Mathematics

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(Chapter – 5) (Complex Numbers and Quadratic Equations)

(Class – XI)

Exercise 5.1

Question 1:

Express the given complex number in the form $a + ib$: $(5i)\left(-\frac{3}{5}i\right)$

Answer 1:

$$(5i)\left(\frac{-3}{5}i\right) = -5 \times \frac{3}{5} \times i \times i$$
$$= -3i^2$$
$$= -3(-1) \qquad [i^2 = -1]$$
$$= 3$$

Question 2:

Express the given complex number in the form $a + ib$: $i^9 + i^{19}$

Answer 2:

$$i^9 + i^{19} = i^{4\times 2+1} + i^{4\times 4+3}$$
$$= (i^4)^2 \cdot i + (i^4)^4 \cdot i^3$$
$$= 1 \times i + 1 \times (-i) \qquad [i^4 = 1,\ i^3 = -i]$$
$$= i + (-i)$$
$$= 0$$

Question 3:

Express the given complex number in the form $a + ib$: i^{-39}

Answer 3:

$$i^{-39} = i^{-4\times 9-3} = \left(i^4\right)^{-9}\cdot i^{-3}$$

$$= (1)^{-9}\cdot i^{-3} \qquad \left[i^4 = 1\right]$$

$$= \frac{1}{i^3} = \frac{1}{-i} \qquad \left[i^3 = -i\right]$$

$$= \frac{-1}{i}\times\frac{i}{i}$$

$$= \frac{-i}{i^2} = \frac{-i}{-1} = i \qquad \left[i^2 = -1\right]$$

Question 4:

Express the given complex number in the form $a + ib$:

$3(7 + i7) + i(7 + i7)$

Answer 4:

$$3(7+i7)+i(7+i7) = 21+21i+7i+7i^2$$

$$= 21+28i+7\times(-1) \qquad \left[\because i^2 = -1\right]$$

$$= 14+28i$$

Question 5:

Express the given complex number in the form $a + ib$: $(1 – i) – (–1 + i6)$

Answer 5:

$$(1-i)-(-1+i6) = 1-i+1-6i$$

$$= 2-7i$$

Question 6:

Express the given complex number in the form $a + ib$: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Answer 6:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

Question 7:

Express the given complex number in the form $a + ib$:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$$

Answer 7:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Question 8:

Express the given complex number in the form $a + ib$: $(1 - i)^4$

Answer 8:

$$(1-i)^4 = \left[(1-i)^2\right]^2$$
$$= \left[1^2 + i^2 - 2i\right]^2$$
$$= \left[1-1-2i\right]^2$$
$$= (-2i)^2$$
$$= (-2i)\times(-2i)$$
$$= 4i^2 = -4 \qquad [i^2 = -1]$$

Question 9:

Express the given complex number in the form $a + ib$: $\left(\frac{1}{3}+3i\right)^3$

Answer 9:

$$\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$$
$$= \frac{1}{27} + 27(-i) + i + 9i^2 \qquad [i^3 = -i]$$
$$= \frac{1}{27} - 27i + i - 9 \qquad [i^2 = -1]$$
$$= \left(\frac{1}{27} - 9\right) + i(-27+1)$$
$$= \frac{-242}{27} - 26i$$

Question 10:

Express the given complex number in the form $a + ib$: $\left(-2-\frac{1}{3}i\right)^3$

Answer 10:

$$\left(-2-\frac{1}{3}i\right)^3 = (-1)^3\left(2+\frac{1}{3}i\right)^3$$

$$= -\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$

$$= -\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$

$$= -\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad [i^3 = -i]$$

$$= -\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad [i^2 = -1]$$

$$= -\left[\frac{22}{3}+\frac{107i}{27}\right]$$

$$= -\frac{22}{3}-\frac{107}{27}i$$

Question 11:

Find the multiplicative inverse of the complex number $4 - 3i$.

Answer 11:

Let $z = 4 - 3i$

Then,

$\bar{z} = 4 + 3i$ and $|z|^2 = 4^2+(-3)^2 = 16+9 = 25$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25}+\frac{3}{25}i$$

Question 12:

Find the multiplicative inverse of the complex number $\sqrt{5}+3i$

Answer 12:

Let $z = \sqrt{5}+3i$

Then, $\bar{z} = \sqrt{5}-3i$ and $|z|^2 = \left(\sqrt{5}\right)^2 + 3^2 = 5+9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5}+3i$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5}-3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

Question 13:

Find the multiplicative inverse of the complex number $-i$

Answer 13:

Let $z = -i$

Then, $\bar{z} = i$ and $|z|^2 = 1^2 = 1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

Question 14:

Express the following expression in the form of $a + ib$.

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Answer 14:

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$= \frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad [(a+b)(a-b)=a^2-b^2]$$

$$= \frac{9-5i^2}{2\sqrt{2}i}$$

$$= \frac{9-5(-1)}{2\sqrt{2}i} \qquad [i^2=-1]$$

$$= \frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$= \frac{14i}{2\sqrt{2}i^2}$$

$$= \frac{14i}{2\sqrt{2}(-1)}$$

$$= \frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}i}{2}$$