Question 10.21:

A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. compute the force necessary to keep the door close.

Answer 10.21:

Base area of the given tank, $A = 1.0\ m^2$
Area of the hinged door, $a = 20\ cm^2 = 20 \times 10^{-4}\ m^2$
Density of water, $\rho_1 = 10^3\ kg/m^3$
Density of acid, $\rho_2 = 1.7 \times 10^3\ kg/m^3$
Height of the water column, $h_1 = 4$ m
Height of the acid column, $h_2 = 4$ m
Acceleration due to gravity, g = 9.8 Pressure due to water is given as:

$$P_1 = h_1 \rho_1 g$$
$$= 4 \times 10^3 \times 9.8$$
$$= 3.92 \times 10^4 \text{ Pa}$$

Pressure due to acid is given as:

$$P_2 = h_2 \rho_2 g$$
$$= 4 \times 1.7 \times 10^3 \times 9.8$$
$$= 6.664 \times 10^4 \text{ Pa}$$

Pressure difference between the water and acid columns:

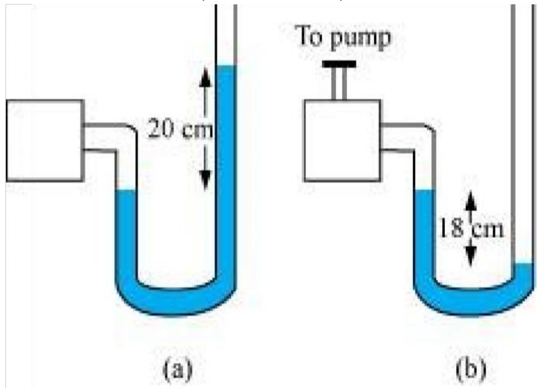
$$\Delta P = P_2 - P_1$$
$$= 6.664 \times 10^4 - 3.92 \times 10^4$$
$$= 2.744 \times 10^4 \text{ Pa}$$

Hence, the force exerted on the door = $\Delta P \times a$
$= 2.744 \times 10^4 \times 20 \times 10^{-4}$
= 54.88 N
Therefore, the force necessary to keep the door closed is 54.88 N.

Question 10.22:

A manometer reads the pressure of a gas in an enclosure as shown in Fig. 10.25 (a) When a pump removes some of the gas, the manometer reads as in Fig. 10.25 (b) The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.

a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.
b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in the volume of the gas).



Answer 10.22:

(a) 96 cm of Hg & 20 cm of Hg; 58 cm of Hg & –18 cm of Hg
19 cm
For figure (a)
Atmospheric pressure, P_0 = 76 cm of Hg
Difference between the levels of mercury in the two limbs gives gauge pressure
Hence, gauge pressure is 20 cm of Hg.
Absolute pressure = Atmospheric pressure + Gauge pressure
= 76 + 20 = 96 cm of Hg
For figure (b)
Difference between the levels of mercury in the two limbs = –18 cm
Hence, gauge pressure is –18 cm of Hg.
Absolute pressure = Atmospheric pressure + Gauge pressure
= 76 cm – 18 cm = 58 cm
13.6 cm of water is poured into the right limb of figure (b).
Relative density of mercury = 13.6
Hence, a column of 13.6 cm of water is equivalent to 1 cm of mercury.

b) Let h be the difference between the levels of mercury in the two limbs.
The pressure in the right limb is given as:
$P_R =$ Atmospheric pressure + 1 cm of Hg
= 76 + 1 = 77 cm of Hg ……………… (i)
The mercury column will rise in the left limb.
Hence, pressure in the left limb, $P_L = 58 + h$ ... (ii)
Equating equations (i) and (ii), we get:
77 = 58 + h
$\therefore h$ = 19 cm Hence, the difference between the levels of mercury in the two limbs will be 19 cm.

Question 10.23:

Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?

