Additional Exercises

Question 5.24:
Figure 5.17 shows the position-time graph of a body of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the body? What is the magnitude of each impulse?

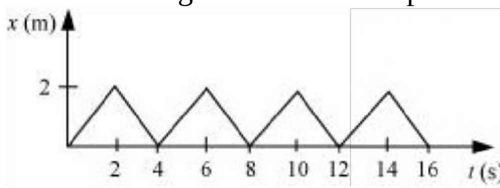


Figure 5.17

Answer 5.24:
A ball rebounding between two walls located between at $x = 0$ and $x = 2$ cm; after every 2 s, the ball receives an impulse of magnitude 0.08×10^{-2} kg m/s from the walls

The given graph shows that a body changes its direction of motion after every 2 s. Physically, this situation can be visualized as a ball rebounding to and fro between two stationary walls situated between positions $x = 0$ and $x = 2$ cm. Since the slope of the *x*-*t* graph reverses after every 2 s, the ball collides with a wall after every 2 s. Therefore, ball receives an impulse after every 2 s.
Mass of the ball, $m = 0.04$ kg

The slope of the graph gives the velocity of the ball. Using the graph, we can calculate initial velocity (u) as:

$$u = \frac{(2-0)\times 10^{-2}}{(2-0)} = 10^{-2} \text{ m/s}$$

Velocity of the ball before collision, $u = 10^{-2}$ m/s
Velocity of the ball after collision, $v = -10^{-2}$ m/s

(Here, the negative sign arises as the ball reverses its direction of motion.)
Magnitude of impulse = Change in momentum

$$= |mv - mu|$$
$$= |0.04(v-u)|$$
$$= |0.04(-10^{-2} - 10^{-2})|$$

$= 0.08 \times 10^{-2}$ kg m/s

Question 5.25:

Figure 5.18 shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 m s^{-2}. What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, up to what acceleration of the belt can the man continue to be stationary relative to the belt? (Mass of the man = 65 kg.)

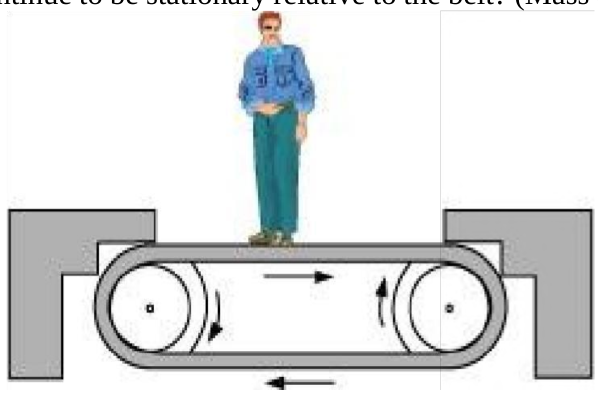


Figure 5.18

Answer 5.25:

Mass of the man, m = 65 kg

Acceleration of the belt, a = 1 m/s^2

Coefficient of static friction, μ = 0.2

The net force F, acting on the man is given by Newton's second law of motion as:

$F_{net} = ma$ = 65 × 1 = 65 N

The man will continue to be stationary with respect to the conveyor belt until the net force on the man is less than or equal to the frictional force f_s, exerted by the belt, i.e.,

$F'_{net} = f_s$

$ma' = \mu mg$

$\therefore a'$ = 0.2 × 10 = 2 m/s^2

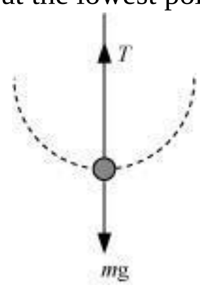
Therefore, the maximum acceleration of the belt up to which the man can stand stationary is 2 m/s^2.

Question 5.26:
A stone of mass m tied to the end of a string revolves in a vertical circle of radius R. The net forces at the lowest and highest points of the circle directed vertically downwards are: [Choose the correct alternative]

	Lowest Point	Highest Point
(a)	$mg - T_1$	$mg + T_2$
(b)	$mg + T_1$	$mg - T_2$
(c)	$mg + T_1 - \frac{mv_1^2}{R}$	$mg - T_2 + \frac{mv_1^2}{R}$
(d)	$mg - T_1 - \frac{mv_1^2}{R}$	$mg + T_2 + \frac{mv_1^2}{R}$

T_1 and v_1 denote the tension and speed at the lowest point. T_2 and v_2 denote corresponding values at the highest point.

