

Mathematics

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(Chapter 10)(Vector Algebra)

XII

Exercise 10.3

Question 1:

Find the angle between two vectors \vec{a} and \vec{b} with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

Answer 1:

It is given that,

$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2 \text{ and } \vec{a} \cdot \vec{b} = \sqrt{6}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

Now, we know that

$$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, the angle between the given vectors \vec{a} and \vec{b} is $\frac{\pi}{4}$

Question 2:

Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

Answer 2:

The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.

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$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9+4+1} = \sqrt{14}$$

$$\begin{aligned}\text{Now, } \vec{a} \cdot \vec{b} &= (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k}) \\ &= 1 \cdot 3 + (-2)(-2) + 3 \cdot 1 \\ &= 3 + 4 + 3 \\ &= 10\end{aligned}$$

Also, we know that $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$

$$\therefore 10 = \sqrt{14} \sqrt{14} \cos \theta$$

$$\Rightarrow \cos \theta = \frac{10}{14}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{5}{7} \right)$$

Question 3:

Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

Answer 3:

Let $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$

Now, projection of vector \vec{a} on \vec{b} is given by,

$$\frac{1}{|\vec{b}|} (\vec{a} \cdot \vec{b}) = \frac{1}{\sqrt{1+1}} \{1 \cdot 1 + (-1)(1)\} = \frac{1}{\sqrt{2}} (1-1) = 0$$

Hence, the projection of vector \vec{a} on \vec{b} is 0.

Question 4:

Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

Answer 4:

Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$.

Now, projection of vector \vec{a} on \vec{b} is given by,

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$$\frac{1}{|\vec{b}|}(\vec{a} \cdot \vec{b}) = \frac{1}{\sqrt{7^2 + (-1)^2 + 8^2}} \{1(7) + 3(-1) + 7(8)\} = \frac{7-3+56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$$

Question 5:

Show that each of the given three vectors is a unit vector:

$$\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}), \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$$

Also, show that they are mutually perpendicular to each other.

Answer 5:

$$\text{Let } \vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k},$$

$$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k},$$

$$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}.$$

$$|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = 1$$

Thus, each of the given three vectors is a unit vector.

$$\vec{a} \cdot \vec{b} = \frac{2}{7} \times \frac{3}{7} + \frac{3}{7} \times \left(-\frac{6}{7}\right) + \frac{6}{7} \times \frac{2}{7} = \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = 0$$

$$\vec{b} \cdot \vec{c} = \frac{3}{7} \times \frac{6}{7} + \left(-\frac{6}{7}\right) \times \frac{2}{7} + \frac{2}{7} \times \left(-\frac{3}{7}\right) = \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = 0$$

$$\vec{c} \cdot \vec{a} = \frac{6}{7} \times \frac{2}{7} + \frac{2}{7} \times \frac{3}{7} + \left(-\frac{3}{7}\right) \times \frac{6}{7} = \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = 0$$

Hence, the given three vectors are mutually perpendicular to each other.

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Question 6:

Find

$$|\vec{a}| \text{ and } |\vec{b}|, \text{ if } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8 \text{ and } |\vec{a}| = 8|\vec{b}|.$$

Answer 6:

$$(\vec{a} \cdot \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$$

$$\Rightarrow \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 8$$

$$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8 \quad [|\vec{a}| = 8|\vec{b}|]$$

$$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$$

$$\Rightarrow 63|\vec{b}|^2 = 8$$

$$\Rightarrow |\vec{b}|^2 = \frac{8}{63}$$

$$\Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} \quad [\text{Magnitude of a vector is non-negative}]$$

$$\Rightarrow |\vec{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}$$

$$|\vec{a}| = 8|\vec{b}| = \frac{8 \times 2\sqrt{2}}{3\sqrt{7}} = \frac{16\sqrt{2}}{3\sqrt{7}}$$

Question 7:

Evaluate the product. $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$

Answer 7:

$$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$$

$$= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b}$$

$$= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b}$$

$$= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$$

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Question 8:

Find the magnitude of two vectors \vec{a} and \vec{b} , having the same magnitude and such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

Answer 8:

Let θ be the angle between the vectors \vec{a} and \vec{b} .

It is given that $|\vec{a}| = |\vec{b}|$, $\vec{a} \cdot \vec{b} = \frac{1}{2}$, and $\theta = 60^\circ$ (1)

We know that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$$\therefore \frac{1}{2} = |\vec{a}||\vec{a}|\cos 60^\circ \quad [\text{Using (1)}]$$

$$\Rightarrow \frac{1}{2} = |\vec{a}|^2 \times \frac{1}{2}$$

$$\Rightarrow |\vec{a}|^2 = 1$$

$$\Rightarrow |\vec{a}| = |\vec{b}| = 1$$

Question 9:

Find

$|\vec{x}|$, if for a unit vector \vec{a} , $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$.