Answer 10.23:

Yes

Two vessels having the same base area have identical force and equal pressure acting on their common base area. Since the shapes of the two vessels are different, the force exerted on the sides of the vessels has non-zero vertical components. When these vertical components are added, the total force on one vessel comes out to be greater than that on the other vessel. Hence, when these vessels are filled with water to the same height, they give different readings on a weighing scale.

Question 10.24:

During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? [Use the density of whole blood from Table 10.1].

Answer 10.24:

Gauge pressure, P = 2000 Pa

Density of whole blood, $\rho = 1.06 \times 10^3$ kg m^{-3}

Acceleration due to gravity, $g = 9.8$ m/s^2

Height of the blood container = h

Pressure of the blood container, $P = h\rho g$

$$\therefore h = \frac{P}{\rho g}$$

$$= \frac{2000}{1.06 \times 10^3 \times 9.8}$$

$$= 0.1925 \text{ m}$$

The blood may enter the vein if the blood container is kept at a height greater than 0.1925 m, i.e., about 0.2 m.

Question 10.25:

In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) What is the largest average velocity of blood flow in an artery of diameter 2×10^{-3} m if the flow must remain laminar? (b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.

Answer 10.25:

(a) 1.966 m/s (b) Yes

Diameter of the artery, $d = 2 \times 10^{-3}$ m

Viscosity of blood, $\eta = 2.084 \times 10^{-3}$ Pa s

Density of blood, $\rho = 1.06 \times 10^{3}$ kg/m^3

Reynolds' number for laminar flow, $N_R = 2000$

The largest average velocity of blood is given as:

$$V_{\text{arg}} = \frac{N_R \eta}{\rho d}$$

$$= \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^{3} \times 2 \times 10^{-3}}$$

$$= 1.966 \text{ m/s}$$

Therefore, the largest average velocity of blood is 1.966 m/s.

As the fluid velocity increases, the dissipative forces become more important. This is because of the rise of turbulence. Turbulent flow causes dissipative loss in a fluid.

Question 10.26:

What is the largest average velocity of blood flow in an artery of radius 2×10^{-3} m if the flow must remain laminar? (b) What is the corresponding flow rate? (Take viscosity of blood to be 2.084×10^{-3} Pa s).

Answer 10.26:

Radius of the artery, $r = 2 \times 10^{-3}$ m

Diameter of the artery, $d = 2 \times 2 \times 10^{-3}$ m $= 4 \times 10^{-3}$ m

Viscosity of blood, $\eta = 2.084 \times 10^{-3}$ Pa s

Density of blood, $\rho = 1.06 \times 10^{3}$ kg/m^3

Reynolds' number for laminar flow, $N_R = 2000$

The largest average velocity of blood is given by the relation:

$$V_{\text{arg}} = \frac{N_R \eta}{\rho d}$$

$= \dfrac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^{3} \times 4 \times 10^{-3}}$

$= 0.983$ m/s

Therefore, the largest average velocity of blood is 0.983 m/s.
Flow rate is given by the relation:

$R = \pi r^2 V_{avg}$

$= 3.14 \times \left(2 \times 10^{-3}\right)^2 \times 0.983$

$= 1.235 \times 10^{-5}\ m^3\,s^{-1}$

Therefore, the corresponding flow rate is $1.235 \times 10^{-5}\ m^3\,s^{-1}$.

Question 10.27:

A plane is in level flight at constant speed and each of its two wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg m^{-3}).

