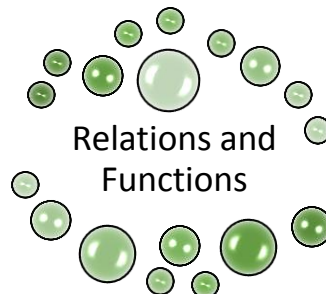


Chapter 1



- Mathematics XII
- Miscellaneous Exercise



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Miscellaneous Exercise

Question 1:

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = 10x + 7$. Find the function $g: \mathbf{R} \rightarrow \mathbf{R}$ such that $g \circ f = f \circ g = \text{Id}$.

Answer 1:

It is given that $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = 10x + 7$.

For one – one

Let $f(x) = f(y)$, where $x, y \in \mathbf{R}$.

$$\Rightarrow 10x + 7 = 10y + 7$$

$$\Rightarrow x = y$$

$\therefore f$ is a one – one function.

For onto

For $y \in \mathbf{R}$, let $y = 10x + 7$.

$$\Rightarrow x = \frac{y - 7}{10} \in \mathbf{R}$$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \frac{y-7}{10} \in \mathbf{R}$ such that

$$f(x) = f\left(\frac{y-7}{10}\right) = 10\left(\frac{y-7}{10}\right) + 7 = y - 7 + 7 = y$$

$\therefore f$ is onto.

Therefore, f is one – one and onto.

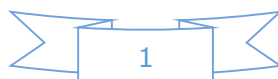
Thus, f is an invertible function.

Let us define $g: \mathbf{R} \rightarrow \mathbf{R}$ as $g(y) = \frac{y-7}{10}$

Now, we have

$$g \circ f(x) = g(f(x)) = g(10x + 7) = \frac{(10x + 7) - 7}{10} = \frac{10x}{10} = x$$

and



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$$f \circ g(y) = f(g(y)) = f\left(\frac{y-7}{10}\right) = 10\left(\frac{y-7}{10}\right) + 7 = y - 7 + 7 = y$$

$$\therefore g \circ f = I_{\mathbb{R}} \text{ and } f \circ g = I_{\mathbb{R}}.$$

Hence, the required function $g: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $g(y) = \frac{y-7}{10}$.

Question 2:

Let $f: \mathbb{W} \rightarrow \mathbb{W}$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f . Here, \mathbb{W} is the set of all whole numbers.

Answer 2:

It is given that:

$$f: \mathbb{W} \rightarrow \mathbb{W} \text{ is defined as } f(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even} \end{cases}$$

For one – one

$$\text{Let } f(n) = f(m).$$

It can be observed that if n is odd and m is even, then we will have $n - 1 = m + 1$.

$$\Rightarrow n - m = 2$$

However, this is impossible.

Similarly, the possibility of n being even and m being odd can also be ignored under a similar argument.

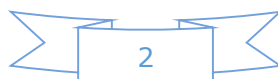
\therefore Both n and m must be either odd or even. Now, if both n and m are odd,

Then, we have

$$f(n) = f(m)$$

$$\Rightarrow n - 1 = m - 1$$

$$\Rightarrow n = m$$



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Again, if both n and m are even,

Then, we have

$$f(n) = f(m)$$

$$\Rightarrow n + 1 = m + 1$$

$$\Rightarrow n = m$$

$\therefore f$ is one – one.

For onto

It is clear that any odd number $2r + 1$ in co-domain \mathbf{N} is the image of $2r$ in domain \mathbf{N} and any even number $2r$ in co-domain \mathbf{N} is the image of $2r + 1$ in domain \mathbf{N} .

$\therefore f$ is onto.

Hence, f is an invertible function.

Let us define $g: W \rightarrow W$ as $g(m) = \begin{cases} m + 1, & \text{if } m \text{ is even} \\ m - 1, & \text{if } m \text{ is odd} \end{cases}$

Now, when n is odd

$$gof(n) = g(f(n)) = g(n - 1) = n - 1 + 1 = n \text{ and}$$

When n is even

$$gof(n) = g(f(n)) = g(n + 1) = n + 1 - 1 = n$$

Similarly,

When m is odd

$$fog(m) = f(g(m)) = f(m - 1) = m - 1 + 1 = m \text{ and}$$

When m is even

$$fog(m) = f(g(m)) = f(m + 1) = m + 1 - 1 = m$$

$\therefore gof = I_W$ and $fog = I_W$

Thus, f is invertible and the inverse of f is given by $f^{-1} = g$, which is the same as f . Hence, the inverse of f is f itself.



