(Chapter 8)(Gravitation)

XI

Additional Exercises

Question 8.22:

As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the earth = 6.0×10^{24} kg, radius = 6400 km.

Answer

Mass of the Earth, $M = 6.0 \times 10^{24}$ kg

Radius of the Earth, $R = 6400$ km $= 6.4 \times 10^{6}$ m

Height of a geostationary satellite from the surface of the Earth,

$h = 36000$ km $= 3.6 \times 10^{7}$ m

Gravitational potential energy due to Earth's gravity at height h,

$$= \frac{-GM}{(R+h)}$$

$$= -\frac{6.67\times10^{-11}\times6.0\times10^{24}}{3.6\times10^{7}+0.64\times10^{7}}$$

$$= -\frac{6.67\times6}{4.24}\times10^{13-7}$$

$$= -9.4\times10^{6} \text{ J/kg}$$

Question 8.23:

A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars.

Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (Mass of the sun = 2×10^{30} kg).

Answer: Yes

A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, $f_g = \frac{GMm}{R^2}$

Where,

M = Mass of the star = $2.5 \times 2 \times 10^{30} = 5 \times 10^{30}$ kg

m = Mass of the body

R = Radius of the star = 12 km = 1.2×10^4 m

$$\therefore f_g = \frac{6.67 \times 10^{-11} \times 5 \times 10^{30} \times m}{\left(1.2 \times 10^4\right)^2} = 2.31 \times 10^{11} m \text{ N}$$

Centrifugal force, $f_c = mr\omega^2$ ω =

Angular speed = $2\pi v$ v = Angular

frequency = 1.2 rev s^{-1} $f_c = mR$

$(2\pi v)^2$

$= m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2 = 1.7 \times 10^5 m$N

Since $f_g > f_c$, the body will remain stuck to the surface of the star.

Question 8.24:

A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the Sun = 2×10^{30} kg; mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; radius of the orbit of mars = 2.28×10^{8}kg; G= 6.67×10^{-11} m^{2}kg^{-2}.

Answer

Mass of the spaceship, m_s = 1000 kg

Mass of the Sun, $M = 2 \times 10^{30}$ kg

Mass of Mars, $m_m = 6.4 \times 10^{23}$ kg

Orbital radius of Mars, $R = 2.28 \times 10^{8}$ kg $= 2.28 \times 10^{11}$m

Radius of Mars, r = 3395 km = 3.395×10^{6} m

Universal gravitational constant, G = 6.67×10^{-11} m^{2}kg^{-2}

Potential energy of the spaceship due to the gravitational attraction of the Sun

$$= \frac{-GMm_s}{R}$$

Potential energy of the spaceship due to the gravitational attraction of Mars $= \frac{-GM_m m_s}{r}$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.

Total energy of the spaceship $= \frac{-GMm_s}{R} - \frac{-GM_s m_m}{r}$

$$= -Gm_s\left(\frac{M}{R} + \frac{m_m}{r}\right)$$

The negative sign indicates that the system is in bound state.

Energy required for launching the spaceship out of the solar system

= – (Total energy of the spaceship)

$$= Gm_s\left(\frac{M}{R}+\frac{m_m}{r}\right)$$

$$= 6.67\times10^{-11}\times10^3\times\left(\frac{2\times10^{30}}{2.28\times10^{11}}+\frac{6.4\times10^{23}}{3.395\times10^6}\right)$$

$$= 6.67\times10^{-8}\left(87.72\times10^{17}+1.88\times10^{17}\right)$$

$$= 6.67\times10^{-8}\times89.50\times10^{17}$$

$$= 596.97\times10^{9}$$

$$= 6\times10^{11}\text{ J}$$

Question 8.25:

A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s–1. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = 6.4× 1023 kg; radius of mars = 3395 km; G = 6.67× 10^{-11} N m^2 kg^{-2}.

Answer

Initial velocity of the rocket, v = 2 km/s = 2 × 10^3 m/s

Mass of Mars, M = 6.4 × 10^{23} kg

Radius of Mars, R = 3395 km = 3.395 × 10^6 m

Universal gravitational constant, G = 6.67× 10^{-11} N m^2 kg^{-2}

Mass of the rocket = m

Initial kinetic energy of the rocket = $\frac{1}{2}mv^2$

Initial potential energy of the rocket $= \frac{-GMm}{R}$

Total initial energy $= \frac{1}{2}mv^2 - \frac{GMm}{R}$

If 20 % of initial kinetic energy is lost due to Martian atmospheric resistance, then only 80 % of its kinetic energy helps in reaching a height.

Total initial energy available $= \frac{80}{100} \times \frac{1}{2}mv^2 - \frac{GMm}{R} = 0.4mv^2 - \frac{GMm}{R}$

Maximum height reached by the rocket = h

At this height, the velocity and hence, the kinetic energy of the rocket will become zero.

Total energy of the rocket at height h $= -\frac{GMm}{(R+h)}$

Applying the law of conservation of energy for the rocket, we can write:

$$0.4mv^2 - \frac{GMm}{R} = \frac{-GMm}{(R+h)}$$

$$0.4v^2 = \frac{GM}{R} - \frac{GM}{R+h}$$

$$= GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= GM\left(\frac{R+h-R}{R(R+h)}\right)$$

$$= \frac{GMh}{R(R+h)}$$

$$\frac{R+h}{h} = \frac{GM}{0.4v^2R}$$

$$\frac{R}{h} + 1 = \frac{GM}{0.4v^2R}$$

$$\frac{R}{h} = \frac{GM}{0.4v^2R} - 1$$

$$h = \frac{R}{\frac{GM}{0.4v^2R} - 1}$$

$$= \frac{0.4R^2v^2}{GM - 0.4v^2R}$$

$$= \frac{0.4 \times (3.395 \times 10^6)^2 \times (2 \times 10^3)^2}{6.67 \times 10^{-11} \times 6.4 \times 10^{23} - 0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)}$$

$$= \frac{18.442 \times 10^{18}}{42.688 \times 10^{12} - 5.432 \times 10^{12}} = \frac{18.442}{37.256} \times 10^6$$

$$= 495 \times 10^3 \text{ m} = 495 \text{ km}$$