Additional Exercises

Question 15.22:
A travelling harmonic wave on a string is described by

$$y(x,t) = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

What are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1 s? Is this velocity equal to the velocity of wave propagation?

Locate the points of the string which have the same transverse displacements and velocity as the x = 1 cm point at t = 2 s, 5 s and 11 s.

Answer
The given harmonic wave is:

$$y(x, t) = 7.5\sin\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

For x = 1 cm and t = 1s,

$$y=(1,1)=7.5\sin\left(0.0050+12+\frac{\pi}{4}\right)$$

$$=7.5\sin\left(12.0050+\frac{\pi}{4}\right)$$

$$=7.5\sin\theta$$

Where, $\theta=12.0050+\frac{\pi}{4}=12.0050+\frac{3.14}{4}=12.79$ rad

$$=\frac{180}{3.14}\times 12.79=732.81^\circ$$

$$\therefore y=(1,1)=7.5\sin(732.81^\circ)$$
$$=7.5\sin(90\times 8+12.81^\circ)=7.5\sin 12.81^\circ$$
$$=7.5\times 0.2217$$
$$=1.6629\approx 1.663 \text{ cm}$$

The velocity of the oscillation at a given point and time is given as:

$$v = \frac{d}{dt} y(x, t) = \frac{d}{dt}\left[7.5 \sin\left(0.0050x + 12t + \frac{\pi}{4}\right)\right]$$

$$= 7.5 \times 12 \cos\left(0.0050x + 12t + \frac{\pi}{4}\right)$$

At $x = 1$ cm and $t = 1$ s:

$$v = y(1,1) = 90 \cos\left(12.005 + \frac{\pi}{4}\right)$$

$$= 90 \cos(732.81°) = 90 \cos(90 \times 8 + 12.81°)$$
$$= 90 \cos(12.81°)$$
$$= 90 \times 0.975 = 87.75 \text{ cm/s}$$

Now, the equation of a propagating wave is given by:

$y(x, t) = a\sin(kx + wt + \phi)$

Where,

$k = \frac{2\pi}{\lambda}$

$\therefore \lambda = \frac{2\pi}{k}$

And $\omega = 2\pi\nu$

$\therefore \nu = \frac{\omega}{2\pi}$

Speed, $v = \nu\lambda = \frac{\omega}{k}$

Where,

$\omega = 12$ rad/s
$k = 0.0050$ m^{-1}

$\therefore v = \frac{12}{0.0050} = 2400$ cm/s

Hence, the velocity of the wave oscillation at $x = 1$ cm and $t = 1$ s is not equal to the velocity of the wave propagation.

Propagation constant is related to wavelength as:

$$k = \frac{2\pi}{\lambda}$$

$$\therefore \lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{0.0050}$$

$$= 1256 \text{ cm} = 12.56 \text{ m}$$

Therefore, all the points at distances $n\lambda$ $(n = \pm 1, \pm 2, \ldots \text{and so on})$, i.e. ± 12.56 m, ± 25.12 m, … and so on for x = 1 cm, will have the same displacement as the x = 1 cm points at t = 2 s, 5 s, and 11 s.

Question 15.23:
A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the requency of the note produced by the whistle equal to $\frac{1}{20}$ or 0.05 Hz?

Answer
(i)No

(ii)No

(iii)Yes

No

Explanation:

The narrow sound pulse does not have a fixed wavelength or frequency. However, the speed of the sound pulse remains the same, which is equal to the speed of sound in that medium.

The short pip produced after every 20 s does not mean that the frequency of the whistle is $\frac{1}{20}$ or 0.05 Hz. It means that 0.05 Hz is the frequency of the repetition of the pip of the whistle.

Question 15.24:
One end of a long string of linear mass density 8.0×10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At $t = 0$, the left end (fork end) of the string $x = 0$ has zero transverse displacement ($y = 0$) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.

