Additional Exercises

Question 4.26:

A vector has magnitude and direction. Does it have a location in space? Can it vary with time? Will two equal vectors **a** and **b** at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

Answer 4.26:

No; Yes; No

Generally speaking, a vector has no definite locations in space. This is because a vector remains invariant when displaced in such a way that its magnitude and direction remain the same. However, a position vector has a definite location in space.

A vector can vary with time. For example, the displacement vector of a particle moving with a certain velocity varies with time.

Two equal vectors located at different locations in space need not produce the same physical effect. For example, two equal forces acting on an object at different points can cause the body to rotate, but their combination cannot produce an equal turning effect.

Question 4.27:

A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axis. Does that make any rotation a vector?

Answer 4.27:

No; No

A physical quantity having both magnitude and direction need not be considered a vector. For example, despite having magnitude and direction, current is a scalar quantity. The essential requirement for a physical quantity to be considered a vector is that it should follow the law of vector addition.

Generally speaking, the rotation of a body about an axis is not a vector quantity as it does not follow the law of vector addition. However, a rotation by a certain small angle follows the law of vector addition and is therefore considered a vector.

Question 4.28:

Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere? Explain.

Answer 4.28:
No; Yes; No

One cannot associate a vector with the length of a wire bent into a loop.
One can associate an area vector with a plane area. The direction of this vector is normal, inward or outward to the plane area.
One cannot associate a vector with the volume of a sphere. However, an area vector can be associated with the area of a sphere.

Question 4.29:
A bullet fired at an angle of 30° with the horizontal hits the ground 3.0 km away. By adjusting its angle of projection, can one hope to hit a target 5.0 km away? Assume the *muzzle* speed to the fixed, and neglect air resistance.

Answer 4.29:
No

Range, R = 3 km

Angle of projection, θ = 30°

Acceleration due to gravity, g = 9.8 m/s^2

Horizontal range for the projection velocity u_0, is given by the relation:

$$R = \frac{u_0^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0^2}{g} \sin 60^\circ$$

$$\frac{u_0^2}{g} = 2\sqrt{3} \qquad \ldots (i)$$

The maximum range (R_{max}) is achieved by the bullet when it is fired at an angle of 45° with the horizontal, that is,

$$R_{max} = \frac{u_0^2}{g} \qquad \ldots (ii)$$

On comparing equations (i) and (ii), we get:

$$R_{max} = 3\sqrt{3} = 2 \times 1.732 = 3.46 \text{ km}$$

Hence, the bullet will not hit a target 5 km away.

Question 4.30:

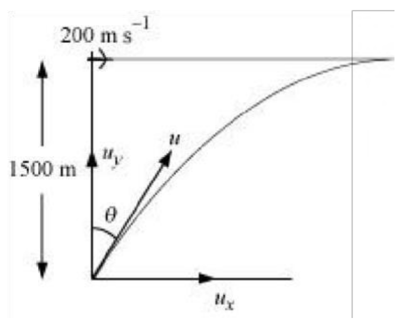
A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 m

Answer 4.30:

Height of the fighter plane = 1.5 km = 1500 m

Speed of the fighter plane, v = 720 km/h = 200 m/s

Let θ be the angle with the vertical so that the shell hits the plane. The situation is shown in the given figure.



Muzzle velocity of the gun, u = 600 m/s
Time taken by the shell to hit the plane = t
Horizontal distance travelled by the shell = $u_x t$
Distance travelled by the plane = vt
The shell hits the plane. Hence, these two distances must be equal.

$$u_x t = vt$$

$$u \sin\theta = v$$

$$\sin\theta = \frac{v}{u}$$

$$= \frac{200}{600} = \frac{1}{3} = \theta.33$$

$$\theta = \sin^{-1}(0.33)$$

$$= 19.5°$$

In order to avoid being hit by the shell, the pilot must fly the plane at an altitude (*H*) higher than the maximum height achieved by the shell.

