Question 9.17:

Anvils made of single crystals of diamond, with the shape as shown in Fig. 9.14, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?

Fig. 9.14

Answer 9.17:

Diameter of the cones at the narrow ends, $d = 0.50\text{ mm} = 0.5 \times 10^{-3}\text{ m}$

Radius, $r = \frac{d}{2} = 0.25 \times 10^{-3}\text{ m}$

Compressional force, $F = 50000$ N

Pressure at the tip of the anvil:

$$p = \frac{\text{Force}}{\text{Area}\pi} = \frac{F}{r^2}$$

$$= \frac{50000}{\pi\left(0.25 \times 10^{-3}\right)^2}$$

$$= 2.55 \times 10^{11}\text{ Pa}$$

Therefore, the pressure at the tip of the anvil is 2.55×10^{11} Pa.

Question 9.18:

A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire **A**) and aluminium (wire **B**) of equal lengths as shown in Fig. 9.15. The crosssectional areas of wires **A** and **B** are 1.0 mm^2 and 2.0 mm^2, respectively. At what point along the rod should a mass *m* be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.

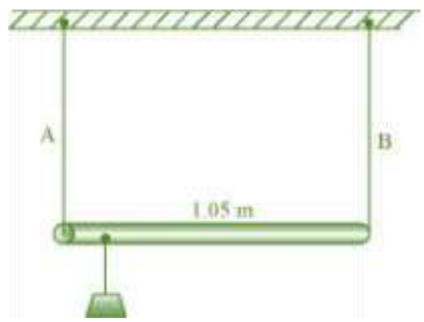


Fig. 9.15

Answer 9.18:

(a) 0.7 m from the steel-wire end 0.432 m from the steel-wire end

Cross-sectional area of wire **A**, $a_1 = 1.0\ mm^2 = 1.0 \times 10^{-6}\ m^2$

Cross-sectional area of wire **B**, $a_2 = 2.0\ mm^2 = 2.0 \times 10^{-6}\ m^2$

Young's modulus for steel, $Y_1 = 2 \times 10^{11}\ Nm^{-2}$

Young's modulus for aluminium, $Y_2 = 7.0 \times 10^{10}\ Nm^{-2}$

Let a small mass *m* be suspended to the rod at a distance *y* from the end where wire **A** is attached.

$$\text{Stress in the wire} = \frac{\text{Force}}{\text{Area}} = \frac{F}{a}$$

If the two wires have equal stresses, then:

$$\frac{F_1}{a_1} = \frac{F_2}{a_2}$$

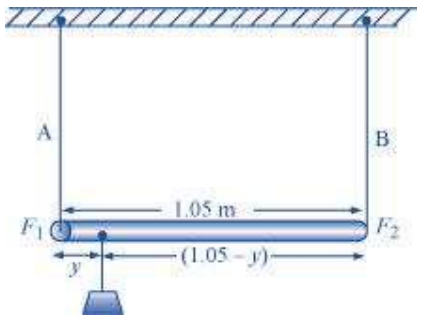
Where,

F_1 = Force exerted on the steel wire

F_2 = Force exerted on the aluminum wire

$$\frac{F_1}{F_2} = \frac{a_1}{a_2} = \frac{1}{2} \qquad \ldots(i)$$

The situation is shown in the following figure.



Taking torque about the point of suspension, we have:

$$F_1 y = F_2 (1.05 - y)$$

$$\frac{F_1}{F_2} = \frac{(1.05 - y)}{y} \qquad \ldots(ii)$$

Using equations (i) and (ii), we can write:

$$\frac{(1.05 - y)}{y} = \frac{1}{2}$$

$$2(1.05 - y) = y$$
$$2.1 - 2y = y$$
$$3y = 2.1$$
$$\therefore y = 0.7\,\text{m}$$

In order to produce an equal stress in the two wires, the mass should be suspended at a distance of 0.7 m from the end where wire **A** is attached.

$$\text{Young's modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Strain} = \frac{\text{Stress}}{\text{Young's modulus}} = \frac{\frac{F}{a}}{Y}$$

If the strain in the two wires is equal, then:

$$\frac{\frac{F_1}{a_1}}{Y_1} = \frac{\frac{F_2}{a_2}}{Y_2}$$

$$\frac{F_1}{F_2} = \frac{a_1}{a_2}\frac{Y_1}{Y_2} = \frac{1}{2} \times \frac{2 \times 10^{11}}{7 \times 10^{10}} = \frac{10}{7} \qquad \ldots(iii)$$

Taking torque about the point where mass m, is suspended at a distance y_1 from the side where wire **A** attached, we get:

$F_1 y_1 = F_2 (1.05 - y_1)$

$$\frac{F_1}{F_2} = \frac{(1.05 - y_1)}{y_1} \qquad \ldots(iii)$$

Using equations (iii) and (iv), we get:

$$\frac{(1.05 - y_1)}{y_1} = \frac{10}{7}$$

$$7(1.05 - y_1) = 10y_1$$
$$17y_1 = 7.35$$
$$\therefore y_1 = 0.432 \text{ m}$$

In order to produce an equal strain in the two wires, the mass should be suspended at a distance of 0.432 m from the end where wire **A** is attached.