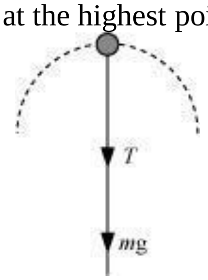
Answer 5.26:
The free body diagram of the stone at the lowest point is shown in the following figure.



According to Newton's second law of motion, the net force acting on the stone at this point is equal to the centripetal force, i.e.,

$$F_{net} = T - mg = \frac{mv_1^2}{R} \quad \text{......................} (i)$$

Where, v_1 = Velocity at the lowest point
The free body diagram of the stone at the highest point is shown in the following figure.



Using Newton's second law of motion, we have:

$$T + mg = \frac{mv_2^2}{R} \quad \text{......................} (ii)$$

Where, v_2 = Velocity at the highest point
It is clear from equations (i) and (ii) that the net force acting at the lowest and the highest points are respectively ($T - m$g) and ($T + m$g).

Question 5.27:
A helicopter of mass 1000 kg rises with a vertical acceleration of 15 m s^{-2}. The crew and the passengers weigh 300 kg. Give the magnitude and direction of the

- **a)** force on the floor by the crew and passengers,
- **b)** action of the rotor of the helicopter on the surrounding air,
- **c)** force on the helicopter due to the surrounding air.

Answer 5.27:
Mass of the helicopter, m_h = 1000 kg
Mass of the crew and passengers, m_p = 300 kg

Total mass of the system, m = 1300 kg

Acceleration of the helicopter, a = 15 m/s^2

a) Using Newton's second law of motion, the reaction force R, on the system by the floor can be calculated as:

$R - m_p g = ma$

$= m_p(g + a)$

$= 300\ (10 + 15) = 300 \times 25$

$= 7500$ N

Since the helicopter is accelerating vertically upward, the reaction force will also be directed upward. Therefore, as per Newton's third law of motion, the force on the floor by the crew and passengers is 7500 N, directed downward.

Using Newton's second law of motion, the reaction force R', experienced by the helicopter can be calculated as:

$R' - mg = ma$

$= m(g + a)$

$= 1300\ (10 + 15) = 1300 \times 25$

= 32500 N

b) The reaction force experienced by the helicopter from the surrounding air is acting upward. Hence, as per Newton's third law of motion, the action of the rotor on the surrounding air will be 32500 N, directed downward.

c) The force on the helicopter due to the surrounding air is 32500 N, directed upward.

Question 5.28:
A stream of water flowing horizontally with a speed of 15 m s^{-1} gushes out of a tube of cross-sectional area 10^{-2} m^2, and hits a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming it does not rebound?

Answer 5.28:
Speed of the water stream, v = 15 m/s

Cross-sectional area of the tube, $A = 10^{-2}\ m^2$

Volume of water coming out from the pipe per second,

$V = Av = 15 \times 10^{-2}\ m^3/s$

Density of water, $\rho = 10^3\ kg/m^3$

Mass of water flowing out through the pipe per second = $\rho \times V$ = 150 kg/s

The water strikes the wall and does not rebound. Therefore, the force exerted by the water on the wall is given by Newton's second law of motion as:

F = Rate of change of momentum $= \frac{\Delta P}{\Delta t}$

$= \frac{mv}{t}$

$= 150 \times 15 = 2250\ N$

Question 5.29:

Ten one-rupee coins are put on top of each other on a table. Each coin has a mass *m*. Give the magnitude and direction of

a) the force on the 7^{th} coin (counted from the bottom) due to all the coins on its top,
b) the force on the 7^{th} coin by the eighth coin,
c) the reaction of the 6th coin on the 7^{th} coin.

Answer 5.29:

a) Force on the seventh coin is exerted by the weight of the three coins on its top.

Weight of one coin = mg

Weight of three coins = $3mg$

Hence, the force exerted on the 7^{th} coin by the three coins on its top is $3mg$. This force acts vertically downward.

b) Force on the seventh coin by the eighth coin is because of the weight of the eighth coin and the other two coins (ninth and tenth) on its top.

Weight of the eighth coin = mg

Weight of the ninth coin = mg

Weight of the tenth coin = mg

Total weight of these three coins = $3mg$

Hence, the force exerted on the 7^{th} coin by the eighth coin is $3mg$. This force acts vertically downward.

c) The 6^{th} coin experiences a downward force because of the weight of the four coins (7^{th}, 8^{th}, 9^{th}, and 10^{th}) on its top.

