Additional Exercises

Question 6.24:

A bullet of mass 0.012 kg and horizontal speed 70 m s^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Answer

Mass of the bullet, m = 0.012 kg

Initial speed of the bullet, u_b = 70 m/s

Mass of the wooden block, M = 0.4 kg

Initial speed of the wooden block, u_B = 0

Final speed of the system of the bullet and the block = ν

Applying the law of conservation of momentum:

$$mu_b + Mu_B = (m + M)v$$
$$0.012 \times 70 + 0.4 \times 0 = (0.012 + 0.4)v$$

$$\therefore v = \frac{0.84}{0.412} = 2.04 \text{ m/s}$$

For the system of the bullet and the wooden block:

Mass of the system, m' = 0.412 kg

Velocity of the system = 2.04 m/s

Height up to which the system rises = *h*

Applying the law of conservation of energy to this system:

Potential energy at the highest point = Kinetic energy at the lowest point

$$m'gh = \frac{1}{2}m'v^2$$

$$\therefore h = \frac{1}{2}\left(\frac{v^2}{g}\right)$$

$$= \frac{1}{2} \times \frac{(2.04)^2}{9.8}$$

= 0.2123 m

The wooden block will rise to a height of 0.2123 m.

Heat produced = Kinetic energy of the bullet – Kinetic energy of the system

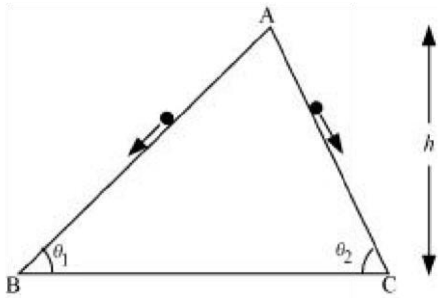
$$= \frac{1}{2}mu^2 - \frac{1}{2}m'v^2$$

$$= \frac{1}{2} \times 0.012 \times (70)^2 - \frac{1}{2} \times 0.412 \times (2.04)^2$$

= 29.4 – 0.857 = 28.54 J

Question 6.25:

Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig. 6.16). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given $\theta_1 = 30°$, $\theta_2 = 60°$, and $h = 10$ m, what are the speeds and times taken by the two stones?



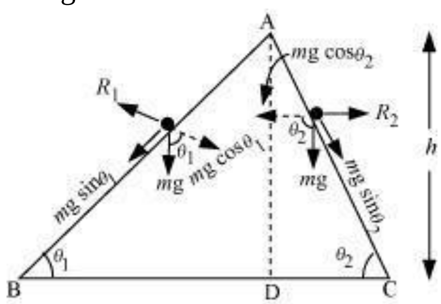
Answer

No; the stone moving down the steep plane will reach the bottom

first Yes; the stones will reach the bottom with the same speed v_B =

v_C = 14 m/s t_1 = 2.86 s; t_2 = 1.65 s

The given situation can be shown as in the following figure:



Here, the initial height (AD) for both the stones is the same (h). Hence, both will have the same potential energy at point A.

As per the law of conservation of energy, the kinetic energy of the stones at points B and C will also be the same, i. e.,

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$$

$v_1 = v_2 = v$, say Where, m = Mass of each

stone v = Speed of each stone at points

B and C

Hence, both stones will reach the bottom with the same speed, v.

For stone I:

Net force acting on this stone is given by:

$$F_{net} = ma_1 = mg\sin\theta_1$$

$$a_1 = g\sin\theta_1$$

For stone II:

$$a_2 = g\sin\theta_2$$

$$\because \theta_2 > \theta_1$$

$$\therefore \sin\theta_2 > \sin\theta_1$$

$$\therefore a_2 > a_1$$

Using the first equation of motion, the time of slide can be obtained as:

$$v = u + at$$

$$\therefore t = \frac{v}{a} \qquad (\because u = 0)$$

For stone I:

$$t_1 = \frac{v}{a_1}$$

For stone II:

$$t_2 = \frac{v}{a_2}$$

$$\because a_2 > a_1$$

$$\therefore t_2 < t_1$$

Hence, the stone moving down the steep plane will reach the bottom first.

