

Mathematics

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(Chapter – 10) (Straight Lines)

(Class – XI)

Exercise 10.3

Question 1:

Reduce the following equations into slope-intercept form and find their slopes and the y intercepts.

(i) $x + 7y = 0$

(ii) $6x + 3y - 5 = 0$

(iii) $y = 0$

Answer 1:

(i) The given equation is $x + 7y = 0$. It can be written as

$$y = -\frac{1}{7}x + 0 \quad \dots(1)$$

This equation is of the form $y = mx + c$, where $m = -\frac{1}{7}$ and $c = 0$.

Therefore, equation (1) is in the slope-intercept form, where the slope and the y-

intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x + 3y - 5 = 0$.

It can be written as

$$y = \frac{1}{3}(-6x + 5)$$

$$y = -2x + \frac{5}{3} \quad \dots(2)$$

This equation is of the form $y = mx + c$, where $m = -2$ and $c = \frac{5}{3}$.

Therefore, equation (2) is in the slope-intercept form, where the slope and the y-

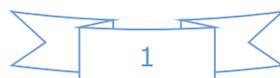
intercept are -2 and $\frac{5}{3}$ respectively.

(iii) The given equation is $y = 0$.

It can be written as

$$y = 0 \cdot x + 0 \quad \dots(3)$$

This equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$.



Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

Therefore, equation (3) is in the slope-intercept form, where the slope and the y-intercept are 0 and 0 respectively.

Question 2:

Reduce the following equations into intercept form and find their intercepts on the axes.

(i) $3x + 2y - 12 = 0$

(ii) $4x - 3y = 6$

(iii) $3y + 2 = 0$.

Answer 2:

(i) The given equation is $3x + 2y - 12 = 0$.

It can be written as

$$3x + 2y = 12$$

$$\frac{3x}{12} + \frac{2y}{12} = 1$$

$$\text{i.e., } \frac{x}{4} + \frac{y}{6} = 1 \quad \dots(1)$$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$,

where $a = 4$ and $b = 6$.

Therefore, equation (1) is in the intercept form, where the intercepts on the x and y axes are 4 and 6 respectively.

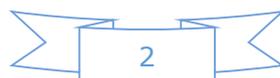
(ii) The given equation is $4x - 3y = 6$.

It can be written as

$$\frac{4x}{6} - \frac{3y}{6} = 1$$

$$\frac{2x}{3} - \frac{y}{2} = 1$$

$$\text{i.e., } \frac{x}{\left(\frac{3}{2}\right)} + \frac{y}{(-2)} = 1 \quad \dots(2)$$



Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = \frac{3}{2}$ and $b = -2$.

Therefore, equation (2) is in the intercept form, where the intercepts on the x and y axes are $\frac{3}{2}$ and -2 respectively.

(iii) The given equation is $3y + 2 = 0$.

It can be written as

$$3y = -2$$

$$\text{i.e., } \frac{y}{\left(-\frac{2}{3}\right)} = 1 \quad \dots(3)$$

The equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 0$ and $b = -\frac{2}{3}$.

Therefore, equation (3) is in the intercept form, where the intercept on the y -axis is $-\frac{2}{3}$ and it has no intercept on the x -axis.

Question 3:

Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x -axis.

(i) $x - \sqrt{3}y + 8 = 0$ **(ii)** $y - 2 = 0$ **(iii)** $x - y = 4$

Answer 3:

(i) The given equation is $x - \sqrt{3}y + 8 = 0$.

It can be reduced as:

$$x - \sqrt{3}y = -8$$

$$\Rightarrow -x + \sqrt{3}y = 8$$

On dividing both sides by $\sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$, we obtain

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(Chapter – 10) (Straight Lines)

(Class – XI)

$$-\frac{x}{2} + \frac{\sqrt{3}}{2}y = \frac{8}{2}$$

$$\Rightarrow \left(-\frac{1}{2}\right)x + \left(\frac{\sqrt{3}}{2}\right)y = 4$$

$$\Rightarrow x \cos 120^\circ + y \sin 120^\circ = 4 \quad \dots(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line $x \cos \omega + y \sin \omega = p$, we obtain $\omega = 120^\circ$ and $p = 4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive x-axis is 120° .