Therefore, the total downward force experienced by the 6^{th} coin is $4mg$.

As per Newton's third law of motion, the 6^{th} coin will produce an equal reaction force on the 7^{th} coin, but in the opposite direction. Hence, the reaction force of the 6^{th} coin on the 7^{th} coin is of magnitude $4mg$. This force acts in the upward direction.

Question 5.30:
An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop?
Answer 5.30:

Speed of the aircraft, v = 720 km/h $= 720 \times \frac{5}{18} = 200$ m/s

Acceleration due to gravity, g = 10 m/s^2

Angle of banking, θ = 15°

For radius r, of the loop, we have the relation:

$$\tan\theta = \frac{v^2}{rg}$$

$$r = \frac{v^2}{g\tan\theta}$$

$$= \frac{200 \times 200}{10 \times \tan 15} = \frac{4000}{0.268}$$

= 14925.37 m

= 14.92 km

Question 5.31:
A train runs along an unbanked circular track of radius 30 m at a speed of 54 km/h. The mass of the train is 10^6 kg. What provides the centripetal force required for this purpose – The engine or the rails? What is the angle of banking required to prevent wearing out of the rail?
Answer 5.31:
Radius of the circular track, r = 30 m

Speed of the train, v = 54 km/h = 15 m/s

Mass of the train, m = 10^6 kg

The centripetal force is provided by the lateral thrust of the rail on the wheel. As per Newton's third law of motion, the wheel exerts an equal and opposite force on the rail.

This reaction force is responsible for the wear and rear of the rail

The angle of banking θ, is related to the radius (r) and speed (v) by the relation:

$$\tan\theta = \frac{v^2}{rg}$$

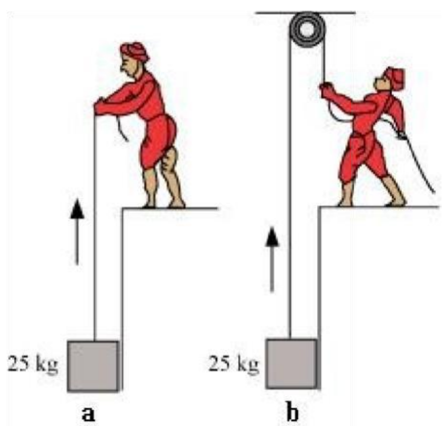
$$= \frac{(15)^2}{30\times 10} = \frac{225}{300}$$

$$\theta = \tan^{-1}(0.75) = 36.87°$$

Therefore, the angle of banking is about 36.87°.

Question 5.32:

A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig.5.19. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding?



Answer 5.32:

750 N and 250 N in the respective cases; Method (b)

Mass of the block, m = 25 kg

Mass of the man, M = 50 kg

Acceleration due to gravity, g = 10 m/s^2

Force applied on the block, $F = 25 \times 10 = 250$ N

Weight of the man, $W = 50 \times 10 = 500$ N

Case (a): When the man lifts the block directly
In this case, the man applies a force in the upward direction. This increases his apparent weight.

$\therefore$ Action on the floor by the man = 250 + 500 = 750 N

Case (b): When the man lifts the block using a pulley
In this case, the man applies a force in the downward direction. This decreases his apparent weight.

$\therefore$ Action on the floor by the man = 500 – 250 = 250 N

If the floor can yield to a normal force of 700 N, then the man should adopt the second method to easily lift the block by applying lesser force.

Question 5.33:
A monkey of mass 40 kg climbs on a rope (Fig. 5.20) which can stand a maximum tension of 600 N. In which of the following cases will the rope break: the monkey

- **a)** climbs up with an acceleration of 6 m s–2
- **b)** climbs down with an acceleration of 4 m s–2
- **c)** climbs up with a uniform speed of 5 m s–1
- **d)** falls down the rope nearly freely under gravity?

(Ignore the mass of the rope).

