Mathematics

(Chapter – 9) (Sequences and Series)

(Class – XI)

Exercise 9.1

Question 1:

Write the first five terms of the sequences whose nth term is $a_n = n(n + 2)$.

Answer 1:

$a_n = n(n + 2)$

Substituting n = 1, 2, 3, 4, and 5, we obtain

$a_1 = 1(1+2) = 3$

$a_2 = 2(2+2) = 8$

$a_3 = 3(3+2) = 15$

$a_4 = 4(4+2) = 24$

$a_5 = 5(5+2) = 35$

Therefore, the required terms are 3, 8, 15, 24, and 35.

Question 2:

Write the first five terms of the sequences whose nth term is $a_n = \frac{n}{n+1}$

Answer 2:

$a_n = \frac{n}{n+1}$

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = \frac{1}{1+1} = \frac{1}{2},\ a_2 = \frac{2}{2+1} = \frac{2}{3},\ a_3 = \frac{3}{3+1} = \frac{3}{4},\ a_4 = \frac{4}{4+1} = \frac{4}{5},\ a_5 = \frac{5}{5+1} = \frac{5}{6}$$

Therefore, the required terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, and $\frac{5}{6}$

Question 3:

Write the first five terms of the sequences whose n^{th} term is $a_n = 2^n$

Answer 3:

$a_n = 2^n$

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = 2^1 = 2$$
$$a_2 = 2^2 = 4$$
$$a_3 = 2^3 = 8$$
$$a_4 = 2^4 = 16$$
$$a_5 = 2^5 = 32$$

Therefore, the required terms are 2, 4, 8, 16, and 32.

Question 4:

Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{2n-3}{6}$

Answer 4:

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$$
$$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$$
$$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$$
$$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$$
$$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$$

Therefore, the required terms are. $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$, and $\frac{7}{6}$

Question 5:

Write the first five terms of the sequences whose n^{th} term is $a_n = (-1)^{n-1} 5^{n+1}$

Answer 5:

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Therefore, the required terms are 25, –125, 625, –3125, and 15625.

Question 6:

Write the first five terms of the sequences whose n^{th} term is $a_n = n\frac{n^2+5}{4}$

Answer 6:

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = 1\cdot\frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$$
$$a_2 = 2\cdot\frac{2^2+5}{4} = 2\cdot\frac{9}{4} = \frac{9}{2}$$
$$a_3 = 3\cdot\frac{3^2+5}{4} = 3\cdot\frac{14}{4} = \frac{21}{2}$$
$$a_4 = 4\cdot\frac{4^2+5}{4} = 21$$
$$a_5 = 5\cdot\frac{5^2+5}{4} = 5\cdot\frac{30}{4} = \frac{75}{2}$$

Therefore, the required terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21,$ and $\frac{75}{2}$.

Question 7:

Find the 17th term in the following sequence whose n^{th} term is

$a_n = 4n - 3; a_{17}, a_{24}$

Answer 7:

Substituting n = 17, we obtain

$a_{17} = 4(17) - 3 = 68 - 3 = 65$

Substituting n = 24, we obtain

$a_{24} = 4(24) - 3 = 96 - 3 = 93$

Question 8:

Find the 7th term in the following sequence whose n^{th} term is $a_n = \frac{n^2}{2n}; a_7$

Answer 8:

Substituting n = 7, we obtain

$a_7 = \frac{7^2}{2^7} = \frac{49}{128}$

Question 9:

Find the 9th term in the following sequence whose n^{th} term is

$a_n = (-1)^{n-1} n^3; a_9$

Answer 9:

Substituting n = 9, we obtain

$a_9 = (-1)^{9-1}(9)^3 = (9)^3 = 729$

Question 10:

Find the 20th term in the following sequence whose n^{th} term is

$$a_n = \frac{n(n-2)}{n+3}; a_{20}$$

Answer 10:

Substituting n = 20, we obtain

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20(18)}{23} = \frac{360}{23}$$

Question 11:

Write the first five terms of the following sequence and obtain the corresponding series:

$$a_1 = 3, a_n = 3a_{n-1} + 2 \text{ for all } n > 1$$

Answer 11:

$$a_1 = 3, a_n = 3a_{n-1} + 2 \text{ for all } n > 1$$

$$\Rightarrow a_2 = 3a_1 + 2 = 3(3) + 2 = 11$$

$$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$$

$$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$$

$$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$$

Hence, the first five terms of the sequence are 3, 11, 35, 107, and 323.

The corresponding series is 3 + 11 + 35 + 107 + 323 + …

Question 12:

Write the first five terms of the following sequence and obtain the corresponding series:

$$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$$

Answer 12:

$$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$$

$$\Rightarrow a_2 = \frac{a_1}{2} = \frac{-1}{2}$$

$$a_3 = \frac{a_2}{3} = \frac{-1}{6}$$

$$a_4 = \frac{a_3}{4} = \frac{-1}{24}$$

$$a_5 = \frac{a_4}{4} = \frac{-1}{120}$$

Hence, the first five terms of the sequence are $-1, \frac{-1}{2}, \frac{-1}{6}, \frac{-1}{24}$, and $\frac{-1}{120}$.

The corresponding series is $(-1)+\left(\frac{-1}{2}\right)+\left(\frac{-1}{6}\right)+\left(\frac{-1}{24}\right)+\left(\frac{-1}{120}\right)+\ldots$

Question 13:

Write the first five terms of the following sequence and obtain the corresponding series:

$$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$$

Answer 13:

$$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$$

$$\Rightarrow a_3 = a_2 - 1 = 2 - 1 = 1$$
$$a_4 = a_3 - 1 = 1 - 1 = 0$$
$$a_5 = a_4 - 1 = 0 - 1 = -1$$

Hence, the first five terms of the sequence are 2, 2, 1, 0, and −1.

The corresponding series is 2 + 2 + 1 + 0 + (−1) + …

Question 14:

The Fibonacci sequence is defined by $1 = a_1 = a_2 \text{ and } a_n = a_{n-1} + a_{n-2}, n > 2$

Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5

Answer 14:

$$1 = a_1 = a_2$$
$$a_n = a_{n-1} + a_{n-2}, n > 2$$
$$\therefore a_3 = a_2 + a_1 = 1 + 1 = 2$$
$$a_4 = a_3 + a_2 = 2 + 1 = 3$$
$$a_5 = a_4 + a_3 = 3 + 2 = 5$$
$$a_6 = a_5 + a_4 = 5 + 3 = 8$$

$$\therefore \text{For } n = 1, \ \frac{a_n + 1}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$$

$$\text{For } n = 2, \ \frac{a_n + 1}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2$$

$$\text{For } n = 3, \frac{a_n + 1}{a_n} = \frac{a_4}{a_3} = \frac{3}{2}$$

$$\text{For } n = 4, \ \frac{a_n + 1}{a_n} = \frac{a_5}{a_4} = \frac{5}{3}$$

$$\text{For } n = 5, \ \frac{a_n + 1}{a_n} = \frac{a_6}{a_5} = \frac{8}{5}$$