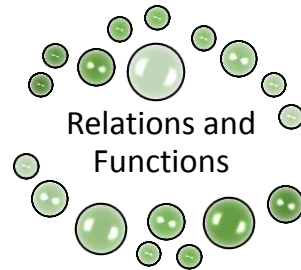


Chapter 1



- Mathematics XII
- Exercise 1.2

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Exercise 1.2

Question 1:

Show that the function $f: \mathbf{R}_* \rightarrow \mathbf{R}_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where \mathbf{R}_* is the set of all non-zero real numbers. Is the result true, if the domain \mathbf{R}_* is replaced by \mathbf{N} with co-domain being same as \mathbf{R}_* ?

Answer 1:

It is given that $f: \mathbf{R}_* \rightarrow \mathbf{R}_*$ is defined by $f(x) = \frac{1}{x}$

For one – one:

Let $x, y \in \mathbf{R}_*$ such that $f(x) = f(y)$

$$\Rightarrow \frac{1}{x} = \frac{1}{y}$$

$$\Rightarrow x = y$$

$\therefore f$ is one – one.

For onto:

It is clear that for $y \in \mathbf{R}_*$, there exists $x = \frac{1}{y} \in \mathbf{R}_*$ [as $y \neq 0$] such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$$

$\therefore f$ is onto.

Thus, the given function f is one – one and onto.

Now, consider function $g: \mathbf{N} \rightarrow \mathbf{R}_*$ defined by $g(x) = \frac{1}{x}$

We have,

$$g(x_1) = g(x_2) \quad \Rightarrow \frac{1}{x_1} = \frac{1}{x_2} \quad \Rightarrow x_1 = x_2$$



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$\therefore g$ is one – one.

Further, it is clear that g is not onto as for $1.2 \in \mathbb{R}^*$ there does not exist any x in \mathbb{N} such that $g(x) = \frac{1}{1.2}$.

Hence, function g is one-one but not onto.

Question 2:

Check the injectivity and surjectivity of the following functions:

(i) $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^2$

(ii) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x^2$

(iii) $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$

(iv) $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3$

(v) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x^3$

Answer 2:

(i) $f: \mathbb{N} \rightarrow \mathbb{N}$ is given by $f(x) = x^2$

It is seen that for $x, y \in \mathbb{N}$, $f(x) = f(y) \Rightarrow x^2 = y^2 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbb{N}$. But, there does not exist any x in \mathbb{N} such that $f(x) = x^2 = 2$.

$\therefore f$ is not surjective.

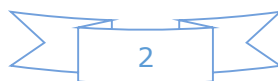
Hence, function f is injective but not surjective.

(ii) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore f$ is not injective.

Now, $-2 \in \mathbb{Z}$. But, there does not exist any element $x \in \mathbb{Z}$ such that



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$$f(x) = -2 \quad \text{or} \quad x^2 = -2.$$

$\therefore f$ is not surjective.

Hence, function f is neither injective nor surjective.

(iii) $f: \mathbf{R} \rightarrow \mathbf{R}$ is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$, but $-1 \neq 1$.

$\therefore f$ is not injective.

Now, $-2 \in \mathbf{R}$. But, there does not exist any element $x \in \mathbf{R}$ such that

$$f(x) = -2 \quad \text{or} \quad x^2 = -2.$$

$\therefore f$ is not surjective.

Hence, function f is neither injective nor surjective.

(iv) $f: \mathbf{N} \rightarrow \mathbf{N}$ given by $f(x) = x^3$

It is seen that for $x, y \in \mathbf{N}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{N}$. But, there does not exist any element $x \in \mathbf{N}$ such that

$$f(x) = 2 \quad \text{or} \quad x^3 = 2.$$

$\therefore f$ is not surjective

Hence, function f is injective but not surjective.

(v) $f: \mathbf{Z} \rightarrow \mathbf{Z}$ is given by $f(x) = x^3$

It is seen that for $x, y \in \mathbf{Z}$, $f(x) = f(y) \Rightarrow x^3 = y^3 \Rightarrow x = y$.

$\therefore f$ is injective.

Now, $2 \in \mathbf{Z}$. But, there does not exist any element $x \in \mathbf{Z}$ such that

$$f(x) = 2 \quad \text{or} \quad x^3 = 2.$$

$\therefore f$ is not surjective.

Hence, function f is injective but not surjective.