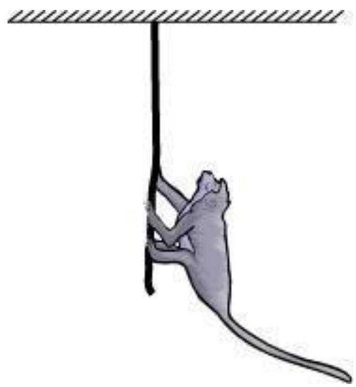


Fig. 5.20

Answer 5.33:
Case (a)
Mass of the monkey, $m = 40$ kg

Acceleration due to gravity, $g = 10$ m/s

Maximum tension that the rope can bear, $T_{max} = 600$ N

Acceleration of the monkey, $a = 6$ m/s^2 upward

Using Newton's second law of motion, we can write the equation of motion as:

$T - mg = ma$

$\therefore T = m(g + a)$

$= 40 (10 + 6)$

$= 640$ N

Since $T > T_{max}$, the rope will break in this case.

Case (b)
Acceleration of the monkey, $a = 4$ m/s^2 downward

Using Newton's second law of motion, we can write the equation of motion as:

$mg - T = ma$

$\therefore T = m (g - a)$

$= 40(10 - 4)$

$= 240$ N

Since $T < T_{max}$, the rope will not break in this case.

Case (c)
The monkey is climbing with a uniform speed of 5 m/s. Therefore, its acceleration is zero, i.e., $a = 0$.

Using Newton's second law of motion, we can write the equation of motion as:

$T - mg = ma$

$T - mg = 0$

$\therefore T = mg$

$= 40 \times 10$

$= 400$ N

Since $T < T_{max}$, the rope will not break in this case.

Case (d)
When the monkey falls freely under gravity, its will acceleration become equal to the acceleration due to gravity, i.e., $a = g$

Using Newton's second law of motion, we can write the equation of motion as:

$mg - T = mg$

$\therefore T = m(g - g) = 0$

Since $T < T_{max}$, the rope will not break in this case.

Question 5.34:
Two bodies *A* and *B* of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid wall (Fig. 5.21). The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally to *A*. What are (a) the reaction of the partition (b) the action-reaction forces between *A* and *B*? What happens when the wall is removed? Does the answer to (b) change, when the bodies are in motion? Ignore the difference between μ_s and μ_k.

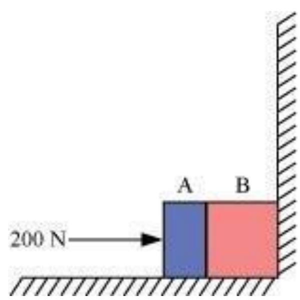


Fig. 5.21

Answer 5.34:
Mass of body A, $m_A = 5$ kg

Mass of body B, $m_B = 10$ kg

Applied force, F = 200 N

Coefficient of friction, μ_s = 0.15

The force of friction is given by the relation:

$f_s = \mu\,(m_A + m_B)g$

= 0.15 (5 + 10) × 10

= 1.5 × 15 = 22.5 N leftward

Net force acting on the partition = 200 – 22.5 = 177.5 N rightward

As per Newton's third law of motion, the reaction force of the partition will be in the direction opposite to the net applied force.

Hence, the reaction of the partition will be 177.5 N, in the leftward direction.

Force of friction on mass A:

$f_A = \mu m_A g$

= 0.15 × 5 × 10 = 7.5 N leftward

Net force exerted by mass A on mass B = 200 – 7.5 = 192.5 N rightward

As per Newton's third law of motion, an equal amount of reaction force will be exerted by mass B on mass A, i.e., 192.5 N acting leftward.

When the wall is removed, the two bodies will move in the direction of the applied force.

Net force acting on the moving system = 177.5 N

The equation of motion for the system of acceleration a, can be written as:

$$\text{Net force} = (m_A + m_B)a$$

$$\therefore a = \frac{\text{Net force}}{m_A + m_B}$$

$$= \frac{177.5}{5+10} = \frac{177.5}{15} = 11.83 \text{ m/s}^2$$

Net force causing mass A to move:

$F_A = m_A a$

$= 5 \times 11.83 = 59.15$ N

Net force exerted by mass A on mass B = 192.5 – 59.15 = 133.35 N

This force will act in the direction of motion. As per Newton's third law of motion, an equal amount of force will be exerted by mass B on mass A, i.e., 133.3 N, acting opposite to the direction of motion.