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Question 3:

If $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

Answer 3:

It is given that $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = x^2 - 3x + 2$.

$$\begin{aligned} f(f(x)) &= f(x^2 - 3x + 2) \\ &= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2 \\ &= (x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2) + (-3x^2 + 9x - 6) + 2 \\ &= x^4 - 6x^3 + 10x^2 - 3x \end{aligned}$$

Question 4:

Show that function $f: \mathbf{R} \rightarrow \{x \in \mathbf{R}: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbf{R}$ is one – one and onto function.

Answer 4:

It is given that $f: \mathbf{R} \rightarrow \{x \in \mathbf{R}: -1 < x < 1\}$ is defined as $f(x) = \frac{x}{1+|x|}$, $x \in \mathbf{R}$.

For one – one

Suppose $f(x) = f(y)$, where $x, y \in \mathbf{R}$.

$$\Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$$

It can be observed that if x is positive and y is negative,

Then, we have

$$\frac{x}{1+x} = \frac{y}{1-y} \Rightarrow 2xy = x - y$$

Since, x is positive and y is negative

$$x > y \Rightarrow x - y > 0$$

But, $2xy$ is negative.

Then $2xy \neq x - y$



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Thus, the case of x being positive and y being negative can be ruled out.

Under a similar argument, x being negative and y being positive can also be ruled out.

$\therefore x$ and y have to be either positive or negative.

When x and y are both positive, we have

$$f(x) = f(y) \Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$$

When x and y are both negative, we have

$$f(x) = f(y) \Rightarrow \frac{x}{1-x} = \frac{y}{1-y} \Rightarrow x - xy = y - xy \Rightarrow x = y$$

$\therefore f$ is one – one.

For onto

Now, let $y \in \mathbf{R}$ such that $-1 < y < 1$.

If y is negative, then, there exists $x = \frac{y}{1+y} \in \mathbf{R}$ such that

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\left(\frac{y}{1+y}\right)}{1 + \left|\frac{y}{1+y}\right|} = \frac{\frac{y}{1+y}}{1 + \left(\frac{-y}{1+y}\right)} = \frac{y}{1+y-y} = y$$

If y is positive, then, there exists $x = \frac{y}{1-y} \in \mathbf{R}$ such that

$$f(x) = f\left(\frac{y}{1-y}\right) = \frac{\left(\frac{y}{1-y}\right)}{1 + \left|\frac{y}{1-y}\right|} = \frac{\frac{y}{1-y}}{1 + \left(\frac{y}{1-y}\right)} = \frac{y}{1-y+y} = y$$

$\therefore f$ is onto.

Hence, f is one – one and onto.



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Question 5:

Show that the function $f: \mathbf{R} \rightarrow \mathbf{R}$ given by $f(x) = x^3$ is injective.

Answer 5:

$f: \mathbf{R} \rightarrow \mathbf{R}$ is given as $f(x) = x^3$.

For one – one

Suppose $f(x) = f(y)$, where $x, y \in \mathbf{R}$.

$$\Rightarrow x^3 = y^3 \dots\dots\dots (1)$$

Now, we need to show that $x = y$.

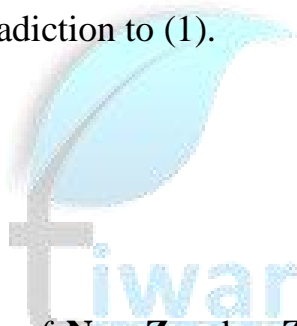
Suppose $x \neq y$, their cubes will also not be equal.

$$\Rightarrow x^3 \neq y^3$$

However, this will be a contradiction to (1).

$$\therefore x = y$$

Hence, f is injective.



Question 6:

Give examples of two functions $f: \mathbf{N} \rightarrow \mathbf{Z}$ and $g: \mathbf{Z} \rightarrow \mathbf{Z}$ such that $g \circ f$ is injective but g is not injective.