(ii) The given equation is $y - 2 = 0$.

It can be reduced as $0.x + 1.y = 2$

On dividing both sides by $\sqrt{0^2 + 1^2} = 1$, we obtain $0.x + 1.y = 2$

$$\Rightarrow x \cos 90^\circ + y \sin 90^\circ = 2 \quad \dots (1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line $x \cos \omega + y \sin \omega = p$, we obtain $\omega = 90^\circ$ and $p = 2$.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive x-axis is 90° .

(iii) The given equation is $x - y = 4$.

It can be reduced as $1.x + (-1)y = 4$

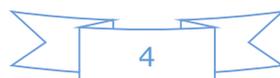
On dividing both sides by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}}x + \left(-\frac{1}{\sqrt{2}}\right)y = \frac{4}{\sqrt{2}}$$

$$\Rightarrow x \cos \left(2\pi - \frac{\pi}{4}\right) + y \sin \left(2\pi - \frac{\pi}{4}\right) = 2\sqrt{2}$$

$$\Rightarrow x \cos 315^\circ + y \sin 315^\circ = 2\sqrt{2} \quad \dots(1)$$

Equation (1) is in the normal form.



Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 315^\circ$ and $p = 2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive x-axis is 315° .

Question 4:

Find the distance of the point $(-1, 1)$ from the line $12(x + 6) = 5(y - 2)$.

Answer 4:

The given equation of the line is $12(x + 6) = 5(y - 2)$.

$$\Rightarrow 12x + 72 = 5y - 10$$

$$\Rightarrow 12x - 5y + 82 = 0 \dots (1)$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 12$, $B = -5$, and $C = 82$.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point

$$(x_1, y_1) \text{ is given by } d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

The given point is $(x_1, y_1) = (-1, 1)$.

Therefore, the distance of point $(-1, 1)$ from the given line

$$= \frac{|12(-1) + (-5)(1) + 82|}{\sqrt{(12)^2 + (-5)^2}} \text{ units} = \frac{|-12 - 5 + 82|}{\sqrt{169}} \text{ units} = \frac{|65|}{13} \text{ units} = 5 \text{ units}$$

Question 5:

Find the points on the x-axis, whose distances from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

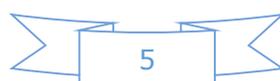
Answer 5:

The given equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$

$$\text{or, } 4x + 3y - 12 = 0 \dots (1)$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$,



Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

we obtain $A = 4$, $B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the x -axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point

(x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

Therefore,

$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$\Rightarrow 4 = \frac{|4a - 12|}{5}$$

$$\Rightarrow |4a - 12| = 20$$

$$\Rightarrow \pm(4a - 12) = 20$$

$$\Rightarrow (4a - 12) = 20 \text{ or } -(4a - 12) = 20$$

$$\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12$$

$$\Rightarrow a = 8 \text{ or } -2$$

Thus, the required points on the x -axis are $(-2, 0)$ and $(8, 0)$.

Question 6:

Find the distance between parallel lines

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$

Answer 6:

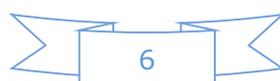
It is known that the distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

(i) The given parallel lines are $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Here, $A = 15$, $B = 8$, $C_1 = -34$, and $C_2 = 31$.

Therefore, the distance between the parallel lines is



Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} \text{ units} = \frac{|-65|}{17} \text{ units} = \frac{65}{17} \text{ units}$$

(ii) The given parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$.

$lx + ly + p = 0$ and $lx + ly - r = 0$ Here,

$A = l, B = l, C_1 = p,$ and $C_2 = -r$.