Question 9.19:

A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the midpoint.

Answer 9.19:

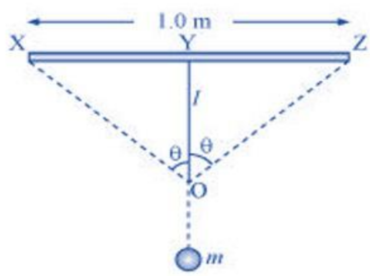
Length of the steel wire = 1.0 m

Area of cross-section, $A = 0.50 \times 10^{-2}\ cm^2 = 0.50 \times 10^{-6}\ m^2$

A mass 100 g is suspended from its midpoint.

m = 100 g = 0.1 kg

Hence, the wire dips, as shown in the given figure.



Original length = XZ

Depression = l

The length after mass m, is attached to the wire = XO + OZ

Increase in the length of the wire:

Δl = (XO + OZ) – XZ

Where,

XO = OZ = $\left[(0.5)^2 + l^2\right]^{\frac{1}{2}}$

$$\therefore \Delta l = 2\left[(0.5)^2 + (l)^2\right]^{\frac{1}{2}} - 1.0$$

$$= 2 \times 0.5\left[1 + \left(\frac{l}{0.5}\right)^2\right]^{\frac{1}{2}} - 1.0$$

Expanding and neglecting higher terms, we get:

$$\Delta l = \frac{l^2}{0.5}$$

$$\text{Strain} = \frac{\text{Increase in length}}{\text{Original length}}$$

Let T be the tension in the wire.

$\therefore mg = 2T\cos\theta$

Using the figure, it can be written as:

$$\cos\theta = \frac{l}{\left((0.5)^2 + l^2\right)^{\frac{1}{2}}}$$

$$= \frac{l}{(0.5)\left(1 + \left(\frac{l}{0.5}\right)^2\right)^{\frac{1}{2}}}$$

Expanding the expression and eliminating the higher terms:

$$\cos\theta = \frac{l}{(0.5)\left(1 + \frac{l^2}{2(0.5)^2}\right)}$$

$$\left(1 + \frac{l^2}{0.5}\right) \simeq 1 \text{ for small } l$$

$$\therefore \cos\theta = \frac{l}{0.5}$$

$$\therefore T = \frac{mg}{2\left(\frac{l}{0.5}\right)} = \frac{mg \times 0.5}{2l} = \frac{mg}{4l}$$

$$\text{Stress} = \frac{\text{Tension}}{\text{Area}} = \frac{mg}{4l \times A}$$

$$\text{Young's modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \frac{mg \times 0.5}{4l \times A \times l^2}$$

$$l = \sqrt[3]{\frac{mg \times 0.5}{4YA}}$$

Young's modulus of steel, $Y = 2 \times 10^{11}$ Pa

$$\therefore l = \sqrt{\frac{0.1 \times 9.8 \times 0.5}{4 \times 2 \times 10^{11} \times 0.50 \times 10^{-6}}}$$

$$= 0.0106 \text{ m}$$

Hence, the depression at the midpoint is 0.0106 m.

Question 9.20:

Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one quarter of the load.

Answer 9.20:

Diameter of the metal strip, $d = 6.0$ mm $= 6.0 \times 10^{-3}$ m

Radius, $r = \frac{d}{2} = 3.0 \times 10^{-3}$ m

Maximum shearing stress = 6.9×10^7 Pa

$$\text{Maximum stress} = \frac{\text{Maximum load or force}}{\text{Area}}$$

Maximum force = Maximum stress × Area

$= 6.9 \times 10^7 \times \pi \times (r)^2$

$= 6.9 \times 10^7 \times \pi \times (3 \times 10^{-3})^2$

$= 1949.94$ N

Each rivet carries one quarter of the load.

$\therefore$ Maximum tension on each rivet $= 4 \times 1949.94 = 7799.76$ N

Question 9.21:

The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1 $\times 10^8$ Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?

Answer 9.21:

Water pressure at the bottom, $p = 1.1 \times 10^8$ Pa

Initial volume of the steel ball, $V = 0.32$ m^3

Bulk modulus of steel, $B = 1.6 \times 10^{11}$ Nm^{-2}

The ball falls at the bottom of the Pacific Ocean, which is 11 km beneath the surface.

Let the change in the volume of the ball on reaching the bottom of the trench be ΔV.

$$\text{Bulk modulus, } B = \frac{p}{\frac{\Delta V}{V}}$$

$$\Delta V = \frac{B}{pV}$$

$$= \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}}$$

$$= 2.2 \times 10^{-4} \text{ m}^3$$

Therefore, the change in volume of the ball on reaching the bottom of the trench is 2.2×10^{-4} m^3.