(Hint: Consider $f(x) = x$ and $g(x) = |x|$)

Answer 6:

Define $f: \mathbf{N} \rightarrow \mathbf{Z}$ as $f(x) = x$ and $g: \mathbf{Z} \rightarrow \mathbf{Z}$ as $g(x) = |x|$.

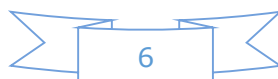
We first show that g is not injective.

It can be observed that

$$g(-1) = |-1| = 1$$

$$g(1) = |1| = 1$$

$$\therefore g(-1) = g(1), \text{ but } -1 \neq 1.$$



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$\therefore g$ is not injective.

Now, g of: $\mathbf{N} \rightarrow \mathbf{Z}$ is defined as $gof(x) = g(f(x)) = g(x) = |x|$.

Let $x, y \in \mathbf{N}$ such that g of(x) = g of(y).

$$\Rightarrow |x| = |y|$$

Since x and $y \in \mathbf{N}$, both are positive.

$$\therefore |x| = |y| \Rightarrow x = y$$

Hence, gof is injective

Question 7:

Given examples of two functions $f: \mathbf{N} \rightarrow \mathbf{N}$ and $g: \mathbf{N} \rightarrow \mathbf{N}$ such that g of is onto but f is not onto.

(Hint: Consider $f(x) = x + 1$ and $g(x) = \begin{cases} x - 1, & \text{if } x > 1 \\ 1, & \text{if } x = 1 \end{cases}$

Answer 7:

Define $f: \mathbf{N} \rightarrow \mathbf{N}$ by $f(x) = x + 1$

and $g: \mathbf{N} \rightarrow \mathbf{N}$ by $g(x) = \begin{cases} x - 1, & \text{if } x > 1 \\ 1, & \text{if } x = 1 \end{cases}$

We first show that g is not onto.

For this, consider element 1 in co-domain \mathbf{N} . It is clear that this element is not an image of any of the elements in domain \mathbf{N} .

$\therefore f$ is not onto.

Now, $gof: \mathbf{N} \rightarrow \mathbf{N}$ is defined by

$$gof(x) = g(f(x)) = g(x + 1) = x + 1 - 1 = x \quad [x \in \mathbf{N} \Rightarrow x + 1 > 1]$$

Then, it is clear that for $y \in \mathbf{N}$, there exists $x = y \in \mathbf{N}$ such that $gof(x) = y$.

Hence, gof is onto.



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Question 8:

Given a non-empty set X , consider $P(X)$ which is the set of all subsets of X .

Define the relation R in $P(X)$ as follows:

For subsets A, B in $P(X)$, ARB if and only if $A \subset B$. Is R an equivalence relation on $P(X)$? Justify your answer.

Answer 8:

Since every set is a subset of itself, ARA for all $A \in P(X)$.

$\therefore R$ is reflexive.

Let $ARB \Rightarrow A \subset B$.

This cannot be implied to $B \subset A$.

For instance, if $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then it cannot be implied that B is related to A .

$\therefore R$ is not symmetric.

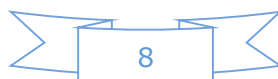
Further, if ARB and BRC , then $A \subset B$ and $B \subset C$.

$\Rightarrow A \subset C$

$\Rightarrow ARC$

$\therefore R$ is transitive.

Hence, R is not an equivalence relation as it is not symmetric.



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Question 9:

Given a non-empty set X , consider the binary operation $*$: $P(X) \times P(X) \rightarrow P(X)$ given by $A * B = A \cap B \forall A, B$ in $P(X)$ is the power set of X . Show that X is the identity element for this operation and X is the only invertible element in $P(X)$ with respect to the operation $*$.

Answer 9:

It is given the binary operation $*$:

$P(X) \times P(X) \rightarrow P(X)$ given by $A * B = A \cap B \forall A, B$ in $P(X)$

We know that $A \cap X = A = X \cap A$ for all $A \in P(X)$

$\Rightarrow A * X = A = X * A$ for all $A \in P(X)$

Thus, X is the identity element for the given binary operation $*$.

Now, an element $A \in P(X)$ is invertible if there exists $B \in P(X)$ such that

$$A * B = X = B * A \quad [\text{As } X \text{ is the identity element}]$$

or

$$A \cap B = X = B \cap A$$

This case is possible only when $A = X = B$.

