Additional Exercises

Question 3.23:

A three-wheeler starts from rest, accelerates uniformly with 1 m s^{-2} on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the n^{th} second (n = 1, 2, 3….) versus n. What do you expect this plot to be during accelerated motion: a straight line or a parabola?

Answer 3.23:

Straight line

Distance covered by a body in n^{th} second is given by the relation

$$D_n = u + \frac{a}{2}(2n-1) \quad \text{... (i)}$$

Where,

u = Initial velocity

a = Acceleration

n = Time = 1, 2, 3, ... , n

In the given case, u = 0 and a = 1 m/s^2

$$\therefore D_n = \frac{1}{2}(2n-1) \quad \text{... (ii)}$$

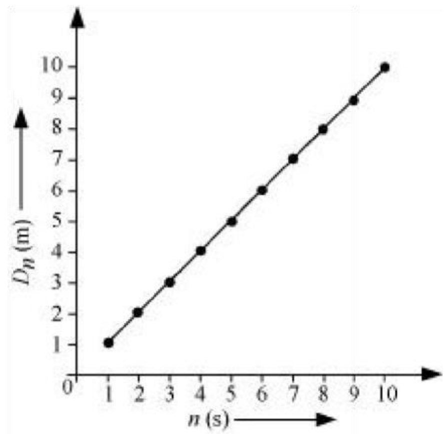
This relation shows that:

$D_n \propto n$ … (iii)

Now, substituting different values of n in equation (iii), we get the following table:

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
D_n	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

The plot between n and D_n will be a straight line as shown:



Since the given three-wheeler acquires uniform velocity after 10 s, the line will be parallel to the time-axis after n = 10 s.

Question 3.24:

A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m/s. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m/s and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands?

Answer 3.24:

Initial velocity of the ball, u = 49 m/s

Acceleration, $a = -g = -9.8$ m/s^2

Case I:

When the lift was stationary, the boy throws the ball.

Taking upward motion of the ball,

Final velocity, v of the ball becomes zero at the highest point.

From first equation of motion, time of ascent (t) is given as:

$$v = u + at$$
$$t = \frac{v-u}{a}$$
$$= \frac{-49}{-9.8} = 5 \text{ s}$$

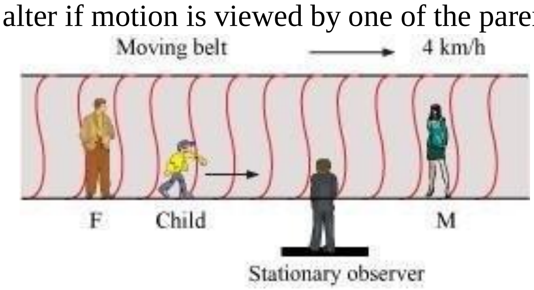
But, the time of ascent is equal to the time of descent.
Hence, the total time taken by the ball to return to the boy's hand = 5 + 5 = 10 s.

Case II:
The lift was moving up with a uniform velocity of 5 m/s. In this case, the relative velocity of the ball with respect to the boy remains the same i.e., 49 m/s. Therefore, in this case also, the ball will return back to the boy's hand after 10 s.

Question 3.25:
On a long horizontally moving belt (Fig. 3.26), a child runs to and fro with a speed 9 km h^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km h^{-1}. For an observer on a stationary platform outside, what is the

a) speed of the child running in the direction of motion of the belt?
b) speed of the child running opposite to the direction of motion of the belt?
c) time taken by the child in (a) and (b) ?

Which of the answers alter if motion is viewed by one of the parents?