Question 5.35:
A block of mass 15 kg is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 m s^{-2} for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (a) a stationary observer on the ground, (b) an observer moving with the trolley.
Answer 5.35:
Mass of the block, m = 15 kg

Coefficient of static friction, $\mu = 0.18$

Acceleration of the trolley, $a = 0.5$ m/s^2

As per Newton's second law of motion, the force (F) on the block caused by the motion of the trolley is given by the relation:

$F = ma = 15 \times 0.5 = 7.5$ N

This force is acted in the direction of motion of the trolley.

Force of static friction between the block and the trolley:

$f = \mu m$g

$= 0.18 \times 15 \times 10 = 27$ N

The force of static friction between the block and the trolley is greater than the applied external force. Hence, for an observer on the ground, the block will appear to be at rest.

When the trolley moves with uniform velocity there will be no applied external force. Only the force of friction will act on the block in this situation.

An observer, moving with the trolley, has some acceleration. This is the case of non-inertial frame of reference. The frictional force, acting on the trolley backward, is opposed by a pseudo force of the same magnitude. However, this force acts in the opposite direction. Thus, the trolley will appear to be at rest for the observer moving with the trolley.

Question 5.36:

The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. 5.22. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms^{-2}. At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).

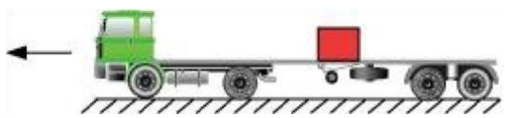


Fig. 5.22

Answer 5.36:

Mass of the box, m = 40 kg

Coefficient of friction, μ = 0.15

Initial velocity, u = 0

Acceleration, a = 2 m/s^2

Distance of the box from the end of the truck, s' = 5 m

As per Newton's second law of motion, the force on the box caused by the accelerated motion of the truck is given by:

$F = ma$

= 40 × 2 = 80 N

As per Newton's third law of motion, a reaction force of 80 N is acting on the box in the backward direction. The backward motion of the box is opposed by the force of friction *f*, acting between the box and the floor of the truck. This force is given by:

$f = \mu m g$

$= 0.15 \times 40 \times 10 = 60$ N

$\therefore$Net force acting on the block:

$F_{net} = 80 - 60 = 20$ N backward

The backward acceleration produced in the box is given by:

$$a_{back} = \frac{F_{net}}{m} = \frac{20}{40} = 0.5 \text{ m/s}^2$$

Using the second equation of motion, time *t* can be calculated as:

$$s' = ut + \frac{1}{2}a_{back}t^2$$

$$5 = 0 + \frac{1}{2} \times 0.5 \times t^2$$

$$\therefore t = \sqrt{20} \text{ s}$$

Hence, the box will fall from the truck after $\sqrt{20}$ s from start.

The distance *s*, travelled by the truck in $\sqrt{20}$ s is given by the relation:

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2} \times 2 \times \left(\sqrt{20}\right)^2$$

= 20 m

Question 5.37:

A disc revolves with a speed of $33\frac{1}{3}$ rev/min, and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the co-efficient of friction between the coins and the record is 0.15, which of the coins will revolve with the record?

Answer 5.37:

Coin placed at 4 cm from the centre

Mass of each coin = m

Radius of the disc, r = 15 cm = 0.15 m

Frequency of revolution, $\nu = 33\frac{1}{3}$ rev/min $= \frac{100}{3 \times 60} = \frac{5}{9}$ rev/s

Coefficient of friction, $\mu = 0.15$

In the given situation, the coin having a force of friction greater than or equal to the centripetal force provided by the rotation of the disc will revolve with the disc. If this is not the case, then the coin will slip from the disc.

Coin placed at 4 cm:
Radius of revolution, r' = 4 cm = 0.04 m

Angular frequency, $\omega = 2\pi\nu = 2 \times \frac{22}{7} \times \frac{5}{9} = 3.49\ s^{-1}$

Frictional force, $f = \mu m g = 0.15 \times m \times 10 = 1.5m$ N

Centripetal force on the coin:

$$F_{cent.} = mr'\omega^2$$
$$= m \times 0.04 \times (3.49)^2$$

$= 0.49m$ N

Since $f > F_{cent}$, the coin will revolve along with the record.