Thus, X is the only invertible element in $P(X)$ with respect to the given operation $*$.

Hence, the given result is proved.

Question 10:

Find the number of all onto functions from the set $\{1, 2, 3, \dots, n\}$ to itself.

Answer 10:

Onto functions from the set $\{1, 2, 3, \dots, n\}$ to itself is simply a permutation on n symbols $1, 2, \dots, n$.

Thus, the total number of onto maps from $\{1, 2, \dots, n\}$ to itself is the same as the total number of permutations on n symbols $1, 2, \dots, n$, which is $n!$.



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Question 11:

Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find F^{-1} of the following functions F from S to T , if it exists.

(i) $F = \{(a, 3), (b, 2), (c, 1)\}$

(ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

Answer 11:

$$S = \{a, b, c\}, \quad T = \{1, 2, 3\}$$

(i) $F: S \rightarrow T$ is defined as $F = \{(a, 3), (b, 2), (c, 1)\}$

$$\Rightarrow F(a) = 3, F(b) = 2, F(c) = 1$$

Therefore, $F^{-1}: T \rightarrow S$ is given by $F^{-1} = \{(3, a), (2, b), (1, c)\}$.

(ii) $F: S \rightarrow T$ is defined as $F = \{(a, 2), (b, 1), (c, 1)\}$

Since $F(b) = F(c) = 1$, F is not one – one.

Hence, F is not invertible i.e., F^{-1} does not exist.

Question 12:

Consider the binary operations $*$: $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ and \circ : $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ defined as $a * b = |a - b|$ and $a \circ b = a$, $\forall a, b \in \mathbf{R}$. Show that $*$ is commutative but not associative, \circ is associative but not commutative. Further, show that $\forall a, b, c \in \mathbf{R}$, $a*(b \circ c) = (a * b) \circ (a * c)$. [If it is so, we say that the operation $*$ distributes over the operation \circ]. Does \circ distribute over $*$? Justify your answer.

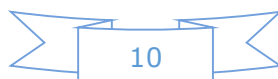
Answer 12:

It is given that $*$: $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ and \circ : $\mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ is defined as $a * b = |a - b|$ and $a \circ b = a$, $\forall a, b \in \mathbf{R}$

For $a, b \in \mathbf{R}$, we have

$$a * b = |a - b| \text{ and } b * a = |b - a| = |-(a - b)| = |a - b|$$

$$\therefore a * b = b * a$$



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Hence, the operation $*$ is commutative.

It can be observed that

$$(1 * 2) * 3 = (|1 - 2|) * 3 = 1 * 3 = |1 - 3| = 2$$

and

$$1 * (2 * 3) = 1 * (|2 - 3|) = 1 * 1 = |1 - 1| = 0$$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$ where $1, 2, 3 \in \mathbf{R}$.

Hence, the operation $*$ is not associative.

Now, consider the operation o

It can be observed that $1 o 2 = 1$ and $2 o 1 = 2$.

$\therefore 1 o 2 \neq 2 o 1$ where $1, 2 \in \mathbf{R}$.

Hence, the operation o is not commutative.

Let $a, b, c \in \mathbf{R}$. Then, we have

$$(a o b) o c = a o c = a$$

and

$$a o (b o c) = a o b = a$$

$\therefore (a o b) o c = a o (b o c)$, where $a, b, c \in \mathbf{R}$

Hence, the operation o is associative.

Now, let $a, b, c \in \mathbf{R}$, then we have

$$a * (b o c) = a * b = |a - b|$$

$$(a * b) o (a * c) = (|a - b|) o (|a - c|) = |a - b|$$

Hence, $a * (b o c) = (a * b) o (a * c)$.

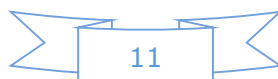
Now,

$$1 o (2 * 3) = 1 o (|2 - 3|) = 1 o 1 = 1$$

$$(1 o 2) * (1 o 3) = 1 * 1 = |1 - 1| = 0$$

$\therefore 1 o (2 * 3) \neq (1 o 2) * (1 o 3)$ where $1, 2, 3 \in \mathbf{R}$

Hence, the operation o does not distribute over $*$.



