Chapter 1

Relations and Functions

- Mathematics XII
- Exercise 1.4

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Exercise 1.4

Question 1:

Determine whether or not each of the definition of given below gives a binary operation.

In the event that * is not a binary operation, give justification for this.

(i) On $\mathbf{Z}^+$, define * by $a * b = a - b$

(ii) On $\mathbf{Z}^+$, define * by $a * b = ab$

(iii) On **R**, define * by $a * b = ab^2$

(iv) On $\mathbf{Z}^+$, define * by $a * b = |a - b|$

(v) On $\mathbf{Z}^+$, define * by $a * b = a$

Answer 1:

(i) On $\mathbf{Z}^+$, * is defined by $a * b = a - b$.

It is not a binary operation

as the image of (1, 2) under * is $1 * 2 = 1 - 2 = -1 \notin \mathbf{Z}^+$.

(ii) On $\mathbf{Z}^+$, * is defined by $a * b = ab$.

It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element ab in $\mathbf{Z}^+$.

This means that * carries each pair (a, b) to a unique element $a * b = ab$ in $\mathbf{Z}^+$.

Therefore, * is a binary operation.

(iii) On **R**, * is defined by $a * b = ab^2$.

It is seen that for each $a, b \in \mathbf{R}$, there is a unique element ab^2 in **R**.

This means that * carries each pair (a, b) to a unique element $a * b = ab^2$ in **R**.

Therefore, * is a binary operation.

(iv) On $\mathbf{Z}^+$, * is defined by $a * b = |a - b|$.

It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element $|a - b|$ in $\mathbf{Z}^+$.

This means that * carries each pair (a, b) to a unique element $a * b = |a - b|$ in $\mathbf{Z}^+$.

Therefore, * is a binary operation.

(v) On $\mathbf{Z}^+$, * is defined by $a * b = a$.

It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element a in $\mathbf{Z}^+$.

This means that * carries each pair (a, b) to a unique element $a * b = a$ in $\mathbf{Z}^+$.

Therefore, * is a binary operation.

Question 2:

For each binary operation * defined below, determine whether * is commutative or associative.

(i) On $\mathbf{Z}$, define $a * b = a - b$

(ii) On $\mathbf{Q}$, define $a * b = ab + 1$

(iii) On $\mathbf{Q}$, define $a * b = \frac{ab}{2}$

(iv) On $\mathbf{Z}^+$, define $a * b = 2^{ab}$

(v) On $\mathbf{Z}^+$, define $a * b = a^b$

(vi) On $\mathbf{R} - \{-1\}$, define $a * b = \frac{a}{b+1}$

Answer 2:

(i) On $\mathbf{Z}$, * is defined by $a * b = a - b$.

It can be observed that $1 * 2 = 1 - 2 = -1$ and $2 * 1 = 2 - 1 = 1$.

$\therefore 1 * 2 \neq 2 * 1$, where $1, 2 \in \mathbf{Z}$

Hence, the operation * is not commutative.

Also, we have

$(1 * 2) * 3 = (1 - 2) * 3 = -1 * 3 = -1 - 3 = -4$

$1 * (2 * 3) = 1 * (2 - 3) = 1 * -1 = 1 - (-1) = 2$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in \mathbf{Z}$

Hence, the operation * is not associative.

(ii) On **Q**, * is defined by $a * b = ab + 1$.

It is known that: $ab = ba$ for all $a, b \in \mathbf{Q}$

$\Rightarrow ab + 1 = ba + 1$ for all $a, b \in \mathbf{Q}$

$\Rightarrow a * b = a * b$ for all $a, b \in \mathbf{Q}$

Therefore, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = (1 \times 2 + 1) * 3 = 3 * 3 = 3 \times 3 + 1 = 10$

$1 * (2 * 3) = 1 * (2 \times 3 + 1) = 1 * 7 = 1 \times 7 + 1 = 8$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in \mathbf{Q}$

Therefore, the operation * is not associative.

(iii) On **Q**, * is defined by $a * b = \frac{ab}{2}$

It is known that: $ab = ba$ for all $a, b \in \mathbf{Q}$

$\Rightarrow \frac{ab}{2} = \frac{ba}{2}$ for all $a, b \in \mathbf{Q}$

$\Rightarrow a * b = b * a$ for all $a, b \in \mathbf{Q}$

Therefore, the operation * is commutative.