(Fig: 3.26)

Answer 3.25:
Speed of the belt, $v_B = 4$ km/h
Speed of the boy, $v_b = 9$ km/h

a) Since the boy is running in the same direction of the motion of the belt, his speed (as observed by the stationary observer) can be obtained as:
$v_{bB} = v_b + v_B = 9 + 4 = 13$ km/h

b) Since the boy is running in the direction opposite to the direction of the motion of the belt, his speed (as observed by the stationary observer) can be obtained as:
$v_{bB} = v_b + (-v_B) = 9 - 4 = 5$ km/h

c) Distance between the child's parents = 50 m

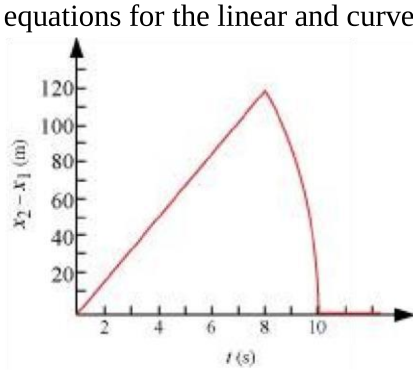
As both parents are standing on the moving belt, the speed of the child in either direction as observed by the parents will remain the same i.e., 9 km/h = 2.5 m/s.

Hence, the time taken by the child to move towards one of his parents is 50/2.5 = 20s.

If the motion is viewed by any one of the parents, answers obtained in (a) and (b) get altered. This is because the child and his parents are standing on the same belt and hence, are equally affected by the motion of the belt. Therefore, for both parents (irrespective of the direction of motion) the speed of the child remains the same i.e., 9 km/h.

For this reason, it can be concluded that the time taken by the child to reach any one of his parents remains unaltered.

Question 3.26:
Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m/s and 30 m/s. Verify that the graph shown in Fig. 3.27 correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10$ m/s^2. Give the equations for the linear and curved parts of the plot.



Answer 3.26:

For first stone:

Initial velocity, $u_I = 15$ m/s

Acceleration, $a = -g = -10$ m/s^2

Using the relation,

$$x_1 = x_0 + u_1 t + \frac{1}{2}at^2$$

Where, height of the cliff, $x_0 = 200$ m

$$x_1 = 200 + 15t - 5t^2 \quad \text{... (i)}$$

When this stone hits the ground, $x_1 = 0$

$\therefore -5t^2 + 15t + 200 = 0$

$t^2 - 3t - 40 = 0$

$t^2 - 8t + 5t - 40 = 0$

$t(t - 8) + 5(t - 8) = 0$

$t = 8$ s or $t = -5$ s

Since the stone was projected at time $t = 0$, the negative sign before time is meaningless.

$\therefore t = 8$ s

For second stone:

Initial velocity, $u_{II} = 30$ m/s

Acceleration, $a = -g = -10$ m/s^2

Using the relation,

$$x_2 = x_0 + u_{II}t + \frac{1}{2}at^2$$
$$= 200 + 30t - 5t^2 \quad \text{... (ii)}$$

At the moment when this stone hits the ground; $x_2 = 0$

$5t^2 + 30\,t + 200 = 0$

$t^2 - 6t - 40 = 0$

$t^2 - 10t + 4t + 40 = 0$

$t\,(t - 10) + 4\,(t - 10) = 0$

$t\,(t - 10)\,(t + 4) = 0$

$t = 10$ s or $t = -4$ s

Here again, the negative sign is meaningless.

$\therefore t = 10$ s

Subtracting equations (i) and (ii), we get

$$x_2 - x_1 = (200 + 30t - 5t^2) - (200 + 15t - 5t^2)$$
$$x_2 - x_1 = 15t \quad \text{... (iii)}$$

Equation (iii) represents the linear path of both stones. Due to this linear relation between $(x_2 - x_1)$ and t, the path remains a straight line till 8 s.

Maximum separation between the two stones is at $t = 8$ s.

$(x_2 - x_1)_{max} = 15 \times 8 = 120$ m

This is in accordance with the given graph.

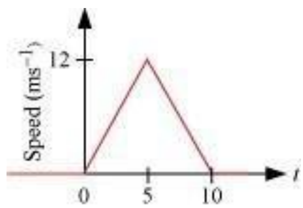
After 8 s, only second stone is in motion whose variation with time is given by the quadratic equation: $x_2 - x_1 = 200 + 30t - 5t^2$

Hence, the equation of linear and curved path is given by

$x_2 - x_1 = 15t$ (Linear path)

$x_2 - x_1 = 200 + 30t - 5t^2$ (Curved path)

Question 3.27:
The speed-time graph of a particle moving along a fixed direction is shown in Fig. 3.28. Obtain the distance traversed by the particle between (a) $t = 0$ s to 10 s, (b) $t = 2$ s to 6 s.



