Physics

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(Chapter 13)(Kinetic Theory)

XI

Exercises

Question 13.1:

Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3Å.

Answer

Diameter of an oxygen molecule, d = 3Å

Radius, $r = \frac{d}{2} = \frac{3}{2}$ = 1.5 Å = 1.5 × 10^{-8} cm

Actual volume occupied by 1 mole of oxygen gas at STP = 22400 cm^3

Molecular volume of oxygen gas, $V = \frac{4}{3}\pi r^3 \cdot N$

Where, N is Avogadro's number = 6.023 × 10^{23} molecules/mole

$$\therefore V = \frac{4}{3} \times 3.14 \times \left(1.5 \times 10^{-8}\right)^3 \times 6.023 \times 10^{23} = 8.51 \text{ cm}^3$$

Ratio of the molecular volume to the actual volume of oxygen $= \frac{8.51}{22400}$

= 3.8 × 10^{-4}

Question 13.2:

Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP: 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres.

Answer

The ideal gas equation relating pressure (P), volume (V), and absolute temperature (T) is given as:

$PV = nRT$

Where,

R is the universal gas constant = 8.314 J mol^{-1} K^{-1}

n = Number of moles = 1

T = Standard temperature = 273 K

P = Standard pressure = 1 atm = 1.013×10^5 Nm^{-2}

$$\therefore V = \frac{nRT}{P}$$

$$= \frac{1 \times 8.314 \times 273}{1.013 \times 10^5}$$

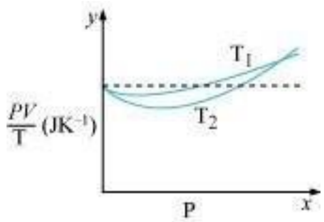
= 0.0224 m^3

= 22.4 litres

Hence, the molar volume of a gas at STP is 22.4 litres.

Question 13.3:

Figure 13.8 shows plot of PV/T versus Pfor 1.00×10^{-3} kg of oxygen gas at two different temperatures.



What does the dotted plot signify?

Which is true: $T_1 > T_2$ or $T_1 < T_2$?

What is the value of PV/T where the curves meet on the y-axis?

If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot)? (Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mo1^{-1} K^{-1}.)

Answer

The dotted plot in the graph signifies the ideal behaviour of the gas, i.e., the ratio $\frac{PV}{T}$ is equal. μR (μ is the number of moles and R is the universal gas constant) is a constant quality. It is not dependent on the pressure of the gas.

The dotted plot in the given graph represents an ideal gas. The curve of the gas at temperature T_1 is closer to the dotted plot than the curve of the gas at temperature T_2. A real gas approaches the behaviour of an ideal gas when its temperature increases. Therefore, $T_1 > T_2$ is true for the given plot.

The value of the ratio PV/T, where the two curves meet, is μR. This is because the ideal gas equation is given as:

$PV = \mu RT$

$\frac{PV}{T} = \mu R$

Where,

P is the pressure

T is the temperature V

is the volume μ is the

number of moles

R is the universal constant

Molecular mass of oxygen = 32.0 g

Mass of oxygen = 1×10^{-3} kg = 1 g

$R = 8.314$ J mole^{-1} K^{-1}

$$\therefore \frac{PV}{T} = \frac{1}{32} \times 8.314$$

$= 0.26$ J K^{-1}

Therefore, the value of the ratio PV/T, where the curves meet on the y-axis, is

0.26 J K^{-1}.

If we obtain similar plots for 1.00×10^{-3} kg of hydrogen, then we will not get the same value of PV/T at the point where the curves meet the y-axis. This is because the molecular mass of hydrogen (2.02 u) is different from that of oxygen (32.0 u).

We have:

$$\frac{PV}{T} = 0.26 \text{ J K}^{-1}$$

$R = 8.314$ J mole^{-1} K^{-1}

Molecular mass (M) of H_2 = 2.02 u

$$\frac{PV}{T} = \mu R \text{ at constant temperature}$$

Where, $\mu = \frac{m}{M}$

m = Mass of H_2

$$\therefore m = \frac{PV}{T} \times \frac{M}{R}$$

$$= \frac{0.26 \times 2.02}{8.31}$$

$= 6.3 \times 10^{-2}$ g $= 6.3 \times 10^{-5}$ kg

Hence, 6.3×10^{-5} kg of H_2 will yield the same value of PV/T.

Question 13.4:

An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27 °C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder (R = 8.31 J mol^{-1} K^{-1}, molecular mass of O_2 = 32 u).

Answer

Volume of oxygen, V_1 = 30 litres = 30×10^{-3} m^3

Gauge pressure, P_1 = 15 atm = $15 \times 1.013 \times 10^5$ Pa

Temperature, T_1 = 27°C = 300 K

Universal gas constant, R = 8.314 J mole^{-1} K^{-1}

Let the initial number of moles of oxygen gas in the cylinder be n_1.

