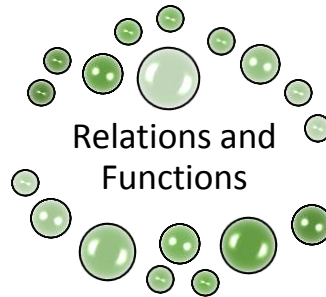


# Chapter 1



- Mathematics XII
- Exercise 1.4

# Mathematics

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(Chapter – 1) (Relations and Functions)

(Class – XII)

## Exercise 1.4

### Question 1:

Determine whether or not each of the definition of given below gives a binary operation.

In the event that  $*$  is not a binary operation, give justification for this.

(i) On  $\mathbf{Z}^+$ , define  $*$  by  $a * b = a - b$

(ii) On  $\mathbf{Z}^+$ , define  $*$  by  $a * b = ab$

(iii) On  $\mathbf{R}$ , define  $*$  by  $a * b = ab^2$

(iv) On  $\mathbf{Z}^+$ , define  $*$  by  $a * b = |a - b|$

(v) On  $\mathbf{Z}^+$ , define  $*$  by  $a * b = a$

### Answer 1:

(i) On  $\mathbf{Z}^+$ ,  $*$  is defined by  $a * b = a - b$ .

It is not a binary operation

as the image of  $(1, 2)$  under  $*$  is  $1 * 2 = 1 - 2 = -1 \notin \mathbf{Z}^+$ .

(ii) On  $\mathbf{Z}^+$ ,  $*$  is defined by  $a * b = ab$ .

It is seen that for each  $a, b \in \mathbf{Z}^+$ , there is a unique element  $ab$  in  $\mathbf{Z}^+$ .

This means that  $*$  carries each pair  $(a, b)$  to a unique element  $a * b = ab$  in  $\mathbf{Z}^+$ .

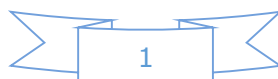
Therefore,  $*$  is a binary operation.

(iii) On  $\mathbf{R}$ ,  $*$  is defined by  $a * b = ab^2$ .

It is seen that for each  $a, b \in \mathbf{R}$ , there is a unique element  $ab^2$  in  $\mathbf{R}$ .

This means that  $*$  carries each pair  $(a, b)$  to a unique element  $a * b = ab^2$  in  $\mathbf{R}$ .

Therefore,  $*$  is a binary operation.



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(iv) On  $\mathbf{Z}^+$ ,  $*$  is defined by  $a * b = |a - b|$ .

It is seen that for each  $a, b \in \mathbf{Z}^+$ , there is a unique element  $|a - b|$  in  $\mathbf{Z}^+$ .

This means that  $*$  carries each pair  $(a, b)$  to a unique element  $a * b = |a - b|$  in  $\mathbf{Z}^+$ .

Therefore,  $*$  is a binary operation.

(v) On  $\mathbf{Z}^+$ ,  $*$  is defined by  $a * b = a$ .

It is seen that for each  $a, b \in \mathbf{Z}^+$ , there is a unique element  $a$  in  $\mathbf{Z}^+$ .

This means that  $*$  carries each pair  $(a, b)$  to a unique element  $a * b = a$  in  $\mathbf{Z}^+$ .

Therefore,  $*$  is a binary operation.

### Question 2:

For each binary operation  $*$  defined below, determine whether  $*$  is commutative or associative.

(i) On  $\mathbf{Z}$ , define  $a * b = a - b$

(ii) On  $\mathbf{Q}$ , define  $a * b = ab + 1$

(iii) On  $\mathbf{Q}$ , define  $a * b = \frac{ab}{2}$

(iv) On  $\mathbf{Z}^+$ , define  $a * b = 2^{ab}$

(v) On  $\mathbf{Z}^+$ , define  $a * b = a^b$

(vi) On  $\mathbf{R} - \{-1\}$ , define  $a * b = \frac{a}{b+1}$

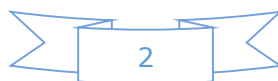
### Answer 2:

(i) On  $\mathbf{Z}$ ,  $*$  is defined by  $a * b = a - b$ .

It can be observed that  $1 * 2 = 1 - 2 = -1$  and  $2 * 1 = 2 - 1 = 1$ .

$\therefore 1 * 2 \neq 2 * 1$ , where  $1, 2 \in \mathbf{Z}$

Hence, the operation  $*$  is not commutative.