(Fig. 3.28)

What is the average speed of the particle over the intervals in (a) and (b)?

Answer 3.27:
Distance travelled by the particle = Area under the given graph

$= \frac{1}{2} \times (10 - 0) \times (12 - 0) = 60$ m

Average speed = $\frac{\text{Distance}}{\text{Time}} = \frac{60}{10} = 6$ m/s

Let s_1 and s_2 be the distances covered by the particle between time

$t = 2$ s to 5 s and $t = 5$ s to 6 s respectively.

Total distance (s) covered by the particle in time $t = 2$ s to 6

s $s = s_1 + s_2$ … (i)

For distance s_1:

Let u' be the velocity of the particle after 2 s and a' be the acceleration of the particle in t

= 0 to t = 5 s.

Since the particle undergoes uniform acceleration in the interval t = 0 to t = 5 s, from first equation of motion, acceleration can be obtained as:

$v = u + at$

Where,

v = Final velocity of the particle

$12 = 0 + a' \times 5$

$$a' = \frac{12}{5} = 2.4\ \text{m/s}^2$$

Again, from first equation of motion, we have

$v = u + at$

$= 0 + 2.4 \times 2 = 4.8$ m/s

Distance travelled by the particle between time 2 s and 5 s i.e., in 3 s

$$s_1 = u't + \frac{1}{2}a't^2$$
$$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times (3)^2$$
$$= 25.2\ \text{m} \qquad \text{... (ii)}$$

For distance s_2:

Let a'' be the acceleration of the particle between time t = 5 s and t = 10 s.
From first equation of motion, $v = u + at$ (where v = 0 as the particle finally comes to rest)
$0 = 12 + a'' \times 5$

$$a'' = \frac{-12}{5}$$
$$= -2.4\ \text{m/s}^2$$

Distance travelled by the particle in 1s (i.e., between t = 5 s and t = 6 s)

$$s_2 = u''t + \frac{1}{2}at^2$$
$$= 12 \times a + \frac{1}{2}(-2.4) \times (1)^2$$
$$= 12 - 1.2 = 10.8 \text{ m} \qquad \ldots \text{(iii)}$$

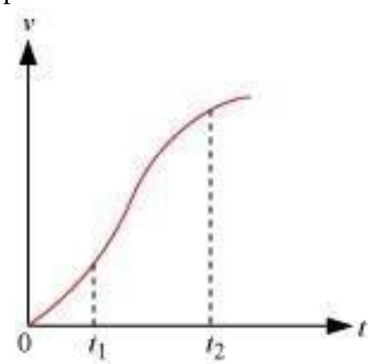
From equations (i), (ii), and (iii), we get

$s = 25.2 + 10.8 = 36$ m

$$\therefore \text{Average speed} = \frac{36}{4} = 9 \text{ m/s}$$

Question 3.28:

The velocity-time graph of a particle in one-dimensional motion is shown in Fig. 3.29:



Which of the following formulae are correct for describing the motion of the particle over the time-interval t_2 to t_1?

a) $x(t_2) = x(t_1) + v(t_1)(t_2–t_1) + (1/2)a(t_2–t_1)^2$
b) $v(t_2)= v(t_1) + a(t_2–t_1)$
c) $v_{Average} = (x(t_2) – x(t_1)) / (t_2 – t_1)$
d) $a_{Average} = (v(t_2) – v(t_1)) / (t_2 – t_1)$
e) $x(t_2) = x(t_1) + v_{Average}(t_2 – t_1) + (1/2)a_{Average}(t_2 – t_1)_2$
f) $x(t_2) – x(t_1)$ = area under the $v–t$ curve bounded by the t-axis and the dotted line shown.

Answer 3.28:
The correct formulae describing the motion of the particle are (c), (d) and, (f)

The given graph has a non-uniform slope.

Hence, the formulae given in (a), (b), and (e) cannot describe the motion of the particle.

Only relations given in (c), (d), and (f) are correct equations of motion.