The gas equation is given as:

$P_1V_1 = n_1RT_1$

$$\therefore n_1 = \frac{P_1V_1}{RT_1}$$

$$= \frac{15.195 \times 10^5 \times 30 \times 10^{-3}}{(8.314) \times 300} = 18.276$$

But, $n_1 = \frac{m_1}{M}$ Where, m_1 =

Initial mass of oxygen

M = Molecular mass of oxygen = 32 g $\therefore$

$m_1 = n_1M = 18.276 \times 32 = 584.84$ g

After some oxygen is withdrawn from the cylinder, the pressure and temperature reduces.

Volume, V_2 = 30 litres = 30×10^{-3} m^3

Gauge pressure, P_2 = 11 atm = $11 \times 1.013 \times 10^5$ Pa

Temperature, T_2 = 17°C = 290 K

Let n_2 be the number of moles of oxygen left in the cylinder.

The gas equation is given as:

$P_2V_2 = n_2RT_2$

$$\therefore n_2 = \frac{P_2V_2}{RT_2}$$

$$= \frac{11.143 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290} = 13.86$$

But, $n_2 = \frac{m_2}{M}$ Where, m_2 is the mass of oxygen

remaining in the cylinder $\therefore m_2 = n_2M = 13.86 \times 32$

= 453.1 g

The mass of oxygen taken out of the cylinder is given by the relation:

Initial mass of oxygen in the cylinder – Final mass of oxygen in the cylinder

$= m_1 - m_2$

= 584.84 g – 453.1 g

= 131.74 g

= 0.131 kg

Therefore, 0.131 kg of oxygen is taken out of the cylinder.

Question 13.5:

An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12 °C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C?

Answer

Volume of the air bubble, $V_1 = 1.0\ cm^3 = 1.0 \times 10^{-6}\ m^3$

Bubble rises to height, $d = 40$ m

Temperature at a depth of 40 m, $T_1 = 12°C = 285$ K

Temperature at the surface of the lake, $T_2 = 35°C = 308$ K

The pressure on the surface of the lake:

$P_2 = 1\ atm = 1 \times 1.013 \times 10^5$ Pa

The pressure at the depth of 40 m:

$P_1 = 1\ atm + d\rho g$ Where, ρ is the density of

water = $10^3\ kg/m^3$ g is the acceleration due

to gravity = $9.8\ m/s^2$

$\therefore P_1 = 1.013 \times 10^5 + 40 \times 10^3 \times 9.8 = 493300$ Pa

We have: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Where, V_2 is the volume of the air bubble when it reaches the surface

$$V_2 = \frac{P_1V_1T_2}{T_1P_2}$$

$$= \frac{(493300)(1.0 \times 10^{-6})308}{285 \times 1.013 \times 10^5}$$

$= 5.263 \times 10^{-6}\ m^3$ or $5.263\ cm^3$

Therefore, when the air bubble reaches the surface, its volume becomes 5.263 cm^3.

Question 13.6:

Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure.

Answer

Volume of the room, $V = 25.0\ m^3$

Temperature of the room, $T = 27°C = 300\ K$

Pressure in the room, $P = 1\ atm = 1 \times 1.013 \times 10^5\ Pa$

The ideal gas equation relating pressure (P), Volume (V), and absolute temperature (T) can be written as:

$PV = k_B NT$

Where,

K_B is Boltzmann constant $= 1.38 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$

N is the number of air molecules in the room

$\therefore N = \frac{PV}{k_B T}$

$= \frac{1.013 \times 10^5 \times 25}{1.38 \times 10^{-23} \times 300} = 6.11 \times 10^{26}$ molecules

Therefore, the total number of air molecules in the given room is 6.11×10^{26}.

Question 13.7:

Estimate the average thermal energy of a helium atom at (i) room temperature (27 °C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million Kelvin (the typical core temperature in the case of a star).

Answer

At room temperature, T = 27°C = 300 K

Average thermal energy $= \frac{3}{2} kT$

Where k is Boltzmann constant = $1.38 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$

$$\therefore \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-38} \times 300$$

$= 6.21 \times 10^{-21}$ J

Hence, the average thermal energy of a helium atom at room temperature (27°C) is 6.21×10^{-21} J.

On the surface of the sun, T = 6000 K

Average thermal energy $= \frac{3}{2} kT$

$$= \frac{3}{2} \times 1.38 \times 10^{-38} \times 6000$$

$= 1.241 \times 10^{-19}$ J

Hence, the average thermal energy of a helium atom on the surface of the sun is 1.241×10^{-19} J.

At temperature, $T = 10^7$ K

Average thermal energy $= \frac{3}{2}kT$

$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^{7}$

$= 2.07 \times 10^{-16}$ J

Hence, the average thermal energy of a helium atom at the core of a star is 2.07×10^{-16} J.