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Also, we have

$$(1 * 2) * 3 = (1 - 2) * 3 = -1 * 3 = -1 - 3 = -4$$

$$1 * (2 * 3) = 1 * (2 - 3) = 1 * -1 = 1 - (-1) = 2$$

$$\therefore (1 * 2) * 3 \neq 1 * (2 * 3), \text{ where } 1, 2, 3 \in \mathbf{Z}$$

Hence, the operation  $*$  is not associative.

(ii) On  $\mathbf{Q}$ ,  $*$  is defined by  $a * b = ab + 1$ .

It is known that:  $ab = ba$  for all  $a, b \in \mathbf{Q}$

$$\Rightarrow ab + 1 = ba + 1 \text{ for all } a, b \in \mathbf{Q}$$

$$\Rightarrow a * b = b * a \text{ for all } a, b \in \mathbf{Q}$$

Therefore, the operation  $*$  is commutative.

It can be observed that

$$(1 * 2) * 3 = (1 \times 2 + 1) * 3 = 3 * 3 = 3 \times 3 + 1 = 10$$

$$1 * (2 * 3) = 1 * (2 \times 3 + 1) = 1 * 7 = 1 \times 7 + 1 = 8$$

$$\therefore (1 * 2) * 3 \neq 1 * (2 * 3), \text{ where } 1, 2, 3 \in \mathbf{Q}$$

Therefore, the operation  $*$  is not associative.

(iii) On  $\mathbf{Q}$ ,  $*$  is defined by  $a * b = \frac{ab}{2}$

It is known that:  $ab = ba$  for all  $a, b \in \mathbf{Q}$

$$\Rightarrow \frac{ab}{2} = \frac{ba}{2} \text{ for all } a, b \in \mathbf{Q}$$

$$\Rightarrow a * b = b * a \text{ for all } a, b \in \mathbf{Q}$$

Therefore, the operation  $*$  is commutative.

For all  $a, b, c \in \mathbf{Q}$ , we have

$$(a * b) * c = \left(\frac{ab}{2}\right) * c = \frac{\left(\frac{ab}{2}\right)c}{2} = \frac{abc}{4}$$

and



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$$a * (b * c) = a * \left(\frac{bc}{2}\right) = \frac{a\left(\frac{bc}{2}\right)}{2} = \frac{abc}{4}$$

$\therefore (a*b)*c = a*(b*c)$ , where  $a, b, c \in \mathbf{Q}$

Therefore, the operation  $*$  is associative.

(iv) On  $\mathbf{Z}^+$ ,  $*$  is defined by  $a * b = 2^{ab}$ .

It is known that:  $ab = ba$  for all  $a, b \in \mathbf{Z}^+$

$$\Rightarrow 2^{ab} = 2^{ba} \text{ for all } a, b \in \mathbf{Z}^+$$

$$\Rightarrow a * b = b * a \text{ for all } a, b \in \mathbf{Z}^+$$

Therefore, the operation  $*$  is commutative.

It can be observed that

$$(1 * 2) * 3 = 2^{1 \times 2} * 3 = 4 * 3 = 2^{4 \times 3} = 2^{12} \quad \text{and}$$

$$1 * (2 * 3) = 1 * 2^{2 \times 3} = 1 * 2^6 = 1 * 64 = 2^{1 \times 64} = 2^{64}$$

$$\therefore (1 * 2) * 3 \neq 1 * (2 * 3), \text{ where } 1, 2, 3 \in \mathbf{Z}^+$$

Therefore, the operation  $*$  is not associative.

(v) On  $\mathbf{Z}^+$ ,  $*$  is defined by  $a * b = a^b$ .

It can be observed that

$$1 * 2 = 1^2 = 1 \text{ and } 2 * 1 = 2^1 = 2$$

$$\therefore 1 * 2 \neq 2 * 1, \text{ where } 1, 2 \in \mathbf{Z}^+$$

Therefore, the operation  $*$  is not commutative.

It can also be observed that

$$(2 * 3) * 4 = 2^3 * 4 = 8 * 4 = 8^4 = 2^{12} \quad \text{and}$$

$$2 * (3 * 4) = 2 * 3^4 = 2 * 81 = 2^{81}$$



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$\therefore (2 * 3) * 4 \neq 2 * (3 * 4)$ , where  $2, 3, 4 \in \mathbb{Z}^+$

Therefore, the operation  $*$  is not associative.