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Question 3:

Prove that the Greatest Integer Function $f: \mathbf{R} \rightarrow \mathbf{R}$ given by $f(x) = [x]$, is neither one – one nor onto, where $[x]$ denotes the greatest integer less than or equal to x .

Answer 3:

$f: \mathbf{R} \rightarrow \mathbf{R}$ is given by, $f(x) = [x]$

It is seen that $f(1.2) = [1.2] = 1$, $f(1.9) = [1.9] = 1$.

$\therefore f(1.2) = f(1.9)$, but $1.2 \neq 1.9$.

$\therefore f$ is not one – one.

Now, consider $0.7 \in \mathbf{R}$.

It is known that $f(x) = [x]$ is always an integer. Thus, there does not exist any element $x \in \mathbf{R}$ such that $f(x) = 0.7$.

$\therefore f$ is not onto.

Hence, the greatest integer function is neither one – one nor onto.

Question 4:

Show that the Modulus Function $f: \mathbf{R} \rightarrow \mathbf{R}$ given by $f(x) = |x|$, is neither one – one nor onto, where $|x|$ is x , if x is positive or 0 and $|x|$ is $-x$, if x is negative.

Answer 4:

$f: \mathbf{R} \rightarrow \mathbf{R}$ is given by $f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

It is clear that $f(-1) = |-1| = 1$ and $f(1) = |1| = 1$

$\therefore f(-1) = f(1)$, but $-1 \neq 1$.

$\therefore f$ is not one – one.

Now, consider $-1 \in \mathbf{R}$.

It is known that $f(x) = |x|$ is always non-negative. Thus, there does not exist any element x in domain \mathbf{R} such that $f(x) = |x| = -1$.



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$\therefore f$ is not onto.

Hence, the modulus function is neither one-one nor onto.

Question 5:

Show that the Signum Function $f: \mathbf{R} \rightarrow \mathbf{R}$, given by $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

is neither one-one nor onto.

Answer 5:

$f: \mathbf{R} \rightarrow \mathbf{R}$ is given by $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

It is seen that $f(1) = f(2) = 1$, but $1 \neq 2$.

$\therefore f$ is not one – one.

Now, as $f(x)$ takes only 3 values (1, 0, or -1) for the element -2 in co-domain \mathbf{R} , there does not exist any x in domain \mathbf{R} such that $f(x) = -2$.

$\therefore f$ is not onto.

Hence, the Signum function is neither one – one nor onto.

Question 6:

Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B . Show that f is one – one.

Answer 6:

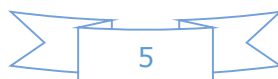
It is given that $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$.

$f: A \rightarrow B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$.

$\therefore f(1) = 4, \quad f(2) = 5, \quad f(3) = 6$

It is seen that the images of distinct elements of A under f are distinct.

Hence, function f is one – one.



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Question 7:

In each of the following cases, state whether the function is one – one, onto or bijective.

Justify your answer.

(i) $f: \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = 3 - 4x$

(ii) $f: \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = 1 + x^2$

Answer 7:

(i) $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = 3 - 4x$.

Let $x_1, x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$

$$\Rightarrow 3 - 4x_1 = 3 - 4x_2$$

$$\Rightarrow -4x_1 = -4x_2$$

$$\Rightarrow x_1 = x_2$$

$\therefore f$ is one – one.

For any real number (y) in \mathbf{R} , there exists $\frac{3-y}{4}$ in \mathbf{R} such that

$$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = y$$

$\therefore f$ is onto.

Hence, f is bijective.

(ii) $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = 1 + x^2$

Let $x_1, x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$

$$\Rightarrow 1 + x_1^2 = 1 + x_2^2$$

$$\Rightarrow x_1^2 = x_2^2$$

$$\Rightarrow x_1 = \pm x_2$$

$\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$



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For example $f(1) = f(-1) = 2$

$\therefore f$ is not one – one.

Consider an element -2 in co-domain \mathbf{R} .

It is seen that $f(x) = 1 + x^2$ is positive for all $x \in \mathbf{R}$.

Thus, there does not exist any x in domain \mathbf{R} such that $f(x) = -2$.

$\therefore f$ is not onto.

Hence, f is neither one – one nor onto.

Question 8:

Let A and B be sets. Show that $f: A \times B \rightarrow B \times A$ such that $(a, b) = (b, a)$ is bijective function.

Answer 8:

$f: A \times B \rightarrow B \times A$ is defined as $f(a, b) = (b, a)$.