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Question 13:

Given a non -empty set X , let $*$: $P(X) \times P(X) \rightarrow P(X)$ be defined as $A * B = (A - B) \cup (B - A)$, $\forall A, B \in P(X)$. Show that the empty set Φ is the identity for the operation $*$ and all the elements A of $P(X)$ are invertible with $A^{-1} = A$.

(Hint: $(A - \Phi) \cup (\Phi - A) = A$ and $(A - A) \cup (A - A) = A * A = \Phi$).

Answer 13:

It is given that $*$: $P(X) \times P(X) \rightarrow P(X)$ is defined

as $A * B = (A - B) \cup (B - A) \forall A, B \in P(X)$.

Let $A \in P(X)$. Then, we have

$$A * \Phi = (A - \Phi) \cup (\Phi - A) = A \cup \Phi = A$$

$$\Phi * A = (\Phi - A) \cup (A - \Phi) = \Phi \cup A = A$$

$$\therefore A * \Phi = A = \Phi * A \text{ for all } A \in P(X)$$

Thus, Φ is the identity element for the given operation*.

Now, an element $A \in P(X)$ will be invertible if there exists $B \in P(X)$ such that $A * B = \Phi = B * A$.

[As Φ is the identity element]

Now, we observed that

$$A * A = (A - A) \cup (A - A) = \Phi \cup \Phi = \Phi \text{ for all } A \in P(X).$$

Hence, all the elements A of $P(X)$ are invertible with $A^{-1} = A$.

Question 14:

Define a binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ as

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with $6 - a$ being the inverse of a .

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Answer 14:

Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation $*$ on X is defined as $a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$

An element $e \in X$ is the identity element for the operation $*$, if

$$a * e = a = e * a \text{ for all } a \in X$$

For $a \in X$, we have

$$a * 0 = a + 0 = a \quad [a \in X \Rightarrow a + 0 < 6]$$

$$0 * a = 0 + a = a \quad [a \in X \Rightarrow 0 + a < 6]$$

$$\therefore a * 0 = a = 0 * a \text{ for all } a \in X$$

Thus, 0 is the identity element for the given operation $*$.

An element $a \in X$ is invertible if there exists $b \in X$ such that $a * b = 0 = b * a$.

$$\text{i.e., } \begin{cases} a + b = 0 = b + a, & \text{if } a + b < 6 \\ a + b - 6 = 0 = b + a - 6, & \text{if } a + b \geq 6 \end{cases}$$

$$\Rightarrow a = -b \text{ or } b = 6 - a$$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$.

$\therefore b = 6 - a$ is the inverse of a for all $a \in X$.

Hence, the inverse of an element $a \in X$, $a \neq 0$ is $6 - a$ i.e., $a^{-1} = 6 - a$.

Question 15:

Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \rightarrow B$ be functions defined by $f(x) = x^2 - x$, $x \in A$ and $g(x) = 2 \left| x - \frac{1}{2} \right| - 1$, $x \in A$. Are f and g equal? Justify your answer. (Hint: One may note that two function $f: A \rightarrow B$ and $g: A \rightarrow B$ such that $f(a) = g(a) \forall a \in A$, are called equal functions).

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Answer 15:

It is given that $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$.

Also, it is given that $f, g: A \rightarrow B$ are defined by

$$f(x) = x^2 - x, x \in A \text{ and } g(x) = 2 \left| x - \frac{1}{2} \right| - 1, x \in A$$

It is observed that

$$f(-1) = (-1)^2 - (-1) = 1 + 1 = 2$$

$$\text{and } g(-1) = 2 \left| (-1) - \frac{1}{2} \right| - 1 = 2 \left(\frac{3}{2} \right) - 1 = 3 - 1 = 2$$

$$\Rightarrow f(-1) = g(-1)$$

$$f(0) = (0)^2 - (0) = 0$$

$$\text{and } g(0) = 2 \left| 0 - \frac{1}{2} \right| - 1 = 2 \left(\frac{1}{2} \right) - 1 = 1 - 1 = 0$$