(vi) On  $\mathbb{R}$ ,  $*$  on  $\mathbb{R} - \{-1\}$  is defined by  $a * b = \frac{a}{b+1}$

It can be observed that

$$1 * 2 = \frac{1}{2+1} = \frac{1}{3} \text{ and } 2 * 1 = \frac{2}{1+1} = \frac{2}{2} = 1$$

$\therefore 1 * 2 \neq 2 * 1$ , where  $1, 2 \in \mathbb{R} - \{-1\}$

Therefore, the operation  $*$  is not commutative.

It can also be observed that

$$(1 * 2) * 3 = \frac{1}{2+1} * 3 = \frac{1}{3} * 3 = \frac{\frac{1}{3}}{3+1} = \frac{1}{12}$$

and

$$1 * (2 * 3) = 1 * \frac{2}{3+1} = 1 * \frac{2}{4} = 1 * \frac{1}{2} = \frac{1}{\frac{1}{2}+1} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$ , where  $1, 2, 3 \in \mathbb{R} - \{-1\}$

Therefore, the operation  $*$  is not associative.

### Question 3:

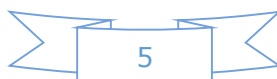
Consider the binary operation  $\wedge$  on the set  $\{1, 2, 3, 4, 5\}$  defined by  $a \wedge b = \min \{a, b\}$ .

Write the operation table of the operation  $\wedge$ .

### Answer 3:

The binary operation  $\wedge$  on the set  $\{1, 2, 3, 4, 5\}$  is defined as  $a \wedge b = \min \{a, b\}$  for all  $a, b \in \{1, 2, 3, 4, 5\}$ .

Thus, the operation table for the given operation  $\wedge$  can be given as:



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| $\wedge$ | 1 | 2 | 3 | 4 | 5 |
|----------|---|---|---|---|---|
| 1        | 1 | 1 | 1 | 1 | 1 |
| 2        | 1 | 2 | 2 | 2 | 2 |
| 3        | 1 | 2 | 3 | 3 | 3 |
| 4        | 1 | 2 | 3 | 4 | 4 |
| 5        | 1 | 2 | 3 | 4 | 5 |

## Question 4:

Consider a binary operation  $*$  on the set  $\{1, 2, 3, 4, 5\}$  given by the following multiplication table.

(i) Compute  $(2 * 3) * 4$  and  $2 * (3 * 4)$

(ii) Is  $*$  commutative?

(iii) Compute  $(2 * 3) * (4 * 5)$ .

(Hint: use the following table)

| $*$ | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|
| 1   | 1 | 1 | 1 | 1 | 1 |
| 2   | 1 | 2 | 1 | 2 | 1 |
| 3   | 1 | 1 | 3 | 1 | 1 |
| 4   | 1 | 2 | 1 | 4 | 1 |
| 5   | 1 | 1 | 1 | 1 | 5 |

## Answer 4:

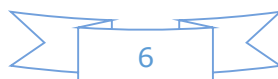
(i)  $(2 * 3) * 4 = 1 * 4 = 1$

$$2 * (3 * 4) = 2 * 1 = 1$$

(ii) For every  $a, b \in \{1, 2, 3, 4, 5\}$ , we have  $a * b = b * a$ . Therefore, the operation  $*$  is commutative.

(iii)  $(2 * 3) = 1$  and  $(4 * 5) = 1$

$$\therefore (2 * 3) * (4 * 5) = 1 * 1 = 1$$



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## Question 5:

Let  $*$ ' be the binary operation on the set  $\{1, 2, 3, 4, 5\}$  defined by  $a *' b = \text{H.C.F. of } a \text{ and } b$ . Is the operation  $*$ ' same as the operation  $*$  defined in Exercise 4 above? Justify your answer.

## Answer 5:

The binary operation  $*$ ' on the set  $\{1, 2, 3, 4, 5\}$  is defined as  $a *' b = \text{H.C.F. of } a \text{ and } b$ .

The operation table for the operation  $*$ ' can be given as:

| $*$ ' | 1 | 2 | 3 | 4 | 5 |
|-------|---|---|---|---|---|
| 1     | 1 | 1 | 1 | 1 | 1 |
| 2     | 1 | 2 | 1 | 2 | 1 |
| 3     | 1 | 1 | 3 | 1 | 1 |
| 4     | 1 | 2 | 1 | 4 | 1 |
| 5     | 1 | 1 | 1 | 1 | 5 |

We observe that the operation tables for the operations  $*$  and  $*$ ' are the same.