Answer
The equation of a travelling wave propagating along the positive y-direction is given by the displacement equation: $y(x, t) = a \sin(wt - kx)$ … (i)

Linear mass density, $\mu = 8.0 \times 10^{-3}\ \text{kg m}^{-1}$

Frequency of the tuning fork, ν = 256 Hz

Amplitude of the wave, a = 5.0 cm = 0.05 m … (ii)

Mass of the pan, m = 90 kg

Tension in the string, $T = mg = 90 \times 9.8 = 882$ N

The velocity of the transverse wave v, is given by the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{882}{8.0 \times 10^{-3}}} = 332 \text{ m/s}$$

Angular frequency, $\omega = 2\pi\nu$

$$= 2 \times 3.14 \times 256$$

$$= 1608.5 = 1.6 \times 10^3 \text{ rad/s} \quad ...(iii)$$

Wavelength, $\lambda = \frac{v}{\nu} = \frac{332}{256}$ m

$\therefore$ Propagation constant, $k = \frac{2\pi}{\lambda}$

$$= \frac{2 \times 3.14}{\frac{332}{256}} = 4.84 \text{ m}^{-1} \quad ...(iv)$$

Substituting the values from equations (*ii*), (*iii*), and (*iv*) in equation (*i*), we get the displacement equation:

$y\,(x, t) = 0.05 \sin (1.6 \times 10^3 t - 4.84\, x)$ m

Question 15.25:
A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 m s^{-1}.

Answer
Operating frequency of the SONAR system, ν = 40 kHz

Speed of the enemy submarine, v_e = 360 km/h = 100 m/s

Speed of sound in water, v = 1450 m/s

The source is at rest and the observer (enemy submarine) is moving toward it. Hence, the apparent frequency (ν') received and reflected by the submarine is given by the relation:

$$\nu' = \left(\frac{v + v_e}{v}\right)\nu$$

$$= \left(\frac{1450 + 100}{1450}\right) \times 40 = 42.76 \text{ kHz}$$

The frequency (ν'') received by the enemy submarine is given by the relation:

$$\nu'' = \left(\frac{v}{v + v_s}\right)\nu'$$

Where, $v_s = 100 \text{ m/s}$

$$\therefore \nu'' = \left(\frac{1450}{1450 - 100}\right) \times 42.76 = 45.93 \text{ kHz}$$

Question 15.26:
Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}, and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Answer
Let v_S and v_P be the velocities of S and P waves respectively.

Let L be the distance between the epicentre and the seismograph.

We have:
$L = v_S t_S$ (*i*)

$L = v_P t_P$ (*ii*) Where,

t_S and t_P are the respective times taken by the *S* and *P* waves to reach the seismograph from the epicentre It is given that:

$v_P = 8$ km/s $v_S = 4$ km/s

From equations (*i*) and (*ii*), we have:

$v_S t_S = v_P t_P$ $4t_S = 8 t_P$ $t_S = 2 t_P$ (*iii*)

It is also given that: $t_S - t_P = 4$

min = 240 s $2t_P - t_P = 240$ $t_P =$

240

And $t_S = 2 \times 240 = 480$ s

From equation (*ii*), we get:

$L = 8 \times 240$

= 1920 km

Hence, the earthquake occurs at a distance of 1920 km from the seismograph.

Question 15.27:
A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

Answer
Ultrasonic beep frequency emitted by the bat, $\nu = 40$ kHz
Velocity of the bat, $v_b = 0.03\ v$

Where, v = velocity of sound in air
The apparent frequency of the sound striking the wall is given as:

$$\nu' = \left(\frac{v}{v - v_b}\right)\nu$$

$$= \left(\frac{v}{v - 0.03v}\right) \times 40$$

$$= \frac{40}{0.97} \text{ kHz}$$

This frequency is reflected by the stationary wall ($v_s = 0$) toward the bat.
The frequency (ν'') of the received sound is given by the relation:

$$\nu'' = \left(\frac{v + v_b}{v}\right)\nu'$$

$$= \left(\frac{v + 0.03v}{v}\right) \times \frac{40}{0.97}$$

$$= \frac{1.03 \times 40}{0.97} = 42.47 \text{ kHz}$$