Coin placed at 14 cm:
Radius, r'' = 14 cm = 0.14 m

Angular frequency, $\omega = 3.49\ s^{-1}$

Frictional force, $f' = 1.5m$ N

Centripetal force is given as:

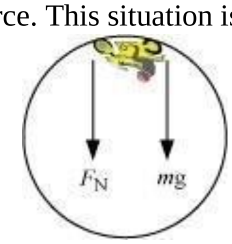
$F_{cent.} = mr''\omega^2$

$= m \times 0.14 \times (3.49)^2$

$= 1.7m$ N

Since $f < F_{cent.}$, the coin will slip from the surface of the record.

Question 5.38:
You may have seen in a circus a motorcyclist driving in vertical loops inside a 'deathwell' (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?
Answer 5.38:
In a death-well, a motorcyclist does not fall at the top point of a vertical loop because both the force of normal reaction and the weight of the motorcyclist act downward and are balanced by the centripetal force. This situation is shown in the following figure.



The net force acting on the motorcyclist is the sum of the normal force (F_N) and the force due to gravity ($F_g = mg$).

The equation of motion for the centripetal acceleration a_c, can be written as:

$F_{net} = ma_c$

$F_N + F_g = ma_c$

$F_N + mg = \frac{mv^2}{r}$

Normal reaction is provided by the speed of the motorcyclist. At the minimum speed (v_{min}), $F_N = 0$

$$mg = \frac{mv_{min}^2}{r}$$

$$\therefore v_{min} = \sqrt{rg}$$
$$= \sqrt{25 \times 10} = 15.8 \text{ m/s}$$

Question 5.39:

A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed?

Answer 5.39:

Mass of the man, $m = 70$ kg

Radius of the drum, $r = 3$ m

Coefficient of friction, $\mu = 0.15$

Frequency of rotation, $\nu = 200$ rev/min $= 200/60 = 10/3$ rev/s

The necessary centripetal force required for the rotation of the man is provided by the normal force (F_N).

When the floor revolves, the man sticks to the wall of the drum. Hence, the weight of the man (mg) acting downward is balanced by the frictional force ($f = \mu F_N$) acting upward.

Hence, the man will not fall until:

$mg < f$ $mg < \mu F_N$

$= \mu m r \omega^2$ $g < \mu r \omega^2$

$$\omega > \sqrt{\frac{g}{\mu r}}$$

The minimum angular speed is given as:

$$\omega_{\min} = \sqrt{\frac{g}{\mu r}}$$

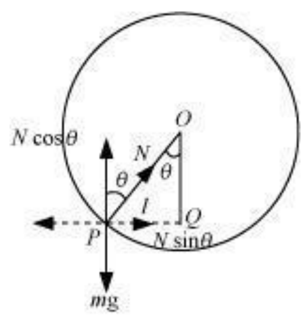
$$= \sqrt{\frac{10}{0.15 \times 3}} = 4.71 \text{ rad s}^{-1}$$

Question 5.40:

A thin circular loop of radius *R* rotates about its vertical diameter with an angular frequency ω. Show that a small bead on the wire loop remains at its lowermost point for $\omega \leq \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{g/R}$? Neglect friction.

Answer 5.40:

Let the radius vector joining the bead with the centre make an angle θ, with the vertical downward direction.



OP = *R* = Radius of the circle

N = Normal reaction

The respective vertical and horizontal equations of forces can be written as:

$Mg = N\cos\theta$ …….…..... (*i*)

$ml\omega^2 = N\sin\theta$ ………... (*ii*)

In ΔOPQ, we have:

$$\sin\theta = \frac{l}{R}$$

$l = R\sin\theta$ ……………………… (*iii*)

Substituting equation (*iii*) in equation (*ii*), we get:

$m(R\sin\theta)\,\omega^2 = N\sin\theta$

$mR\,\omega^2 = N$ ………..................(*iv*)

Substituting equation (*iv*) in equation (*i*), we get:

$mg = mR\,\omega^2\cos\theta$

$\cos\theta = \frac{g}{R\omega^2}$ ………………... (*v*)

Since $\cos\theta \leq 1$, the bead will remain at its lowermost point for $\frac{g}{R\omega^2} \leq 1$, i.e., for $\omega \leq \sqrt{\frac{g}{R}}$

For $\omega = \sqrt{\frac{2g}{R}}$ or $\omega^2 = \frac{2g}{R}$ ... (*vi*)

On equating equations (*v*) and (*vi*), we get:

$\frac{2g}{R} = \frac{g}{R\cos\theta}$

$\cos\theta = \frac{1}{2}$

$\therefore \theta = \cos^{-1}(0.5) = 60°$