$$
$$\Rightarrow \tan 2x = \tan 0$$
$$\Rightarrow 2x = n\pi + 0, \text{ where } n \in Z$$
$$\Rightarrow x = \frac{n\pi}{2}, \text{ where } n \in Z$$
$$\tan 2x + 1 = 0$$
$$\Rightarrow \tan 2x = -1 = -\tan\frac{\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$$
$$\Rightarrow 2x = n\pi + \frac{3\pi}{4}, \text{ where } n \in Z$$
$$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}, \text{ where } n \in Z$$

Therefore, the general solution is $\frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{3\pi}{8}, n \in Z$

Question 9:

Find the general solution of the equation $\sin x + \sin 3x + \sin 5x = 0$

Answer 9:

$$\sin x + \sin 3x + \sin 5x = 0$$

$$(\sin x + \sin 5x) + \sin 3x = 0$$

$$\Rightarrow \left[2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right] + \sin 3x = 0 \qquad \left[\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$\Rightarrow 2\sin 3x \cos(-2x) + \sin 3x = 0$$

$$\Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$$

$$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2\cos 2x + 1 = 0$$

$$\text{Now, } \sin 3x = 0 \Rightarrow 3x = n\pi, \text{ where } n \in Z$$

$$\text{i.e., } x = \frac{n\pi}{3}, \text{ where } n \in Z$$

$$2\cos 2x + 1 = 0$$

$$\Rightarrow \cos 2x = \frac{-1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right)$$

$$\Rightarrow \cos 2x = \cos\frac{2\pi}{3}$$

$$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in Z$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in Z$$

Therefore, the general solution is $\frac{n\pi}{3}$ or $n\pi \pm \frac{\pi}{3}, n \in Z$