Thus, the operation  $*$ ' is same as the operation  $*$ .

## Question 6:

Let  $*$  be the binary operation on  $\mathbf{N}$  given by  $a * b = \text{L.C.M. of } a \text{ and } b$ . Find

(i)  $5 * 7, 20 * 16$

(ii) Is  $*$  commutative?

(iii) Is  $*$  associative?

(iv) Find the identity of  $*$  in  $\mathbf{N}$

(v) Which elements of  $\mathbf{N}$  are invertible for the operation  $*$ ?

## Answer 6:

The binary operation  $*$  on  $\mathbf{N}$  is defined as  $a * b = \text{L.C.M. of } a \text{ and } b$ .

(i)  $5 * 7 = \text{L.C.M. of } 5 \text{ and } 7 = 35$

$20 * 16 = \text{L.C.M of } 20 \text{ and } 16 = 80$





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(ii) It is known that

L.C.M of  $a$  and  $b = \text{L.C.M of } b \text{ and } a$  for all  $a, b \in \mathbf{N}$ .

$$\therefore a * b = b * a$$

Thus, the operation  $*$  is commutative.

(iii) For  $a, b, c \in \mathbf{N}$ , we have

$$(a * b) * c = (\text{L.C.M of } a \text{ and } b) * c = \text{LCM of } a, b, \text{ and } c$$

$$a * (b * c) = a * (\text{LCM of } b \text{ and } c) = \text{L.C.M of } a, b, \text{ and } c$$

$$\therefore (a * b) * c = a * (b * c)$$

Thus, the operation  $*$  is associative.

(iv) It is known that:

L.C.M. of  $a$  and  $1 = a = \text{L.C.M. } 1 \text{ and } a$  for all  $a \in \mathbf{N}$

$$\Rightarrow a * 1 = a = 1 * a \text{ for all } a \in \mathbf{N}$$

Thus, 1 is the identity of  $*$  in  $\mathbf{N}$ .

(v) An element  $a$  in  $\mathbf{N}$  is invertible with respect to the operation  $*$  if there exists an element  $b$  in  $\mathbf{N}$ , such that  $a * b = e = b * a$ .

Here,  $e = 1$

This means that

L.C.M of  $a$  and  $b = 1 = \text{L.C.M of } b \text{ and } a$

This case is possible only when  $a$  and  $b$  are equal to 1.

Thus, 1 is the only invertible element of  $\mathbf{N}$  with respect to the operation  $*$ .

### Question 7:

Is  $*$  defined on the set  $\{1, 2, 3, 4, 5\}$  by  $a * b = \text{L.C.M. of } a \text{ and } b$  a binary operation? Justify your answer.



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## Answer 7:

The operation  $*$  on the set  $A = \{1, 2, 3, 4, 5\}$  is defined as  $a * b = \text{L.C.M. of } a \text{ and } b$ .

Then, the operation table for the given operation  $*$  can be given as:

| $*$ | 1 | 2  | 3  | 4  | 5  |
|-----|---|----|----|----|----|
| 1   | 1 | 2  | 3  | 4  | 5  |
| 2   | 2 | 2  | 6  | 4  | 10 |
| 3   | 3 | 6  | 3  | 12 | 15 |
| 4   | 4 | 4  | 12 | 4  | 20 |
| 5   | 5 | 10 | 15 | 20 | 5  |

It can be observed from the obtained table that

$$3 * 2 = 2 * 3 = 6 \notin A,$$

$$5 * 2 = 2 * 5 = 10 \notin A,$$

$$3 * 4 = 4 * 3 = 12 \notin A,$$

$$3 * 5 = 5 * 3 = 15 \notin A,$$

$$4 * 5 = 5 * 4 = 20 \notin A$$

Hence, the given operation  $*$  is not a binary operation.

## Question 8:

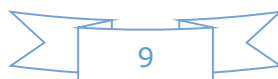
Let  $*$  be the binary operation on  $\mathbf{N}$  defined by  $a * b = \text{H.C.F. of } a \text{ and } b$ . Is  $*$  commutative? Is  $*$  associative? Does there exist identity for this binary operation on  $\mathbf{N}$ ?