Let $(a_1, b_1), (a_2, b_2) \in A \times B$ such that $f(a_1, b_1) = f(a_2, b_2)$

$$\Rightarrow (b_1, a_1) = (b_2, a_2)$$

$$\Rightarrow b_1 = b_2 \text{ and } a_1 = a_2$$

$$\Rightarrow (a_1, b_1) = (a_2, b_2)$$

$\therefore f$ is one – one.

Now, let $(b, a) \in B \times A$ be any element.

Then, there exists $(a, b) \in A \times B$ such that $f(a, b) = (b, a)$. [By definition of f]

$\therefore f$ is onto.

Hence, f is bijective.



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Question 9:

Let $f: \mathbf{N} \rightarrow \mathbf{N}$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in \mathbf{N}$

State whether the function f is bijective. Justify your answer.

Answer 9:

$f: \mathbf{N} \rightarrow \mathbf{N}$ is defined as $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in \mathbf{N}$

It can be observed that:

$$f(1) = \frac{1+1}{2} = 1 \text{ and } f(2) = \frac{2}{2} = 1 \quad [\text{By definition of } f(n)]$$

$$f(1) = f(2), \text{ where } 1 \neq 2$$

$\therefore f$ is not one-one.

Consider a natural number (n) in co-domain \mathbf{N} .

Case I: n is odd

$\therefore n = 2r + 1$ for some $r \in \mathbf{N}$. Then, there exists $4r + 1 \in \mathbf{N}$ such that

$$f(4r + 1) = \frac{4r + 1 + 1}{2} = 2r + 1$$

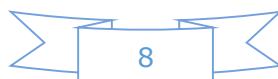
Case II: n is even

$\therefore n = 2r$ for some $r \in \mathbf{N}$. Then, there exists $4r \in \mathbf{N}$ such that

$$f(4r) = \frac{4r}{2} = 2r.$$

$\therefore f$ is onto.

Hence, f is not a bijective function.



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Question 10:

Let $A = \mathbf{R} - \{3\}$ and $B = \mathbf{R} - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Answer 10:

$A = \mathbf{R} - \{3\}$, $B = \mathbf{R} - \{1\}$ and $f: A \rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x, y \in A$ such that $f(x) = f(y)$

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$

$$\Rightarrow xy - 3x - 2y + 6 = xy - 2x - 3y + 6$$

$$\Rightarrow -3x - 2y = -2x - 3y \Rightarrow x = y$$

$\therefore f$ is one-one.

Let $y \in B = \mathbf{R} - \{1\}$. Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now, $f(x) = y$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow x - 2 = xy - 3y \Rightarrow x(1 - y) = -3y + 2$$

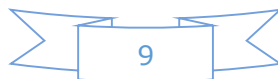
$$\Rightarrow x = \frac{2-3y}{1-y} \in A \quad [y \neq 1]$$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$

$\therefore f$ is onto.

Hence, function f is one – one and onto.



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Question 11:

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = x^4$. Choose the correct answer.

- (A) f is one-one onto
(B) f is many-one onto
(C) f is one-one but not onto
(D) f is neither one-one nor onto

Answer 11:

$f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = x^4$.

Let $x, y \in \mathbf{R}$ such that $f(x) = f(y)$.

$$\Rightarrow x^4 = y^4$$

$$\Rightarrow x = \pm y$$

$\therefore f(x) = f(y)$ does not imply that $x = y$.

For example $f(1) = f(-1) = 1$

$\therefore f$ is not one-one.

Consider an element 2 in co-domain \mathbf{R} . It is clear that there does not exist any x in domain \mathbf{R} such that $f(x) = 2$.

$\therefore f$ is not onto.

Hence, function f is neither one – one nor onto.

The correct answer is D.

Question 12:

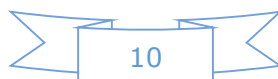
Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = 3x$. Choose the correct answer.

- (A) f is one – one onto
(B) f is many – one onto
(C) f is one – one but not onto
(D) f is neither one – one nor onto

Answer 12:

$f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = 3x$.

Let $x, y \in \mathbf{R}$ such that $f(x) = f(y)$.



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$$\Rightarrow 3x = 3y$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Also, for any real number (y) in co-domain \mathbf{R} , there exists $\frac{y}{3}$ in \mathbf{R} such that $f\left(\frac{y}{3}\right) =$

$$3\left(\frac{y}{3}\right) = y$$

$\therefore f$ is onto.

Hence, function f is one – one and onto.

The correct answer is A.