For all $a, b, c \in \mathbf{Q}$, we have

$$(a * b) * c = \left(\frac{ab}{2}\right) * c = \frac{\left(\frac{ab}{2}\right)c}{2} = \frac{abc}{4}$$

and

$$a * (b * c) = a * \left(\frac{bc}{2}\right) = \frac{a\left(\frac{bc}{2}\right)}{2} = \frac{abc}{4}$$

$\therefore$ (a*b)*c = a*(b*c), where $a, b, c \in \mathbf{Q}$

Therefore, the operation * is associative.

(iv) On $\mathbf{Z}^+$, * is defined by $a * b = 2^{ab}$.

It is known that: $ab = ba$ for all $a, b \in \mathbf{Z}^+$

$\Rightarrow 2^{ab} = 2^{ba}$ for all $a, b \in \mathbf{Z}^+$

$\Rightarrow a * b = b * a$ for all $a, b \in \mathbf{Z}^+$

Therefore, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = 2^{1\times 2} * 3 = 4 * 3 = 2^{4\times 3} = 2^{12}$ and

$1 * (2 * 3) = 1 * 2^{2\times 3} = 1 * 2^6 = 1 * 64 = 2^{1\times 64} = 2^{64}$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in \mathbf{Z}^+$

Therefore, the operation * is not associative.

(v) On $\mathbf{Z}^+$, * is defined by $a * b = a^b$.

It can be observed that

$1*2 = 1^2 = 1$ and $2*1 = 2^1 = 2$

$\therefore 1 * 2 \neq 2 * 1$, where $1, 2 \in \mathbf{Z}^+$

Therefore, the operation * is not commutative.

It can also be observed that

$(2 * 3) * 4 = 2^3 * 4 = 8 * 4 = 8^4 = 2^{12}$ and

$2 * (3 * 4) = 2 * 3^4 = 2 * 81 = 2^{81}$

$\therefore (2 * 3) * 4 \neq 2 * (3 * 4)$, where $2, 3, 4 \in \mathbf{Z}^+$

Therefore, the operation * is not associative.

(vi) On **R**, * − {−1} is defined by $a * b = \frac{a}{b+1}$

It can be observed that

$1 * 2 = \frac{1}{2+1} = \frac{1}{3}$ and $2 * 1 = \frac{2}{1+1} = \frac{2}{2} = 1$

$\therefore 1 * 2 \neq 2 * 1$, where $1, 2 \in \mathbf{R} - \{-1\}$

Therefore, the operation * is not commutative.

It can also be observed that

$$(1 * 2) * 3 = \frac{1}{2+1} * 3 = \frac{1}{3} * 3 = \frac{\frac{1}{3}}{3+1} = \frac{1}{12}$$

and

$$1 * (2 * 3) = 1 * \frac{2}{3+1} = 1 * \frac{2}{4} = 1 * \frac{1}{2} = \frac{1}{\frac{1}{2}+1} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in \mathbf{R} - \{-1\}$

Therefore, the operation * is not associative.

Question 3:

Consider the binary operation ∧ on the set {1, 2, 3, 4, 5} defined by $a \wedge b = \min\{a, b\}$.

Write the operation table of the operation ∧.

Answer 3:

The binary operation ∧ on the set {1, 2, 3, 4, 5} is defined as $a \wedge b = \min\{a, b\}$ for all $a, b \in \{1, 2, 3, 4, 5\}$.

Thus, the operation table for the given operation ∧ can be given as:

∧	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Question 4:

Consider a binary operation * on the set {1, 2, 3, 4, 5} given by the following multiplication table.

(i) Compute (2 * 3) * 4 and 2 * (3 * 4)

(ii) Is * commutative?

(iii) Compute (2 * 3) * (4 * 5).

(Hint: use the following table)

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Answer 4:

(i) (2 * 3) * 4 = 1 * 4 = 1

2 * (3 * 4) = 2 * 1 = 1

(ii) For every $a, b \in \{1, 2, 3, 4, 5\}$, we have $a * b = b * a$. Therefore, the operation * is commutative.

(iii) (2 * 3) = 1 and (4 * 5) = 1

∴ (2 * 3) * (4 * 5) = 1 * 1 = 1

Question 5:

Let *′ be the binary operation on the set {1, 2, 3, 4, 5} defined by a *′ b = H.C.F. of a and b. Is the operation *′ same as the operation * defined in Exercise 4 above? Justify your answer.

Answer 5:

The binary operation *′ on the set {1, 2, 3 4, 5} is defined as a *′ b = H.C.F of a and b.

The operation table for the operation *′ can be given as:

*′	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

We observe that the operation tables for the operations * and *′ are the same.