Question 13.8:

Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

Answer

Yes. All contain the same number of the respective molecules.

No. The root mean square speed of neon is the largest.

Since the three vessels have the same capacity, they have the same volume.

Hence, each gas has the same pressure, volume, and temperature.

According to Avogadro's law, the three vessels will contain an equal number of the respective molecules. This number is equal to Avogadro's number, $N = 6.023 \times 10^{23}$.

The root mean square speed (v_{rms}) of a gas of mass m, and temperature T, is given by the relation:

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Where, k is Boltzmann constant

For the given gases, k and T are constants.

Hence v_{rms} depends only on the mass of the atoms, i.e.,

$$v_{rms} \propto \sqrt{\frac{1}{m}}$$

Therefore, the root mean square speed of the molecules in the three cases is not the same.
Among neon, chlorine, and uranium hexafluoride, the mass of neon is the smallest. Hence, neon has the largest root mean square speed among the given gases.

Question 13.9:

At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at – 20 °C? (atomic mass of Ar = 39.9 u, of He = 4.0 u).

Answer

Temperature of the helium atom, T_{He} = –20°C= 253 K

Atomic mass of argon, M_{Ar} = 39.9 u

Atomic mass of helium, M_{He} = 4.0 u

Let, $(v_{rms})_{Ar}$ be the rms speed of argon.

Let $(v_{rms})_{He}$ be the rms speed of helium.

The rms speed of argon is given by:

$$(v_{rms})_{Ar} = \sqrt{\frac{3RT_{Ar}}{M_{Ar}}} \quad \ldots (i)$$

Where,

R is the universal gas constant

T_{Ar} is temperature of argon gas

The rms speed of helium is given by:

$$(v_{rms})_{He} = \sqrt{\frac{3RT_{He}}{M_{He}}} \quad \ldots (ii)$$

It is given that:

$(v_{rms})_{Ar} = (v_{rms})_{He}$

$$\sqrt{\frac{3RT_{Ar}}{M_{Ar}}} = \sqrt{\frac{3RT_{He}}{M_{He}}}$$

$$\frac{T_{Ar}}{M_{Ar}} = \frac{T_{He}}{M_{He}}$$

$$T_{Ar} = \frac{T_{He}}{M_{He}} \times M_{Ar}$$

$$= \frac{253}{4} \times 39.9$$

$= 2523.675 = 2.52 \times 10^3$ K

Therefore, the temperature of the argon atom is 2.52×10^3 K.

Question 13.10:

Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17 °C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of N_2 = 28.0 u).

Answer

Mean free path = 1.11×10^{-7} m

Collision frequency = 4.58×10^{9} s^{-1}

Successive collision time ≈ 500 × (Collision time)

Pressure inside the cylinder containing nitrogen, P = 2.0 atm = 2.026×10^{5} Pa

Temperature inside the cylinder, T = 17°C =290 K

Radius of a nitrogen molecule, r = 1.0 Å = 1×10^{10} m

Diameter, $d = 2 \times 1 \times 10^{10} = 2 \times 10^{10}$ m

Molecular mass of nitrogen, M = 28.0 g = 28×10^{-3} kg

The root mean square speed of nitrogen is given by the relation:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Where,

R is the universal gas constant = 8.314 J $mole^{-1}$ K^{-1}

$$\therefore v_{\text{rms}} = \sqrt{\frac{3 \times 8.314 \times 290}{28 \times 10^{-3}}} = 508.26 \text{ m/s}$$

The mean free path (l) is given by the relation:

$$l = \frac{kT}{\sqrt{2} \times d^2 \times P}$$

Where, k is the Boltzmann constant = 1.38×10^{-23} kg

$\text{m}^2\,\text{s}^{-2}\text{K}^{-1}$

$$\therefore l = \frac{1.38 \times 10^{-23} \times 290}{\sqrt{2} \times 3.14 \times \left(2 \times 10^{-10}\right)^2 \times 2.026 \times 10^5}$$

$= 1.11 \times 10^{-7}$ m

Collision frequency $= \frac{v_{\text{rms}}}{l}$

$$= \frac{508.26}{1.11 \times 10^{-7}} = 4.58 \times 10^9 \text{ s}^{-1}$$

Collision time is given as:

$$T = \frac{d}{v_{\text{rms}}}$$

$$= \frac{2 \times 10^{-10}}{508.26} = 3.93 \times 10^{-13} \text{ s}$$

Time taken between successive collisions:

$$T' = \frac{l}{v_{\text{rms}}}$$

$$= \frac{1.11 \times 10^{-7}\ \text{m}}{508.26\ \text{m/s}} = 2.18 \times 10^{-10}\ \text{s}$$

$$\therefore \frac{T'}{T} = \frac{2.18 \times 10^{-10}}{3.93 \times 10^{-13}} = 500$$

Hence, the time taken between successive collisions is 500 times the time taken for a collision.