Answer 10.27:

The area of the wings of the plane, $A = 2 \times 25 = 50\ m^2$

Speed of air over the lower wing, $V_1 = 180$ km/h = 50 m/s Speed of air over the upper wing, $V_2 = 234$ km/h = 65 m/s

Density of air, $\rho = 1$ kg m^{-3}

Pressure of air over the lower wing = P_1

Pressure of air over the upper wing= P_2

The upward force on the plane can be obtained using Bernoulli's equation as:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) \qquad \dots (i)$$

The upward force (F) on the plane can be calculated as:

$(P_1 - P_2)A$

$= \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)A$ Using equation (i)

$= \frac{1}{2} \times 1 \times \left((65)^2 - (50)^2\right) \times 50$

$= 43125$ N

Using Newton's force equation, we can obtain the mass (m) of the plane as:

$F = mg$

$\therefore m = \frac{43125}{9.8}$

$= 4400.51$ kg

~ 4400 kg

Hence, the mass of the plane is about 4400 kg.

Question 10.28:

In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^{3} kg m^{-3}? Take the viscosity of air at the temperature of the experiment to be 1.8×10^{-5} Pa s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

Answer 10.28:

Terminal speed = 5.8 cm/s; Viscous force = 3.9×10^{-10} N

Radius of the given uncharged drop, $r = 2.0 \times 10^{-5}$ m

Density of the uncharged drop, $\rho = 1.2 \times 10^{3}$ kg m^{-3}

Viscosity of air, $\eta = 1.8 \times 10^{-5}$ Pa s

Density of air (ρ_o) can be taken as zero in order to neglect buoyancy of air.

Acceleration due to gravity, g = 9.8 m/s^2

Terminal velocity (v) is given by the relation:

$$v = \frac{2r^2 \times (\rho - \rho_0)g}{9\eta}$$

$$= \frac{2 \times (2.0 \times 10^{-5})^2 (1.2 \times 10^3 - 0) \times 9.8}{9 \times 1.8 \times 10^{-5}}$$

$$= 5.807 \times 10^{-2}\ \text{m s}^{-1}$$
$$= 5.8\ \text{cm s}^{-1}$$

Hence, the terminal speed of the drop is 5.8 cm s^{-1}.

The viscous force on the drop is given by:

$$F = 6\pi\eta rv$$
$$\therefore F = 6 \times 3.14 \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-5} \times 5.8 \times 10^{-2}$$
$$= 3.9 \times 10^{-10}\ \text{N}$$

Hence, the viscous force on the drop is 3.9×10^{-10} N.

Question 10.29:

Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 N m^{-1}. Density of mercury = 13.6×10^3 kg m^{-3}.

Answer 10.29:

Angle of contact between mercury and soda lime glass, $\theta = 140°$

Radius of the narrow tube, r = 1 mm = 1×10^{-3} m

Surface tension of mercury at the given temperature, s = 0.465 N m^{-1}

Density of mercury, $\rho = 13.6 \times 10^3$ kg/m^3

Dip in the height of mercury = h

Acceleration due to gravity, g = 9.8 m/s^2

Surface tension is related with the angle of contact and the dip in the height as:

$$s = \frac{h\rho gr}{2\cos\theta}$$

$$\therefore h = \frac{2s\cos\theta}{r\rho g}$$

$$= \frac{2 \times 0.465 \times \cos 140}{1 \times 10^{-3} \times 13.6 \times 10^{3} \times 9.8}$$

$$= -0.00534 \text{ m}$$
$$= -5.34 \text{ mm}$$

Here, the negative sign shows the decreasing level of mercury. Hence, the mercury level dips by 5.34 mm.

Question 10.30:

Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} N m^{-1}. Take the angle of contact to be zero and density of water to be 1.0×10^{3} kg m^{-3} (g = 9.8 m s^{-2}).