## Answer 8:

The binary operation  $*$  on  $\mathbf{N}$  is defined as:  $a * b = \text{H.C.F. of } a \text{ and } b$

It is known that

$\text{H.C.F. of } a \text{ and } b = \text{H.C.F. of } b \text{ and } a \text{ for all } a, b \in \mathbf{N}.$



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$$\therefore a * b = b * a$$

Thus, the operation  $*$  is commutative.

For  $a, b, c \in \mathbf{N}$ , we have

$$(a * b) * c = (\text{H.C.F. of } a \text{ and } b) * c = \text{H.C.F. of } a, b \text{ and } c$$

$$a * (b * c) = a * (\text{H.C.F. of } b \text{ and } c) = \text{H.C.F. of } a, b, \text{ and } c$$

$$\therefore (a * b) * c = a * (b * c)$$

Thus, the operation  $*$  is associative.

Now, an element  $e \in \mathbf{N}$  will be the identity for the operation  $*$  if  $a * e = a = e * a$  for all  $a \in \mathbf{N}$ .

But this relation is not true for any  $a \in \mathbf{N}$ .

Thus, the operation  $*$  does not have any identity in  $\mathbf{N}$ .

### Question 9:

Let  $*$  be a binary operation on the set  $\mathbf{Q}$  of rational numbers as follows:

(i)  $a * b = a - b$

(ii)  $a * b = a^2 + b^2$

(iii)  $a * b = a + ab$

(iv)  $a * b = (a - b)^2$

(v)  $a * b = \frac{ab}{4}$

(vi)  $a * b = ab^2$

Find which of the binary operations are commutative and which are associative.

### Answer 9:

(i) On  $\mathbf{Q}$ , the operation  $*$  is defined as  $a * b = a - b$ . It can be observed that:

$$\frac{1}{2} * \frac{1}{3} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

and

$$\frac{1}{3} * \frac{1}{2} = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$$



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$$\therefore \frac{1}{2} * \frac{1}{3} \neq \frac{1}{3} * \frac{1}{2}, \text{ where } \frac{1}{2}, \frac{1}{3} \in \mathbf{Q}$$

Thus, the operation  $*$  is not commutative.

It can also be observed that

$$\left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} = \left(\frac{1}{2} - \frac{1}{3}\right) * \frac{1}{4} = \left(\frac{3-2}{6}\right) * \frac{1}{4} = \frac{1}{6} * \frac{1}{4} = \frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = \frac{-1}{12}$$

and

$$\frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right) = \frac{1}{2} * \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{2} * \left(\frac{4-3}{12}\right) = \frac{1}{2} * \frac{1}{12} = \frac{1}{2} - \frac{1}{12} = \frac{6-1}{12} = \frac{5}{12}$$

$$\therefore \left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} \neq \frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right), \text{ where } \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \in \mathbf{Q}$$

Thus, the operation  $*$  is not associative.

(ii) On  $\mathbf{Q}$ , the operation  $*$  is defined as  $a * b = a^2 + b^2$ .

For  $a, b \in \mathbf{Q}$ , we have

$$a * b = a^2 + b^2 = b^2 + a^2 = b * a$$

$$\therefore a * b = b * a$$

Thus, the operation  $*$  is commutative.

It can be observed that

$$(1 * 2) * 3 = (1^2 + 2^2) * 3 = (1 + 4) * 3 = 5 * 3 = 5^2 + 3^2 = 34 \quad \text{and}$$

$$1 * (2 * 3) = 1 * (2^2 + 3^2) = 1 * (4 + 9) = 1 * 13 = 1^2 + 13^2 = 170$$

$$\therefore (1 * 2) * 3 \neq 1 * (2 * 3), \text{ where } 1, 2, 3 \in \mathbf{Q}$$

Thus, the operation  $*$  is not associative.

(iii) On  $\mathbf{Q}$ , the operation  $*$  is defined as  $a * b = a + ab$ .



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It can be observed that

$$1 * 2 = 1 + 1 \times 2 = 1 + 2 = 3$$

$$2 * 1 = 2 + 2 \times 1 = 2 + 2 = 4$$

$$\therefore 1 * 2 \neq 2 * 1, \text{ where } 1, 2 \in \mathbf{Q}$$

Thus, the operation  $*$  is not commutative.