Thus, the operation *′ is same as the operation*.

Question 6:

Let * be the binary operation on **N** given by a * b = L.C.M. of a and b. Find

(i) 5 * 7, 20 * 16

(ii) Is * commutative?

(iii) Is * associative?

(iv) Find the identity of * in **N**

(v) Which elements of **N** are invertible for the operation *?

Answer 6:

The binary operation * on **N** is defined as a * b = L.C.M. of a and b.

(i) 5 * 7 = L.C.M. of 5 and 7 = 35

20 * 16 = L.C.M of 20 and 16 = 80

(ii) It is known that

L.C.M of a and b = L.C.M of b and a for all $a, b \in \mathbf{N}$.

$\therefore a * b = b * a$

Thus, the operation * is commutative.

(iii) For $a, b, c \in \mathbf{N}$, we have

$(a * b) * c$ = (L.C.M of a and b) $* c$ = LCM of a, b, and c

$a * (b * c) = a *$ (LCM of b and c) = L.C.M of a, b, and c

$\therefore (a * b) * c = a * (b * c)$

Thus, the operation * is associative.

(iv) It is known that:

L.C.M. of a and 1 = a = L.C.M. 1 and a for all $a \in \mathbf{N}$

$\Rightarrow a * 1 = a = 1 * a$ for all $a \in \mathbf{N}$

Thus, 1 is the identity of * in **N**.

(v) An element a in **N** is invertible with respect to the operation * if there exists an element b in **N**, such that $a * b = e = b * a$.

Here, $e = 1$

This means that

L.C.M of a and b = 1 = L.C.M of b and a

This case is possible only when a and b are equal to 1.

Thus, 1 is the only invertible element of **N** with respect to the operation *.

Question 7:

Is * defined on the set {1, 2, 3, 4, 5} by $a * b$ = L.C.M. of a and b a binary operation? Justify your answer.

Answer 7:

The operation * on the set A = {1, 2, 3, 4, 5} is defined as $a * b$ = L.C.M. of a and b.

Then, the operation table for the given operation * can be given as:

*	1	2	3	4	5
1	1	2	3	4	5
2	2	2	6	4	10
3	3	6	3	12	15
4	4	4	12	4	20
5	5	10	15	20	5

It can be observed from the obtained table that

$3 * 2 = 2 * 3 = 6 \notin A$,

$5 * 2 = 2 * 5 = 10 \notin A$,

$3 * 4 = 4 * 3 = 12 \notin A$,

$3 * 5 = 5 * 3 = 15 \notin A$,

$4 * 5 = 5 * 4 = 20 \notin A$

Hence, the given operation * is not a binary operation.

Question 8:

Let * be the binary operation on **N** defined by $a * b$ = H.C.F. of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on **N**?

Answer 8:

The binary operation * on **N** is defined as: $a * b$ = H.C.F. of a and b

It is known that

H.C.F. of a and b = H.C.F. of b and a for all $a, b \in$ **N**.

$\therefore a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in \mathbf{N}$, we have

$(a * b) * c =$ (H.C.F. of a and b) $* c =$ H.C.F. of a, b and c

$a * (b * c) = a *$(H.C.F. of b and c) = H.C.F. of a, b, and c

$\therefore (a * b) * c = a * (b * c)$

Thus, the operation * is associative.

Now, an element $e \in \mathbf{N}$ will be the identity for the operation * if $a * e = a = e * a$ for all $a \in \mathbf{N}$.

But this relation is not true for any $a \in \mathbf{N}$.

Thus, the operation * does not have any identity in **N**.

Question 9:

Let * be a binary operation on the set **Q** of rational numbers as follows:

(i) $a * b = a - b$

(ii) $a * b = a^2 + b^2$

(iii) $a * b = a + ab$

(iv) $a * b = (a - b)^2$

(v) $a * b = \frac{ab}{4}$

(vi) $a * b = ab^2$

Find which of the binary operations are commutative and which are associative.

Answer 9:

(i) On **Q**, the operation * is defined as $a * b = a - b$. It can be observed that:

$$\frac{1}{2} * \frac{1}{3} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

and

$$\frac{1}{3} * \frac{1}{2} = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$$

$\therefore \frac{1}{2} * \frac{1}{3} \neq \frac{1}{3} * \frac{1}{2}, where\ \frac{1}{2}, \frac{1}{3} \in \boldsymbol{Q}$

Thus, the operation * is not commutative.

