Additional Exercises

Question 2.25:

A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v: tan θ = v and checks that the relation has a correct limit: as $v \rightarrow 0$, θ → 0, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

Answer 2.25:

Answer is Incorrect:

On dimensional ground, the relation is $\tan\theta = v$

Dimension of R.H.S = $M^0 L^1 T^{-1}$

Dimension of L.H.S = $M^0 L^0 T^0$

(∵ The trigonometric function is considered to be a dimensionless quantity)

Dimension of R.H.S is not equal to the dimension of L.H.S. Hence, the given relation is not correct dimensionally.

To make the given relation correct, the R.H.S should also be dimensionless. One way to achieve this is by dividing the R.H.S by the speed of rainfall v'.

Therefore, the relation reduces to $\tan\theta = \frac{v}{v'}$

This relation is dimensionally correct.

Question 2.26:

It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s?

Answer 2.26:

Difference in time of caesium clocks = 0.02 s

Time required for this difference = 100 years

$= 100 \times 365 \times 24 \times 60 \times 60 = 3.15 \times 10^9$ s

In 3.15×10^9 s, the caesium clock shows a time difference of 0.02 s.

In 1s, the clock will show a time difference of $\frac{0.02}{3.15\times10^9}$ s .

Hence, the accuracy of a standard caesium clock in measuring a time interval of 1 s is

$\frac{3.15\times10^9}{0.02} = 157.5\times10^9 \text{ s} \approx 1.5\times10^{11} \text{ s}$.

Question 2.27:

Estimate the average mass density of a sodium atom assuming its size to be about 2.5 (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase: 970 kg m^{-3}. Are the two densities of the same order of magnitude? If so, why?

Answer 2.27:

Diameter of sodium atom = Size of sodium atom = 2.5 $\mathring{A}$

Radius of sodium atom, r = $\frac{1}{2}\times 2.5\ \mathring{A} = 1.25\ \mathring{A}$
= 1.25×10^{-10} m

Volume of sodium atom, $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times 3.14 \times \left(1.25\times 10^{-10}\right)^3$

According to the Avogadro hypothesis, one mole of sodium contains 6.023×10^{23} atoms and has a mass of 23 g or 23×10^{-3} kg.

∴Mass of one atom = $\frac{23\times 10^{-3}}{6.023\times 10^{23}}$ kg

Density of sodium atom, ρ = $\frac{\frac{23\times 10^{-3}}{6.023\times 10^{23}}}{\frac{4}{3}\times 3.14\times\left(1.25\times 10^{-10}\right)^3} = 4.67\times 10^{-3}\ \text{kg m}^{-3}$

It is given that the density of sodium in crystalline phase is 970 kg m^{-3}.

Hence, the density of sodium atom and the density of sodium in its crystalline phase are not in the same order. This is because in solid phase, atoms are closely packed. Thus, the inter-atomic separation is very small in the crystalline phase.

Question 2.28:

The unit of length convenient on the nuclear scale is a fermi: 1 f = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation: $r = r_0 A^{1/3}$

where r is the radius of the nucleus, A its mass number, and r_0 is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise. 2.27.

Answer 2.28:

Radius of nucleus r is given by the relation,

$r = r_0 A^{1/3}$……………. (i)

r_0 = 1.2 f = 1.2 × 10^{-15} m

Volume of nucleus, $V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi\left(r_0 A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi r_0^3 A$$

Now, the mass of a nuclei M is equal to its mass number i.e.

$M = A$ amu $= A \times 1.66 \times 10^{-27}$ kg

Density of nucleus,

$$\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$

$$= \frac{A \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi r_0^3 A} = \frac{3 \times 1.66 \times 10^{-27}}{4\pi r_0^3}\ \text{kg/m}^3$$

This relation shows that nuclear mass depends only on constant r_0. Hence, the nuclear mass densities of all nuclei are nearly the same.
Density of sodium nucleus is given by,

$$\rho_{Sodium} = \frac{3\times1.66\times10^{-27}}{4\times3.14\times\left(1.2\times10^{-15}\right)^3}$$
$$= \frac{4.98}{21.71}\times10^{18} = 2.29\times10^{17}\ \text{kg m}^{-3}$$

Question 2.29:

A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Answer 2.29:

Time taken by the laser beam to return to Earth after reflection from the Moon = 2.56 s
Speed of light = 3×10^8 m/s

Time taken by the laser beam to reach Moon = $\frac{1}{2}\times2.56 = 1.28\ \text{s}$

Radius of the lunar orbit

= Distance between the Earth and the Moon

= $1.28 \times 3 \times 10^8$

= 3.84×10^8 m = 3.84×10^5 km

Question 2.30:

A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).