Answer 10.30:

Diameter of the first bore, d_1 = 3.0 mm = 3×10^{-3} m

Hence, the radius of the first bore, $r_1 = \frac{d_1}{2} = 1.5 \times 10^{-3}$ m

Diameter of the second bore, d_2 = 6.0 mm

Hence, the radius of the second bore, $r_2 = \frac{d_2}{2} = 3 \times 10^{-3}$ m

Surface tension of water, $s = 7.3 \times 10^{-2}$ N m^{-1}

Angle of contact between the bore surface and water, θ= 0

Density of water, ρ =1.0×10^{3} kg/m^{-3}

Acceleration due to gravity, g = 9.8 m/s^{2}

Let h_1 and h_2 be the heights to which water rises in the first and second tubes respectively. These heights are given by the relations:

$$h_1 = \frac{2s\cos\theta}{r_1\rho g} \quad \dots (i)$$

$$h_2 = \frac{2s\cos\theta}{r_2\rho g} \quad \dots (ii)$$

The difference between the levels of water in the two limbs of the tube can be calculated as:

$$= \frac{2s\cos\theta}{r_1\rho g} - \frac{2s\cos\theta}{r_2\rho g}$$

$$= \frac{2s\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2\times 7.3\times 10^{-2}\times 1}{1\times 10^{3}\times 9.8}\left[\frac{1}{1.5\times 10^{-3}} - \frac{1}{3\times 10^{-3}}\right]$$

$$= 4.966\times 10^{-3}\ \text{m}$$

$$= 4.97\ \text{mm}$$

Hence, the difference between levels of water in the two bores is 4.97 mm.

Calculator/Computer – Based Problem

Question 10.31:

a) It is known that density ρ of air decreases with height y as $_0e^{y/y_0}$

Where ρ_0 = 1.25 kg m^{-3} is the density at sea level, and y_0 is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.

b) A large He balloon of volume 1425 m^3 is used to lift a payload of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise?

[Take y_0= 8000 m and ρ_{He} = 0.18 kg m^{-3}].

Answer 10.31:

Volume of the balloon, V = 1425 m^3

Mass of the payload, m = 400 kg

Acceleration due to gravity, g = 9.8 m/s^2

$y_0 = 8000$ m

$\rho_{He} = 0.18$ kg m^{-3}

$\rho_0 = 1.25$ kg/m^3

Density of the balloon = ρ

Height to which the balloon rises = y

Density (ρ) of air decreases with height (y) as:

$$\rho = \rho_0 e^{-y/y_0}$$

$$\frac{\rho}{\rho_0} = e^{-y/y_0} \quad \dots (i)$$

This density variation is called the law of atmospherics.
It can be inferred from equation (i) that the rate of decrease of density with height is directly proportional to ρ, i.e.,

$$-\frac{d\rho}{dy} \propto \rho$$

$$\frac{d\rho}{dy} = -k\rho$$

$$\frac{d\rho}{\rho} = -kdy$$

Where, k is the constant of proportionality

Height changes from 0 to y, while density changes from ρ_0 to ρ.
Integrating the sides between these limits, we get:

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = -\int_0^y kdy$$

$$[\log_e \rho]_{\rho_0}^{\rho} = -ky$$

$$\log_e \frac{\rho}{\rho_0} = -ky$$

$$\frac{\rho}{\rho_0} = e^{-ky} \quad \dots (ii)$$

Comparing equations (i) and (ii), we get:

$$y_0 = \frac{1}{k}$$

$$k = \frac{1}{y_0} \quad \dots (iii)$$

From equations (i) and (iii), we get:

$$\rho = \rho_0 e^{-y/y_0}$$

(b)

$$\text{Density } \rho = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{\text{Mass of the payload } + \text{ Mass of helium}}{\text{Volume}}$$

$$= \frac{m + V\rho_{\text{He}}}{V}$$

$$= \frac{400 + 1425 \times 0.18}{1425}$$

$$= 0.46 \text{ kg/m}^3$$

From equations (*ii*) and (*iii*), we can obtain y as:

$$\rho = \rho_0 e^{-y/y_o}$$

$$\log_e \frac{\rho}{\rho_0} = -\frac{y}{y_o}$$

$$\therefore y = -8000 \times \log_e \frac{0.46}{1.25}$$

$$= -8000 \times -1$$
$$= 8000 \text{ m} = 8 \text{ km}$$

Hence, the balloon will rise to a height of 8 km.