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|p + r|}{\sqrt{l^2 + l^2}} \text{ units} = \frac{|p + r|}{\sqrt{2l^2}} \text{ units} = \frac{|p + r|}{l\sqrt{2}} \text{ units} = \frac{1}{\sqrt{2}} \left| \frac{p + r}{l} \right| \text{ units}$$

Question 7:

Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.

Answer 7:

The equation of the given line is

$$3x - 4y + 2 = 0$$

$$\text{or } y = \frac{3x}{4} + \frac{2}{4}$$

$$\text{or } y = \frac{3}{4}x + \frac{1}{2}$$

which is of the form $y = mx + c$

∴ Slope of the given line = $3/4$

It is known that parallel lines have the same slope.

∴ Slope of the other line = $m = \frac{3}{4}$

Now, the equation of the line that has a slope of $\frac{3}{4}$ and passes through the point $(-2, 3)$ is

$$(y - 3) = \frac{3}{4}\{x - (-2)\}$$

$$4y - 12 = 3x + 6$$

$$\text{i.e., } 3x - 4y + 18 = 0$$



Mathematics

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(Chapter – 10) (Straight Lines)

(Class – XI)

Question 8:

Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

Answer 8:

The given equation of line is $x - 7y + 5 = 0$. Or, $y = \frac{1}{7}x + \frac{5}{7}$

which is of the form $y = mx + c$

∴ Slope of the given line $= \frac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\frac{1}{7}$ is $m = -\frac{1}{\left(\frac{1}{7}\right)} = -7$

The equation of the line with slope -7 and x-intercept 3 is given by y

$$= m(x - d)$$

$$\Rightarrow y = -7(x - 3)$$

$$\Rightarrow y = -7x + 21$$

$$\Rightarrow 7x + y = 21$$

Question 9:

Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

Answer 9:

The given lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

$$y = -\sqrt{3}x + 1 \quad \dots(1) \quad \text{and} \quad y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}} \quad \dots(2)$$

The slope of line (1) is $m_1 = -\sqrt{3}$, while the slope of line (2) is $m_2 = -\frac{1}{\sqrt{3}}$

The acute angle i.e., θ between the two lines is given by



Mathematics

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(Chapter – 10) (Straight Lines)

(Class – XI)

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\tan \theta = \left| \frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + (-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)} \right|$$

$$\tan \theta = \left| \frac{-3+1}{\sqrt{3}} \right| = \left| \frac{-2}{2 \times \sqrt{3}} \right|$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Thus, the angle between the given lines is either 30° or $180^\circ - 30^\circ = 150^\circ$.

Question 10:

The line through the points $(h, 3)$ and $(4, 1)$ intersects the line $7x - 9y - 19 = 0$, at right angle. Find the value of h .

Answer 10:

The slope of the line passing through points $(h, 3)$ and $(4, 1)$ is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of line $7x - 9y - 19 = 0$ or $y = \frac{7}{9}x - \frac{19}{9}$ is $m_2 = \frac{7}{9}$

It is given that the two lines are perpendicular.



Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

$$\therefore m_1 \times m_2 = -1$$

$$\Rightarrow \left(\frac{-2}{4-h}\right) \times \left(\frac{7}{9}\right) = -1$$

$$\Rightarrow \frac{-14}{36-9h} = -1$$

$$\Rightarrow 14 = 36 - 9h$$

$$\Rightarrow 9h = 36 - 14$$

$$\Rightarrow h = \frac{22}{9}$$

Thus, the value of h is $\frac{22}{9}$.

Question 11:

Prove that the line through the point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$.

Answer 11:

The slope of line $Ax + By + C = 0$ or $y = \left(\frac{-A}{B}\right)x + \left(\frac{-C}{B}\right)$ is $m = -\frac{A}{B}$

It is known that parallel lines have the same slope.

\therefore Slope of the other line = $m = -\frac{A}{B}$

The equation of the line passing through point (x_1, y_1) and having a slope $m = -\frac{A}{B}$ is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = -\frac{A}{B}(x - x_1)$$

$$B(y - y_1) = -A(x - x_1)$$

$$A(x - x_1) + B(y - y_1) = 0$$

Hence, the line through point (x_1, y_1) and parallel to line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$

Mathematics

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(Chapter – 10) (Straight Lines)

(Class – XI)

Question 12:

Two lines passing through the point (2, 3) intersect each other at an angle of 60° . If slope of one line is 2, find equation of the other line.