$$\Rightarrow f(0) = g(0)$$

$$f(1) = (1)^2 - (1) = 1 - 1 = 0$$

$$\text{and } g(1) = 2 \left| (1) - \frac{1}{2} \right| - 1 = 2 \left(\frac{1}{2} \right) - 1 = 1 - 1 = 0$$

$$\Rightarrow f(1) = g(1)$$

$$f(2) = (2)^2 - (2) = 4 - 2 = 2$$

$$\text{And } g(2) = 2 \left| (2) - \frac{1}{2} \right| - 1 = 2 \left(\frac{3}{2} \right) - 1 = 3 - 1 = 2$$

$$\Rightarrow f(2) = g(2)$$

$$\therefore f(a) = g(a) \text{ for all } a \in A$$

Hence, the functions f and g are equal.

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Question 16:

Let $A = \{1, 2, 3\}$. Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive is

- (A) 1 (B) 2 (C) 3 (D) 4

Answer 16:

The given set is $A = \{1, 2, 3\}$.

The smallest relation containing $(1, 2)$ and $(1, 3)$ which is reflexive and symmetric, but not transitive is given by:

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (3, 1)\}$$

This is because relation R is reflexive as $(1, 1), (2, 2), (3, 3) \in R$.

Relation R is symmetric since $(1, 2), (2, 1) \in R$ and $(1, 3), (3, 1) \in R$.

But relation R is not transitive as $(3, 1), (1, 2) \in R$, but $(3, 2) \notin R$.

Now, if we add any two pairs $(3, 2)$ and $(2, 3)$ (or both) to relation R , then relation R will become transitive.

Hence, the total number of desired relations is one.

The correct answer is A.

Question 17:

Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing $(1, 2)$ is

- (A) 1 (B) 2 (C) 3 (D) 4

Answer 17:

It is given that $A = \{1, 2, 3\}$.

The smallest equivalence relation containing $(1, 2)$ is given by,

$$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

Now, we are left with only four pairs i.e., $(2, 3), (3, 2), (1, 3)$, and $(3, 1)$.

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If we add any one pair [say (2, 3)] to R_1 , then for symmetry we must add (3, 2).

Also, for transitivity we are required to add (1, 3) and (3, 1).

Hence, the only equivalence relation (bigger than R_1) is the universal relation.

This shows that the total number of equivalence relations containing (1, 2) is two.

The correct answer is B.

Question 18:

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be the Signum Function defined as $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

and $g: \mathbf{R} \rightarrow \mathbf{R}$ be the Greatest Integer Function given by $g(x) = [x]$, where $[x]$ is greatest integer less than or equal to x . Then does $f \circ g$ and $g \circ f$ coincide in $(0, 1]$?

Answer 18:

It is given that,

$f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

Also, $g: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $g(x) = [x]$, where $[x]$ is the greatest integer less than or equal to x .

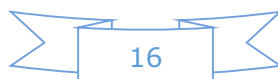
Now, let $x \in (0, 1]$.

Then, we have

$[x] = 1$ if $x = 1$ and $[x] = 0$ if $0 < x < 1$.

$$\begin{aligned} f \circ g(x) &= f(g(x)) = f([x]) = \begin{cases} f(1), & \text{if } x = 1 \\ f(0), & \text{if } x \in (0, 1) \end{cases} \\ &= \begin{cases} 1, & \text{if } x = 1 \\ 0, & \text{if } x \in (0, 1) \end{cases} \end{aligned}$$

$$\begin{aligned} g \circ f(x) &= g(f(x)) = g(1) \quad [\text{as } x > 0] \\ &= [1] = 1 \end{aligned}$$



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Thus, when $x \in (0, 1)$, we have $fog(x) = 0$ and $gof(x) = 1$.

Hence, fog and gof do not coincide in $(0, 1]$.

Question 19:

Number of binary operations on the set $\{a, b\}$ are

(A) 10

(B) 16

(C) 20

(D) 8

Answer 19:

A binary operation $*$ on $\{a, b\}$ is a function from $\{a, b\} \times \{a, b\} \rightarrow \{a, b\}$

i.e., $*$ is a function from $\{(a, a), (a, b), (b, a), (b, b)\} \rightarrow \{a, b\}$.

Hence, the total number of binary operations on the set $\{a, b\}$ is 2^4 i.e., 16.

The correct answer is B.