The speed (v) of each stone at points B and C is given by the relation obtained from the law of conservation of energy.

$$mgh = \frac{1}{2}mv^2$$

$$\begin{aligned}\therefore v &= \sqrt{2gh}\\ &= \sqrt{2 \times 9.8 \times 10}\\ &= \sqrt{196} = 14 \text{ m/s}\end{aligned}$$

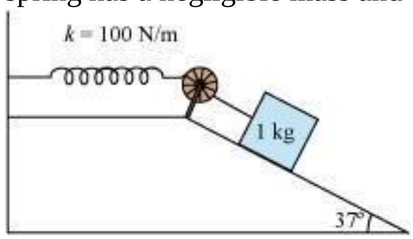
The times are given as:

$$t_1 = \frac{v}{a_1} = \frac{v}{\text{g} \sin\theta_1} = \frac{14}{9.8 \times \sin 30} = \frac{14}{9.8 \times \frac{1}{2}} = 2.86 \text{ s}$$

$$t_2 = \frac{v}{a_2} = \frac{v}{\text{g} \sin\theta_2} = \frac{14}{9.8 \times \sin 60} = \frac{14}{9.8 \times \frac{\sqrt{3}}{2}} = 1.65 \text{ s}$$

Question 6.26:

A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in Fig. 6.17. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.



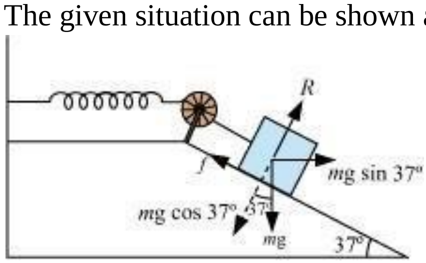
Answer

Mass of the block, m = 1 kg

Spring constant, k = 100 N m^{-1}

Displacement in the block, x = 10 cm = 0.1 m

The given situation can be shown as in the following figure.



At equilibrium:

Normal reaction, $R = mg \cos 37°$

Frictional force, $f = \mu R = mg \sin 37°$

Where, μ is the coefficient of friction

Net force acting on the block = $mg\sin 37° - f$

$= mg\sin 37° - \mu mg\cos 37°$

$= mg(\sin 37° - \mu\cos 37°)$

At equilibrium, the work done by the block is equal to the potential energy of the spring, i.e.,

$$mg(\sin 37° - \mu\cos 37°)x = \frac{1}{2}kx^2$$

$$1 \times 9.8(\sin 37° - \mu\cos 37°) = \frac{1}{2} \times 100 \times 0.1$$

$$0.602 - \mu \times 0.799 = 0.510$$

$$\therefore \mu = \frac{0.092}{0.799} = 0.115$$

Question 6.27:

A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with an uniform speed of 7 m s^{-1}. It hits the floor of the elevator (length of the elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?

Answer

Mass of the bolt, m = 0.3 kg

Speed of the elevator = 7 m/s

Height, $h = 3$ m

Since the relative velocity of the bolt with respect to the lift is zero, at the time of impact, potential energy gets converted into heat energy.

Heat produced = Loss of potential energy

$= mgh = 0.3 \times 9.8 \times 3$

$= 8.82$ J

The heat produced will remain the same even if the lift is stationary. This is because of the fact that the relative velocity of the bolt with respect to the lift will remain zero.

Question 6.28:

A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 m s^{-1} relative to the trolley in a direction opposite to the its motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

Answer

Mass of the trolley, $M = 200$ kg

Speed of the trolley, $v = 36$ km/h $= 10$ m/s

Mass of the boy, $m = 20$ kg

Initial momentum of the system of the boy and the trolley

$= (M + m)v$

$= (200 + 20) \times 10$

= 2200 kg m/s

Let v' be the final velocity of the trolley with respect to the ground.