Answer 12:

It is given that the slope of the first line, $m_1 = 2$.

Let the slope of the other line be m_2 .

The angle between the two lines is 60° .

$$\begin{aligned}\therefore \tan 60^\circ &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ \Rightarrow \sqrt{3} &= \left| \frac{2 - m_2}{1 + 2m_2} \right| \\ \Rightarrow \sqrt{3} &= \pm \left(\frac{2 - m_2}{1 + 2m_2} \right) \\ \Rightarrow \sqrt{3} &= \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2} \right) \\ \Rightarrow \sqrt{3}(1 + 2m_2) &= 2 - m_2 \text{ or } \sqrt{3}(1 + 2m_2) = -(2 - m_2) \\ \Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 &= 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2 \\ \Rightarrow \sqrt{3} + (2\sqrt{3} + 1)m_2 &= 2 \text{ or } \sqrt{3} + (2\sqrt{3} - 1)m_2 = -2 \\ \Rightarrow m_2 &= \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)} \text{ or } m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}\end{aligned}$$

Case I: $m_2 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1} \right)$

The equation of the line passing through point (2, 3) and having a slope of $\frac{(2 - \sqrt{3})}{(2\sqrt{3} + 1)}$ is given by

Mathematics

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(Chapter – 10) (Straight Lines)

(Class – XI)

$$(y-3) = \frac{2-\sqrt{3}}{2\sqrt{3}+1}(x-2)$$

$$(2\sqrt{3}+1)y-3(2\sqrt{3}+1) = (2-\sqrt{3})x-2(2-\sqrt{3})$$

$$(\sqrt{3}-2)x + (2\sqrt{3}+1)y = -4+2\sqrt{3}+6\sqrt{3}+3$$

$$(\sqrt{3}-2)x + (2\sqrt{3}+1)y = -1+8\sqrt{3}$$

In this case, the equation of the other line is $(\sqrt{3}-2)x + (2\sqrt{3}+1)y = -1+8\sqrt{3}$.

Case II: $m_2 = \frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}$

The equation of the line passing through point (2, 3) and having a slope of $\frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}$ is

$$(y-3) = \frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}(x-2)$$

$$(2\sqrt{3}-1)y-3(2\sqrt{3}-1) = -(2+\sqrt{3})x+2(2+\sqrt{3})$$

$$(2\sqrt{3}-1)y + (2+\sqrt{3})x = 4+2\sqrt{3}+6\sqrt{3}-3$$

$$(2+\sqrt{3})x + (2\sqrt{3}-1)y = 1+8\sqrt{3}$$

In this case, the equation of the other line is $(2+\sqrt{3})x + (2\sqrt{3}-1)y = 1+8\sqrt{3}$

Thus, the required equation of the other line is $(\sqrt{3}-2)x + (2\sqrt{3}+1)y = -1+8\sqrt{3}$

or

$$(2+\sqrt{3})x + (2\sqrt{3}-1)y = 1+8\sqrt{3}$$

Question 13:

Find the equation of the right bisector of the line segment joining the points (3, 4) and (-1, 2).

Answer 13:

The right bisector of a line segment bisects the line segment at 90° .

Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

The end-points of the line segment are given as A (3, 4) and B (-1, 2).