It can also be observed that

$$\left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} = \left(\frac{1}{2} - \frac{1}{3}\right) * \frac{1}{4} = \left(\frac{3-2}{6}\right) * \frac{1}{4} = \frac{1}{6} * \frac{1}{4} = \frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = \frac{-1}{12}$$

and

$$\frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right) = \frac{1}{2} * \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{2} * \left(\frac{4-3}{12}\right) = \frac{1}{2} * \frac{1}{12} = \frac{1}{2} - \frac{1}{12} = \frac{6-1}{12} = \frac{5}{12}$$

$\therefore \left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} \neq \frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right), where\ \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \in \boldsymbol{Q}$

Thus, the operation * is not associative.

(ii) On **Q**, the operation * is defined as $a * b = a^2 + b^2$.

For $a, b \in$ **Q**, we have

$a * b = a^2 + b^2 = b^2 + a^2 = b * a$

$\therefore a * b = b * a$

Thus, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = (1^2 + 2^2) * 3 = (1 + 4) * 3 = 5 * 3 = 5^2 + 3^2 = 34$ and

$1 * (2 * 3) = 1 * (2^2 + 3^2) = 1 * (4 + 9) = 1 * 13 = 1^2 + 13^2 = 170$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in$ **Q**

Thus, the operation * is not associative.

(iii) On **Q**, the operation * is defined as $a * b = a + ab$.

It can be observed that

$1 * 2 = 1 + 1\times2 = 1 + 2 = 3$

$2 * 1 = 2 + 2\times1 = 2 + 2 = 4$

$\therefore 1 * 2 \neq 2 * 1$, where $1, 2 \in \mathbf{Q}$

Thus, the operation * is not commutative.

It can also be observed that

$(1 * 2) * 3 = (1+ 1\times2) * 3 = (1 + 2) * 3 = 3 * 3 = 3 + 3\times3 = 3 + 9 = 12$ and

$1 * (2 * 3) = 1 * (2 + 2\times3) = 1 * (2 + 6) = 1 * 8 = 1 + 1\times8 =1 + 8 = 9$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in \mathbf{Q}$

Thus, the operation * is not associative.

(iv) On **Q**, the operation * is defined by $a * b = (a - b)^2$.

For $a, b \in \mathbf{Q}$, we have

$a * b = (a - b)^2$

$b * a = (b - a)^2 = [-(a - b)]^2 = (a - b)^2$

$\therefore a * b = b * a$

Thus, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = (1 - 2)^2 * 3 = (-1)^2 * 3 = 1 * 3 = (1 - 3)^2 = (-2)^2 = 4$

and

$1 * (2 * 3) = 1 * (2 - 3)^2 = 1 * (-1)^2 = 1 * 1 = (1 - 1)^2 = 0$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in \mathbf{Q}$

Thus, the operation * is not associative.

(v) On **Q**, the operation * is defined as $a*b = \frac{ab}{4}$.

For $a, b \in \mathbf{Q}$, we have

$a*b = \frac{ab}{4} = \frac{ba}{4} = b*a$

$\therefore a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in \mathbf{Q}$, we have

$$(a*b)*c = \left(\frac{ab}{4}\right)*c = \frac{\left(\frac{ab}{4}\right).c}{4} = \frac{abc}{16}$$

and

$$a*(b*c) = a*\left(\frac{bc}{4}\right) = \frac{a.\left(\frac{bc}{4}\right)}{4} = \frac{abc}{16}$$

$\therefore (a * b) * c = a * (b * c)$, where $a, b, c \in \mathbf{Q}$

Thus, the operation * is associative.

(vi) On **Q**, the operation * is defined as $a * b = ab^2$

It can be observed that

$$\frac{1}{2}*\frac{1}{3} = \frac{1}{2}.\left(\frac{1}{3}\right)^2 = \frac{1}{2}.\frac{1}{9} = \frac{1}{18}$$

and

$$\frac{1}{3}*\frac{1}{2} = \frac{1}{3}.\left(\frac{1}{2}\right)^2 = \frac{1}{3}.\frac{1}{4} = \frac{1}{12}$$

$\therefore \frac{1}{2}*\frac{1}{3} \neq \frac{1}{3}*\frac{1}{2}$, where $\frac{1}{2}$ and $\frac{1}{3} \in \mathbf{Q}$

Thus, the operation * is not commutative.