It can also be observed that

$$(1 * 2) * 3 = (1 + 1 \times 2) * 3 = (1 + 2) * 3 = 3 * 3 = 3 + 3 \times 3 = 3 + 9 = 12 \text{ and}$$

$$1 * (2 * 3) = 1 * (2 + 2 \times 3) = 1 * (2 + 6) = 1 * 8 = 1 + 1 \times 8 = 1 + 8 = 9$$

$$\therefore (1 * 2) * 3 \neq 1 * (2 * 3), \text{ where } 1, 2, 3 \in \mathbf{Q}$$

Thus, the operation  $*$  is not associative.

**(iv)** On  $\mathbf{Q}$ , the operation  $*$  is defined by  $a * b = (a - b)^2$ .

For  $a, b \in \mathbf{Q}$ , we have

$$a * b = (a - b)^2$$

$$b * a = (b - a)^2 = [- (a - b)]^2 = (a - b)^2$$

$$\therefore a * b = b * a$$

Thus, the operation  $*$  is commutative.

It can be observed that

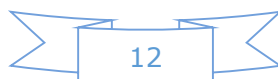
$$(1 * 2) * 3 = (1 - 2)^2 * 3 = (-1)^2 * 3 = 1 * 3 = (1 - 3)^2 = (-2)^2 = 4$$

and

$$1 * (2 * 3) = 1 * (2 - 3)^2 = 1 * (-1)^2 = 1 * 1 = (1 - 1)^2 = 0$$

$$\therefore (1 * 2) * 3 \neq 1 * (2 * 3), \text{ where } 1, 2, 3 \in \mathbf{Q}$$

Thus, the operation  $*$  is not associative.



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(v) On  $\mathbf{Q}$ , the operation  $*$  is defined as  $a*b = \frac{ab}{4}$ .

For  $a, b \in \mathbf{Q}$ , we have

$$a*b = \frac{ab}{4} = \frac{ba}{4} = b*a$$

$$\therefore a * b = b * a$$

Thus, the operation  $*$  is commutative.

For  $a, b, c \in \mathbf{Q}$ , we have

$$(a * b) * c = \left(\frac{ab}{4}\right) * c = \frac{\left(\frac{ab}{4}\right) \cdot c}{4} = \frac{abc}{16}$$

and

$$a * (b * c) = a * \left(\frac{bc}{4}\right) = \frac{a \cdot \left(\frac{bc}{4}\right)}{4} = \frac{abc}{16}$$

$$\therefore (a * b) * c = a * (b * c), \text{ where } a, b, c \in \mathbf{Q}$$

Thus, the operation  $*$  is associative.

(vi) On  $\mathbf{Q}$ , the operation  $*$  is defined as  $a * b = ab^2$

It can be observed that

$$\frac{1}{2} * \frac{1}{3} = \frac{1}{2} \cdot \left(\frac{1}{3}\right)^2 = \frac{1}{2} \cdot \frac{1}{9} = \frac{1}{18}$$

and

$$\frac{1}{3} * \frac{1}{2} = \frac{1}{3} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

$$\therefore \frac{1}{2} * \frac{1}{3} \neq \frac{1}{3} * \frac{1}{2}, \text{ where } \frac{1}{2} \text{ and } \frac{1}{3} \in \mathbf{Q}$$

Thus, the operation  $*$  is not commutative.

It can also be observed that



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$$\left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} = \left[\frac{1}{2} \left(\frac{1}{3}\right)^2\right] * \frac{1}{4} = \frac{1}{18} * \frac{1}{4} = \frac{1}{18} \cdot \left(\frac{1}{4}\right)^2 = \frac{1}{18 \times 16} = \frac{1}{288}$$

and

$$\frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right) = \frac{1}{2} * \left[\frac{1}{3} \left(\frac{1}{4}\right)^2\right] = \frac{1}{2} * \frac{1}{48} = \frac{1}{2} \cdot \left(\frac{1}{48}\right)^2 = \frac{1}{2 \times 2304} = \frac{1}{4608}$$

$$\therefore \left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} \neq \frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right), \text{ where } \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \in \mathbb{Q}$$

Thus, the operation  $*$  is not associative.

Hence, the operations defined in (ii), (iv), (v) are commutative and the operation defined in (v) is associative.

## Question 10:

Find which of the operations given above has identity.



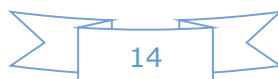
## Answer 10:

An element  $e \in \mathbb{Q}$  will be the identity element for the operation  $*$

if  $a * e = a = e * a$ , for all  $a \in \mathbb{Q}$ .