Accordingly, mid-point of AB = $\left(\frac{3-1}{2}, \frac{4+2}{2}\right) = (1, 3)$

$$\text{Slope of AB} = \frac{2-4}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$$

$$\therefore \text{Slope of the line perpendicular to AB} = -\frac{1}{\left(\frac{1}{2}\right)} = -2$$

The equation of the line passing through (1, 3) and having a slope of -2 is

$$(y - 3) = -2(x - 1)$$

$$-3 = -2x + 2$$

$$2x + y = 5$$

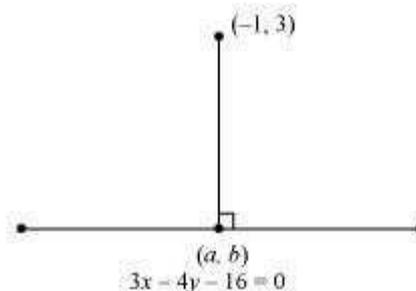
Thus, the required equation of the line is $2x + y = 5$.

Question 14:

Find the coordinates of the foot of perpendicular from the point (-1, 3) to the line $3x - 4y - 16 = 0$.

Answer 14:

Let (a, b) be the coordinates of the foot of the perpendicular from the point (-1, 3) to the line $3x - 4y - 16 = 0$.



$$\text{Slope of the line joining } (-1, 3) \text{ and } (a, b), m_1 = \frac{b-3}{a+1}$$

Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

Slope of the line $3x - 4y - 16 = 0$ or $y = \frac{3}{4}x - 4$, $m_2 = \frac{3}{4}$

Since these two lines are perpendicular, $m_1 m_2 = -1$

$$\therefore \left(\frac{b-3}{a+1}\right) \times \left(\frac{3}{4}\right) = -1$$

$$\Rightarrow \frac{3b-9}{4a+4} = -1$$

$$\Rightarrow 3b-9 = -4a-4$$

$$\Rightarrow 4a+3b = 5 \quad \dots(1)$$

Point (a, b) lies on line $3x - 4y = 16$.

$$\therefore 3a - 4b = 16 \quad \dots(2)$$

On solving equations (1) and (2), we obtain

$$a = \frac{68}{25} \text{ and } b = -\frac{49}{25}$$

Thus, the required coordinates of the foot of the perpendicular are $\left(\frac{68}{25}, -\frac{49}{25}\right)$.

Question 15:

The perpendicular from the origin to the line $y = mx + c$ meets it at the point $(-1, 2)$. Find the values of m and c .

Answer 15:

The given equation of line is $y = mx + c$.

It is given that the perpendicular from the origin meets the given line at $(-1, 2)$.

Therefore, the line joining the points $(0, 0)$ and $(-1, 2)$ is perpendicular to the given line.

$$\therefore \text{Slope of the line joining } (0, 0) \text{ and } (-1, 2) = \frac{2}{-1} = -2$$

The slope of the given line is m .

$$\therefore m \times -2 = -1 \quad [\text{The two lines are perpendicular}]$$

$$\Rightarrow m = \frac{1}{2}$$

Since point $(-1, 2)$ lies on the given line, it satisfies the equation $y = mx + c$.

Mathematics

(www.tiwariacademy.com)

(Chapter – 10) (Straight Lines)

(Class – XI)

$$\therefore 2 = m(-1) + c$$

$$\Rightarrow 2 = \frac{1}{2}(-1) + c$$

$$\Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$.

Question 16:

If p and q are the lengths of perpendiculars from the origin to the lines $x \cos \theta - y \sin \theta = k \cos 2\theta$ and $x \sec \theta + y \operatorname{cosec} \theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

Answer 16:

The equations of given lines are

$$x \cos \theta - y \sin \theta = k \cos 2\theta \dots\dots\dots (1)$$

$$x \sec \theta + y \operatorname{cosec} \theta = k \dots\dots\dots (2)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \cos \theta$, $B = -\sin \theta$, and $C = -k \cos 2\theta$.

It is given that p is the length of the perpendicular from $(0, 0)$ to line (1).

$$\therefore p = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k \cos 2\theta|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = |-k \cos 2\theta| \dots\dots(3)$$

On comparing equation (2) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \sec \theta$, $B = \operatorname{cosec} \theta$, and $C = -k$.