Answer 2.30:

Let the distance between the ship and the enemy submarine be 'S'.

Speed of sound in water = 1450 m/s

Time lag between transmission and reception of Sonar waves = 77 s

In this time lag, sound waves travel a distance which is twice the distance between the ship and the submarine ($2S$).

Time taken for the sound to reach the submarine $= \frac{1}{2} \times 77 = 38.5$ s

∴ Distance between the ship and the submarine (S) = 1450 × 38.5 = 55825 m = 55.8 km

Question 2.31:

The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Answer 2.31:

Time taken by quasar light to reach Earth = 3 billion years $= 3 \times 10^9$ years

$= 3 \times 10^9 \times 365 \times 24 \times 60 \times 60$ s

Speed of light $= 3 \times 10^8$ m/s

Distance between the Earth and quasar

$= (3 \times 10^8) \times (3 \times 10^9 \times 365 \times 24 \times 60 \times 60)$

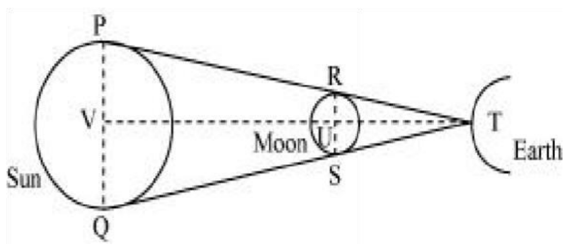
$= 283824 \times 10^{20}$ m $= 2.8 \times 10^{22}$ km

Question 2.32:

It is a well-known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the moon.

Answer 2.32:

The position of the Sun, Moon, and Earth during a lunar eclipse is shown in the given figure.



Distance of the Moon from the Earth = 3.84×10^8 m

Distance of the Sun from the Earth = 1.496×10^{11} m

Diameter of the Sun = 1.39×10^9 m

It can be observed that ΔTRS and ΔTPQ are similar. Hence, it can be written as:

$$\frac{PQ}{RS} = \frac{VT}{UT}$$

$$\frac{1.39 \times 10^9}{RS} = \frac{1.496 \times 10^{11}}{3.84 \times 10^8}$$

$$RS = \frac{1.39 \times 3.84}{1.496} \times 10^6 = 3.57 \times 10^6 \text{ m}$$

Hence, the diameter of the Moon is 3.57×10^6 m.

Question 2.33:

A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (*c*, *e*, mass of electron, mass of proton) and the gravitational constant *G*, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (~15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?

Answer 2.33:

One relation consists of some fundamental constants that give the age of the Universe by:

$$t = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \times \frac{1}{m_p m_e^2 c^3 G}$$

Where,

t = Age of Universe

e = Charge of electrons = 1.6×10^{-19} C

ε_0 = Absolute permittivity

m_p = Mass of protons = 1.67×10^{-27} kg

m_e = Mass of electrons = 9.1×10^{-31} kg

c = Speed of light = 3×10^8 m/s

G = Universal gravitational constant = 6.67×10^{11} Nm^2 kg^{-2}

Also, $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ Nm^2/C^2

Substituting these values in the equation, we get

$$t = \frac{\left(1.6\times10^{-19}\right)^4 \times \left(9\times10^9\right)^2}{\left(9.1\times10^{-31}\right)^2 \times 1.67\times10^{-27} \times \left(3\times10^8\right)^3 \times 6.67\times10^{-11}}$$

$$= \frac{(1.6)^4 \times 81}{9.1\times1.67\times27\times6.67} \times 10^{-76+18+62+27-24+11} \text{ s}$$

$$= \frac{(1.6)^4 \times 81}{9.1\times1.67\times27\times6.67\times365\times24\times3600} \times 10^{-76+18+62+27-24+11} \text{ years}$$

$$\approx 6\times10^{-9}\times10^{18} \text{ years}$$

$$= 6 \text{ billion years}$$