It can also be observed that

$$\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}=\left[\frac{1}{2}\left(\frac{1}{3}\right)^2\right]*\frac{1}{4}=\frac{1}{18}*\frac{1}{4}=\frac{1}{18}.\left(\frac{1}{4}\right)^2=\frac{1}{18\times 16}=\frac{1}{288}$$

and

$$\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)=\frac{1}{2}*\left[\frac{1}{3}\left(\frac{1}{4}\right)^2\right]=\frac{1}{2}*\frac{1}{48}=\frac{1}{2}\left(\frac{1}{48}\right)^2=\frac{1}{2\times 2304}=\frac{1}{4608}$$

$$\therefore \left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}\neq\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right), where\ \frac{1}{2},\frac{1}{3},\frac{1}{4}\in \boldsymbol{Q}$$

Thus, the operation * is not associative.

Hence, the operations defined in (ii), (iv), (v) are commutative and the operation defined in (v) is associative.

Question 10:

Find which of the operations given above has identity.

Answer 10:

An element $e \in \mathbf{Q}$ will be the identity element for the operation *

if $a * e = a = e * a$, for all $a \in \mathbf{Q}$.

However, there is no such element $e \in \mathbf{Q}$ with respect to each of the six operations satisfying the above condition.

Thus, none of the six operations has identity.

Question 11:

Let A = **N** × **N** and * be the binary operation on A defined by

$(a, b) * (c, d) = (a + c, b + d)$

Show that * is commutative and associative. Find the identity element for * on A, if any.

Answer 11:

A = **N** × **N** and * is a binary operation on A and is defined by

$$(a, b) * (c, d) = (a + c, b + d)$$

Let $(a, b), (c, d) \in A$

Then, $a, b, c, d \in \mathbf{N}$

We have:

$(a, b) * (c, d) = (a + c, b + d)$

$(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d)$

[Addition is commutative in the set of natural numbers]

$\therefore (a, b) * (c, d) = (c, d) * (a, b)$

Therefore, the operation * is commutative.

Now, let $(a, b), (c, d), (e, f) \in A$

Then, $a, b, c, d, e, f \in \mathbf{N}$

We have

$[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)$

and

$(a, b) * [(c, d) * (e, f)] = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)$

$\therefore [(a, b) * (c, d)] * (e, f) = (a, b) * [(c, d) * (e, f)]$

Therefore, the operation * is associative.

Let an element $e = (e_1, e_2) \in A$ will be an identity element for the operation * if $a * e = a = e * a$ for all $a = (a_1, a_2) \in A$

i.e., $(a_1 + e_1, a_2 + e_2) = (a_1, a_2) = (e_1 + a_1, e_2 + a_2)$

Which is not true for any element in A.

Therefore, the operation * does not have any identity element.

Question 12:

State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation * on a set **N**, $a * a = a \ \forall \ a \in \mathbf{N}$.

(ii) If * is a commutative binary operation on **N**, then $a * (b * c) = (c * b) * a$

Answer 12:

(i) Define an operation * on **N** as $a * b = a + b \ \forall \ a, b \in \mathbf{N}$

Then, in particular, for $b = a = 3$, we have

$3 * 3 = 3 + 3 = 6 \neq 3$

Therefore, statement (i) is false.

(ii) R.H.S. $= (c * b) * a$

$= (b * c) * a$ [* is commutative]

$= a * (b * c)$ [Again, as * is commutative]

= L.H.S.

$\therefore a * (b * c) = (c * b) * a$

Therefore, statement (ii) is true.

Question 13:

Consider a binary operation * on **N** defined as $a * b = a^3 + b^3$. Choose the correct answer.

(A) Is * both associative and commutative?

(B) Is * commutative but not associative?

(C) Is * associative but not commutative?

(D) Is * neither commutative nor associative?

Answer 13:

On **N**, the operation * is defined as $a * b = a^3 + b^3$.

For, $a, b, \in \mathbf{N}$, we have

$a * b = a^3 + b^3 = b^3 + a^3 = b * a$ [Addition is commutative in **N**]

Therefore, the operation * is commutative.

It can be observed that

$(1 * 2) * 3 = (1^3 + 2^3) * 3 = (1 + 8) * 3 = 9 * 3 = 9^3 + 3^3 = 729 + 27 = 756$ and

$1 * (2 * 3) = 1 * (2^3 + 3^3) = 1 * (8 + 27) = 1 * 35 = 1^3 + 35^3 = 1 + 42875 = 42876$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in \mathbf{N}$

Therefore, the operation * is not associative.

Hence, the operation * is commutative, but not associative.

Thus, the correct answer is B.