Final velocity of the boy with respect to the ground $= v' - 4$

Final momentum $= Mv' + m(v' - 4)$

= 200v' + 20v' – 80

= 220v' – 80

As per the law of conservation of momentum:

Initial momentum = Final momentum

2200 = 220v' – 80

$$\therefore v' = \frac{2280}{220} = 10.36 \text{ m/s}$$

Length of the trolley, l = 10 m

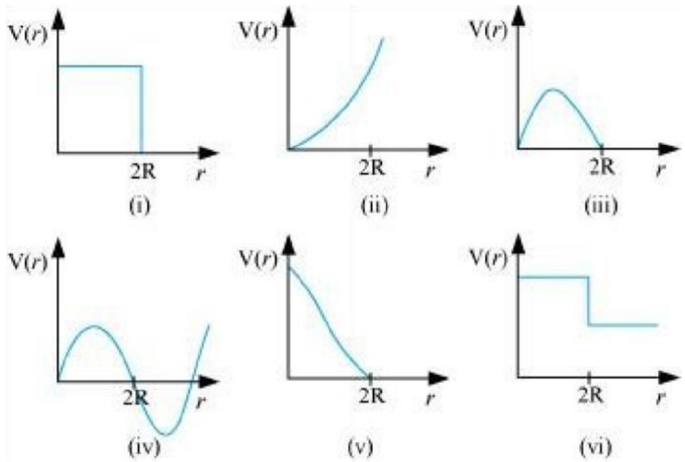
Speed of the boy, v'' = 4 m/s

Time taken by the boy to run, $t = \frac{10}{4} = 2.5 \text{ s}$

$\therefore$ Distance moved by the trolley = v'' × t = 10.36 × 2.5 = 25.9 m

Question 6.29:

Which of the following potential energy curves in Fig. 6.18 cannot possibly describe the elastic collision of two billiard balls? Here r is the distance between centres of the balls.



Answer

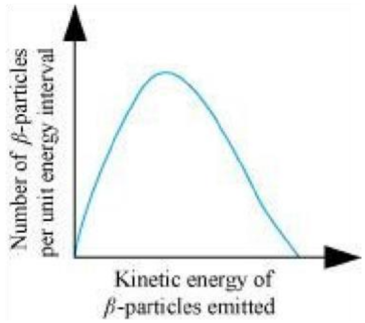
(i), (ii), (iii), (iv), and (vi)

The potential energy of a system of two masses is inversely proportional to the separation between them. In the given case, the potential energy of the system of the two balls will decrease as they come closer to each other. It will become zero (i.e., $V(r) = 0$) when the two balls touch each other, i.e., at $r = 2R$, where R is the radius of each billiard ball. The potential energy curves given in figures (i), (ii), (iii), (iv), and (vi) do not satisfy these two conditions. Hence, they do not describe the elastic collisions between them.

Question 6.30:

Consider the decay of a free neutron at rest: $n \rightarrow p + e^{-}$

Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β-decay of a neutron or a nucleus (Fig. 6.19).



[Note: The simple result of this exercise was one among the several arguments advanced by W. Pauli to predict the existence of a third particle in the decay products of β-decay. This particle is known as neutrino. We now know that it is a particle of intrinsic spin ½ (like e^-, p or n), but is neutral, and either massless or having an extremely small mass (compared to the mass of electron) and which interacts very weakly with matter. The correct decay process of neutron is: $n \rightarrow p + e^- + \nu$]

Answer

The decay process of free neutron at rest is given as:

$$n \rightarrow p + e^-$$

From Einstein's mass-energy relation, we have the energy of electron as Δmc^2

Where,

Δm = Mass defect = Mass of neutron – (Mass of proton + Mass of electron)

c = Speed of light

Δm and c are constants. Hence, the given two-body decay is unable to explain the continuous energy distribution in the β-decay of a neutron or a nucleus. The presence of neutrino νon the LHS of the decay correctly explains the continuous energy distribution.