$$\therefore H = \frac{u^2 \sin^2(90-\theta)}{2g}$$
$$= \frac{(600)^2 \cos^2\theta}{2g}$$
$$= \frac{360000 \times \cos^2 19.5}{2 \times 10}$$
$$= 18000 \times (0.943)^2$$
$$= 16006.482 \text{ m}$$
$$\approx 16 \text{ km}$$

Question 4.31:

A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Answer 4.31:

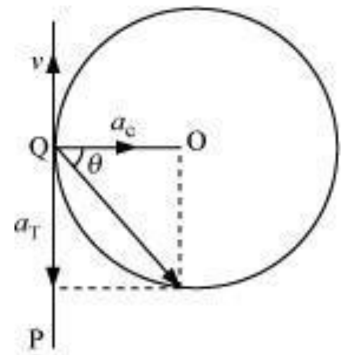
Speed of the cyclist, $v = 27 \text{ km/h} = 7.5 \text{ m/s}$

Radius of the circular turn, r = 80 m

Centripetal acceleration is given as:

$$a_c = \frac{v^2}{r}$$
$$= \frac{(7.5)^2}{80} = 0.7 \text{ m/s}^2$$

The situation is shown in the given figure:



Suppose the cyclist begins cycling from point P and moves toward point Q. At point Q, he applies the breaks and decelerates the speed of the bicycle by 0.5 m/s^2.

This acceleration is along the tangent at Q and opposite to the direction of motion of the cyclist.

Since the angle between a_c and a_T is 90°, the resultant acceleration a is given by:

$$a = \sqrt{a_c^2 + a_T^2}$$

$$= \sqrt{(0.7)^2 + (0.5)^2}$$

$$= \sqrt{0.74} = 0.86 \text{ m/s}^2$$

$$\tan\theta = \frac{a_c}{a_T}$$

Where θ is the angle of the resultant with the direction of velocity

$$\tan\theta = \frac{0.7}{0.5} = 1.4$$

$$\theta = \tan^{-1}(1.4)$$

$$= 54.46°$$

Question 4.32:

Show that for a projectile the angle between the velocity and the *x*-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

Show that the projection angle θ_0 for a projectile launched from the origin is given by

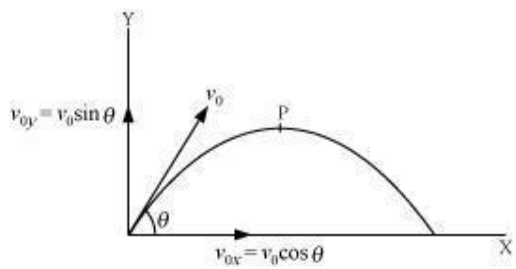
$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

Where the symbols have their usual meaning.

Answer 4.32:

Let v_{0x} and v_{0y} respectively be the initial components of the velocity of the projectile along horizontal (*x*) and vertical (*y*) directions.

Let v_x and v_y respectively be the horizontal and vertical components of velocity at a point P.



Time taken by the projectile to reach point P = *t*

Applying the first equation of motion along the vertical and horizontal directions, we get:

$$v_y = v_{0y} = gt$$

$$\text{And } v_x = v_{0x}$$

$$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{v_{0y} - gt}{v_{0x}}$$

$$\theta = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

Maximum vertical height, $h_m = \dfrac{u_0^2 \sin^2 2\theta}{2g}$ ... (*i*)

Horizontal range, $R = \dfrac{u_0^2 \sin^2 2\theta}{g}$ ... (*ii*)

Solving equations (*i*) and (*ii*), we get:

$$-\frac{h_m}{R} = \frac{\sin^2\theta}{2\sin^2\theta}$$

$$= \frac{\sin\theta \times \sin\theta}{2 \times 2\sin\theta\cos\theta}$$

$$= \frac{1}{4}\frac{\sin\theta}{\cos\theta} = \frac{1}{4}\tan\theta$$

$$\tan\theta = \left(\frac{4h_m}{R}\right)$$

$$\theta = \tan^{-1}\left(\frac{4h_m}{R}\right)$$