However, there is no such element  $e \in \mathbb{Q}$  with respect to each of the six operations satisfying the above condition.

Thus, none of the six operations has identity.



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### Question 11:

Let  $A = \mathbf{N} \times \mathbf{N}$  and  $*$  be the binary operation on  $A$  defined by

$$(a, b) * (c, d) = (a + c, b + d)$$

Show that  $*$  is commutative and associative. Find the identity element for  $*$  on  $A$ , if any.



### Answer 11:

$A = \mathbf{N} \times \mathbf{N}$  and  $*$  is a binary operation on  $A$  and is defined by

$$(a, b) * (c, d) = (a + c, b + d)$$

Let  $(a, b), (c, d) \in A$

Then,  $a, b, c, d \in \mathbf{N}$

We have:

$$(a, b) * (c, d) = (a + c, b + d)$$

$$(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d)$$

[Addition is commutative in the set of natural numbers]

$$\therefore (a, b) * (c, d) = (c, d) * (a, b)$$

Therefore, the operation  $*$  is commutative.

Now, let  $(a, b), (c, d), (e, f) \in A$

Then,  $a, b, c, d, e, f \in \mathbf{N}$

We have

$$[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)$$

and

$$(a, b) * [(c, d) * (e, f)] = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)$$

$$\therefore [(a, b) * (c, d)] * (e, f) = (a, b) * [(c, d) * (e, f)]$$

Therefore, the operation  $*$  is associative.





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Let an element  $e = (e_1, e_2) \in A$  will be an identity element for the operation  $*$  if  $a * e = a = e * a$  for all  $a = (a_1, a_2) \in A$

$$\text{i.e., } (a_1 + e_1, a_2 + e_2) = (a_1, a_2) = (e_1 + a_1, e_2 + a_2)$$

Which is not true for any element in  $A$ .

Therefore, the operation  $*$  does not have any identity element.

### Question 12:

State whether the following statements are true or false. Justify.

- (i) For an arbitrary binary operation  $*$  on a set  $\mathbf{N}$ ,  $a * a = a \forall a \in \mathbf{N}$ .
- (ii) If  $*$  is a commutative binary operation on  $\mathbf{N}$ , then  $a * (b * c) = (c * b) * a$

### Answer 12:

(i) Define an operation  $*$  on  $\mathbf{N}$  as  $a * b = a + b \forall a, b \in \mathbf{N}$

Then, in particular, for  $b = a = 3$ , we have

$$3 * 3 = 3 + 3 = 6 \neq 3$$

Therefore, statement (i) is false.

(ii) R.H.S.  $= (c * b) * a$

$$= (b * c) * a \quad [* \text{ is commutative}]$$

$$= a * (b * c) \quad [\text{Again, as } * \text{ is commutative}]$$

$$= \text{L.H.S.}$$

$$\therefore a * (b * c) = (c * b) * a$$

Therefore, statement (ii) is true.



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## Question 13:

Consider a binary operation  $*$  on  $\mathbf{N}$  defined as  $a * b = a^3 + b^3$ . Choose the correct answer.

- (A) Is  $*$  both associative and commutative?
- (B) Is  $*$  commutative but not associative?
- (C) Is  $*$  associative but not commutative?
- (D) Is  $*$  neither commutative nor associative?



## Answer 13:

On  $\mathbf{N}$ , the operation  $*$  is defined as  $a * b = a^3 + b^3$ .

For,  $a, b, \in \mathbf{N}$ , we have

$$a * b = a^3 + b^3 = b^3 + a^3 = b * a \quad [\text{Addition is commutative in } \mathbf{N}]$$

Therefore, the operation  $*$  is commutative.

It can be observed that

$$(1 * 2) * 3 = (1^3 + 2^3) * 3 = (1 + 8) * 3 = 9 * 3 = 9^3 + 3^3 = 729 + 27 = 756 \text{ and}$$

$$1 * (2 * 3) = 1 * (2^3 + 3^3) = 1 * (8 + 27) = 1 * 35 = 1^3 + 35^3 = 1 + 42875 = 42876$$

$$\therefore (1 * 2) * 3 \neq 1 * (2 * 3), \text{ where } 1, 2, 3 \in \mathbf{N}$$

Therefore, the operation  $*$  is not associative.

Hence, the operation  $*$  is commutative, but not associative.

Thus, the correct answer is B.