It is given that q is the length of the perpendicular from $(0, 0)$ to line (2).

$$\therefore q = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k|}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} \dots\dots(4)$$

From (3) and (4), we have

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(Chapter – 10) (Straight Lines)

(Class – XI)

$$\begin{aligned} p^2 + 4q^2 &= (-k \cos 2\theta)^2 + 4 \left(\frac{|-k|}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} \right)^2 \\ &= k^2 \cos^2 2\theta + \frac{4k^2}{(\sec^2 \theta + \operatorname{cosec}^2 \theta)} \\ &= k^2 \cos^2 2\theta + \frac{4k^2}{\left(\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \right)} \\ &= k^2 \cos^2 2\theta + \frac{4k^2}{\left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} \right)} \\ &= k^2 \cos^2 2\theta + \frac{4k^2}{\left(\frac{1}{\sin^2 \theta \cos^2 \theta} \right)} \\ &= k^2 \cos^2 2\theta + 4k^2 \sin^2 \theta \cos^2 \theta \\ &= k^2 \cos^2 2\theta + k^2 (2 \sin \theta \cos \theta)^2 \\ &= k^2 \cos^2 2\theta + k^2 \sin^2 2\theta \\ &= k^2 (\cos^2 2\theta + \sin^2 2\theta) \\ &= k^2 \end{aligned}$$

Hence, we proved that $p^2 + 4q^2 = k^2$.

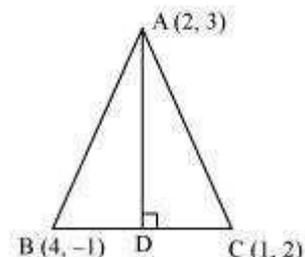
Question 17:

In the triangle ABC with vertices A (2, 3), B (4, -1) and C (1, 2), find the equation and length of altitude from the vertex A.

Answer 17:

Let AD be the altitude of triangle ABC from vertex A.

Accordingly, $AD \perp BC$



Mathematics

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(Chapter – 10) (Straight Lines)

(Class – XI)

The equation of the line passing through point (2, 3) and having a slope of 1 is

$$(y - 3) = 1(x - 2)$$

$$\Rightarrow x - y + 1 = 0$$

$$\Rightarrow y - x = 1$$

Therefore, equation of the altitude from vertex A = $y - x = 1$.

Length of AD = Length of the perpendicular from A (2, 3) to BC The equation of BC is

$$(y+1) = \frac{2+1}{1-4}(x-4)$$

$$\Rightarrow (y+1) = -1(x-4)$$

$$\Rightarrow y+1 = -x+4$$

$$\Rightarrow x+y-3=0 \quad \dots(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = 1$, $B = 1$, and $C = -3$.

$$\therefore \text{Length of AD} = \frac{|1 \times 2 + 1 \times 3 - 3|}{\sqrt{1^2 + 1^2}} \text{ units} = \frac{|2|}{\sqrt{2}} \text{ units} = \frac{2}{\sqrt{2}} \text{ units} = \sqrt{2} \text{ units}$$

Thus, the equation and the length of the altitude from vertex A are $y - x = 1$ and $\sqrt{2}$ units respectively.

Question 18:

If p is the length of perpendicular from the origin to the line whose intercepts on the

axes are a and b , then show that: $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Answer 18:

It is known that the equation of a line whose intercepts on the axes are a and b is

Mathematics

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(Chapter – 10) (Straight Lines)

(Class – XI)

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\text{or } bx + ay = ab$$

$$\text{or } bx + ay - ab = 0 \quad \dots(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = b$, $B = a$, and $C = -ab$.

Therefore, if p is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (1), we obtain

$$p = \frac{|A(0) + B(0) - ab|}{\sqrt{b^2 + a^2}}$$

$$\Rightarrow p = \frac{|-ab|}{\sqrt{a^2 + b^2}}$$

On squaring both sides, we obtain

$$p^2 = \frac{(-ab)^2}{a^2 + b^2}$$

$$\Rightarrow p^2 (a^2 + b^2) = a^2 b^2$$

$$\Rightarrow \frac{a^2 + b^2}{a^2 b^2} = \frac{1}{p^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence, we showed that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

