Additional Exercises

Question 13.11:

A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

Answer

Length of the narrow bore, $L = 1\text{ m} = 100\text{ cm}$

Length of the mercury thread, $l = 76\text{ cm}$

Length of the air column between mercury and the closed end, $l_a = 15\text{ cm}$

Since the bore is held vertically in air with the open end at the bottom, the mercury length that occupies the air space is: $100 - (76 + 15) = 9\text{ cm}$

Hence, the total length of the air column $= 15 + 9 = 24\text{ cm}$

Let h cm of mercury flow out as a result of atmospheric pressure.

$\therefore$Length of the air column in the bore $= 24 + h$ cm

And, length of the mercury column $= 76 - h$ cm

Initial pressure, $P_1 = 76$ cm of mercury

Initial volume, $V_1 = 15\text{ cm}^3$

Final pressure, $P_2 = 76 - (76 - h) = h$ cm of mercury

Final volume, $V_2 = (24 + h)\text{ cm}^3$

Temperature remains constant throughout the process.

$\therefore$ $P_1V_1 = P_2V_2$

$76 \times 15 = h\,(24 + h)$

$h^2 + 24h - 1140 = 0$

$$\therefore h = \frac{-24 \pm \sqrt{(24)^2 + 4 \times 1 \times 1140}}{2 \times 1}$$

= 23.8 cm or –47.8 cm

Height cannot be negative. Hence, 23.8 cm of mercury will flow out from the bore and 52.2 cm of mercury will remain in it. The length of the air column will be 24 + 23.8 = 47.8 cm.

Question 13.12:

From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 $cm^3\ s^{-1}$. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 $cm^3\ s^{-1}$. Identify the gas.

[Hint: Use Graham's law of diffusion: $R_1/R_2 = (M_2/M_1)^{1/2}$, where R_1, R_2 are diffusion rates of gases 1 and 2, and M_1 and M_2 their respective molecular masses. The law is a simple consequence of kinetic theory.]

Answer

Rate of diffusion of hydrogen, R_1 = 28.7 $cm^3\ s^{-1}$

Rate of diffusion of another gas, R_2 = 7.2 $cm^3\ s^{-1}$

According to Graham's Law of diffusion, we have:

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

Where,

M_1 is the molecular mass of hydrogen = 2.020 g

M_2 is the molecular mass of the unknown gas

$$\therefore M_2 = M_1\left(\frac{R_1}{R_2}\right)^2$$

$$= 2.02\left(\frac{28.7}{7.2}\right)^2 = 32.09 \text{ g}$$

32 g is the molecular mass of oxygen. Hence, the unknown gas is oxygen.

Question 13.13:

A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called law of atmospheres $n_2 = n_1 \exp[-mg(h_2 - h_1)/k_BT]$

Where n_2, n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column: $n_2 = n_1 \exp[-mg N_A(\rho - P')(h_2 - h_1)/(\rho RT)]$

Where ρ is the density of the suspended particle, and ρ' that of surrounding medium. [N_A is Avogadro's number, and R the universal gas constant.] [Hint: Use Archimedes principle to find the apparent weight of the suspended particle.]

Answer

According to the law of atmospheres, we have:

$n_2 = n_1 \exp[-mg(h_2 - h_1)/kBT]$ … (i)

Where, n_1 is the number density at height h_1, and n_2 is the number density at

height h_2 mg is the weight of the particle suspended in the gas column

Density of the medium = ρ'

Density of the suspended particle = ρ

Mass of one suspended particle = m'

Mass of the medium displaced = m

Volume of a suspended particle = V

According to Archimedes' principle for a particle suspended in a liquid column, the effective weight of the suspended particle is given as:

Weight of the medium displaced – Weight of the suspended particle

$= mg - m'g$

$$= mg - V\rho' g = mg - \left(\frac{m}{\rho}\right)\rho' g$$

$$= mg\left(1 - \frac{\rho'}{\rho}\right) \qquad \dots(ii)$$

Gas constant, R = $k_B N$

$$k_B = \frac{R}{N} \dots (iii)$$

Substituting equation (*ii*) in place of *mg* in equation (*i*) and then using equation (*iii*), we get: $n_2 = n_1 \exp[-mg(h_2 - h_1)/k_B T]$

$$= n_1 \exp\left[-mg\left(1 - \frac{\rho'}{\rho}\right)(h_2 - h_1)\frac{N}{RT}\right]$$

$$= n_1 \exp\left[-mg(\rho - \rho')(h_2 - h_1)\frac{N}{RT\rho}\right]$$

Question 13.14:

Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms:

Substance	Atomic Mass (u)	Density (10^3 Kg m^{-3})
Carbon (diamond)	12.01	2.22
Gold	197.00	19.32
Nitrogen (liquid)	14.01	1.00
Lithium	6.94	0.53
Fluorine (liquid)	19.00	1.14

[Hint: Assume the atoms to be 'tightly packed' in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few Å].

Answer

Substance	Radius (Å)
Carbon (diamond)	1.29
Gold	1.59
Nitrogen (liquid)	1.77
Lithium	1.73
Fluorine (liquid)	1.88

Atomic mass of a substance = M

Density of the substance = ρ

Avogadro's number = $N = 6.023 \times 10^{23}$

Volume of each atom $= \frac{4}{3}\pi r^3$

Volume of N number of molecules $= \frac{4}{3}\pi r^3 N$ … (i)

Volume of one mole of a substance = $\frac{M}{\rho}$ … (ii)

$\frac{4}{3}\pi r^3 N = \frac{M}{\rho}$

$\therefore r = \sqrt[3]{\frac{3M}{4\pi\rho N}}$

For carbon: M =

12.01×10^{-3} kg, ρ =

2.22×10^{3} kg m^{-3}

$$\therefore r = \left(\frac{3 \times 12.01 \times 10^{-3}}{4\pi \times 2.22 \times 10^{3} \times 6.023 \times 10^{23}} \right)^{\frac{1}{3}} = 1.29 \text{ Å}$$

Hence, the radius of a carbon atom is 1.29 Å.

For gold: M = 197.00

$\times 10^{-3}$ kg $\rho = 19.32 \times$

10^{3} kg m^{-3}

$$\therefore r = \left(\frac{3 \times 197 \times 10^{-3}}{4\pi \times 19.32 \times 10^{3} \times 6.023 \times 10^{23}} \right)^{\frac{1}{3}} = 1.59 \text{ Å}$$

Hence, the radius of a gold atom is 1.59 Å.

For liquid nitrogen:

$M = 14.01 \times 10^{-3}$ kg

$\rho = 1.00 \times 10^{3}$ kg m^{-3}

$$\therefore r = \left(\frac{3 \times 14.01 \times 10^{-3}}{4\pi \times 1.00 \times 10^{3} \times 6.23 \times 10^{23}} \right)^{\frac{1}{3}} = 1.77 \text{ Å}$$

Hence, the radius of a liquid nitrogen atom is 1.77 Å.

For lithium:

$M = 6.94 \times 10^{-3}$ kg ρ

$= 0.53 \times 10^{3}$ kg m^{-3}

$$\therefore r = \left(\frac{3 \times 6.94 \times 10^{-3}}{4\pi \times 0.53 \times 10^{3} \times 6.23 \times 10^{23}}\right)^{\frac{1}{3}} = 1.73 \text{ Å}$$

Hence, the radius of a lithium atom is 1.73 Å.

For liquid fluorine:

$M = 19.00 \times 10^{-3}$ kg

$\rho = 1.14 \times 10^{3}$ kg m^{-}

3

$$\therefore r = \left(\frac{3 \times 19 \times 10^{-3}}{4\pi \times 1.14 \times 10^{3} \times 6.023 \times 10^{23}}\right)^{\frac{1}{3}} = 1.88 \text{ Å}$$

Hence, the radius of a liquid fluorine atom is 1.88 Å.