Answer 9:

$$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$$

$$\Rightarrow \vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$$

$$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$$

$$\Rightarrow |\vec{x}|^2 - 1 = 12 \quad [|\vec{a}| = 1 \text{ as } \vec{a} \text{ is a unit vector}]$$

$$\Rightarrow |\vec{x}|^2 = 13$$

$$\therefore |\vec{x}| = \sqrt{13}$$

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Question 10:

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to \vec{c} , then find the value of λ .

Answer 10:

The given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$.

Now,

$$\vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k}) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$$

If $(\vec{a} + \lambda\vec{b})$ is perpendicular to \vec{c} , then

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0.$$

$$\Rightarrow [(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$$

$$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow -\lambda + 8 = 0$$

$$\Rightarrow \lambda = 8$$

Hence, the required value of λ is 8.

Question 11:

Show that:

$$|\vec{a}|\vec{b} + |\vec{b}|\vec{a} \text{ is perpendicular to } |\vec{a}|\vec{b} - |\vec{b}|\vec{a},$$

for any two nonzero vectors \vec{a} and \vec{b} .

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Answer 11:

$$\begin{aligned} & (|\vec{a}|\vec{b} + |\vec{b}|\vec{a}) \cdot (|\vec{a}|\vec{b} - |\vec{b}|\vec{a}) \\ &= |\vec{a}|^2 \vec{b} \cdot \vec{b} - |\vec{a}||\vec{b}|\vec{b} \cdot \vec{a} + |\vec{b}||\vec{a}|\vec{a} \cdot \vec{b} - |\vec{b}|^2 \vec{a} \cdot \vec{a} \\ &= |\vec{a}|^2 |\vec{b}|^2 - |\vec{b}|^2 |\vec{a}|^2 \\ &= 0 \end{aligned}$$

Hence, $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ and $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ are perpendicular to each other.

Question 12:

If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector \vec{b} ?

Answer 12:

It is given that $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$.

Now,

$$\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$$

$\therefore \vec{a}$ is a zero vector.

Hence, vector \vec{b} satisfying $\vec{a} \cdot \vec{b} = 0$ can be any vector.

Question 13:

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

Answer 13:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$\Rightarrow 0 = 1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = \frac{-3}{2}$$

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Question 14:

If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$. But the converse need not be true. Justify your answer with an example.

Answer 14:

Consider $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$.

Then,

$$\vec{a} \cdot \vec{b} = 2 \cdot 3 + 4 \cdot 3 + 3(-6) = 6 + 12 - 18 = 0$$

We now observe that:

$$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Question 15:

If the vertices A, B, C of a triangle ABC are (1, 2, 3), (-1, 0, 0), (0, 1, 2), respectively, then find $\angle ABC$. [$\angle ABC$ is the angle between the vectors \overrightarrow{BA} and \overrightarrow{BC}]

Answer 15:

The vertices of ΔABC are given as A (1, 2, 3), B (-1, 0, 0), and C (0, 1, 2).

Also, it is given that $\angle ABC$ is the angle between the vectors \overrightarrow{BA} and \overrightarrow{BC} .

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$$\overline{BA} = \{1 - (-1)\}\hat{i} + (2 - 0)\hat{j} + (3 - 0)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{BC} = \{0 - (-1)\}\hat{i} + (1 - 0)\hat{j} + (2 - 0)\hat{k} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \overline{BA} \cdot \overline{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2 \times 1 + 2 \times 1 + 3 \times 2 = 2 + 2 + 6 = 10$$

$$|\overline{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$$

$$|\overline{BC}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$$

Now, it is known that:

$$\overline{BA} \cdot \overline{BC} = |\overline{BA}| |\overline{BC}| \cos(\angle ABC)$$

$$\therefore 10 = \sqrt{17} \times \sqrt{6} \cos(\angle ABC)$$

$$\Rightarrow \cos(\angle ABC) = \frac{10}{\sqrt{17} \times \sqrt{6}}$$

$$\Rightarrow \angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

Question 16:

Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, -1) are collinear.

Answer 16:

The given points are A (1, 2, 7), B (2, 6, 3), and C (3, 10, -1).

$$\therefore \overline{AB} = (2 - 1)\hat{i} + (6 - 2)\hat{j} + (3 - 7)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overline{BC} = (3 - 2)\hat{i} + (10 - 6)\hat{j} + (-1 - 3)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overline{AC} = (3 - 1)\hat{i} + (10 - 2)\hat{j} + (-1 - 7)\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k}$$

$$|\overline{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$|\overline{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$|\overline{AC}| = \sqrt{2^2 + 8^2 + 8^2} = \sqrt{4 + 64 + 64} = \sqrt{132} = 2\sqrt{33}$$

$$\therefore |\overline{AC}| = |\overline{AB}| + |\overline{BC}|$$

Hence, the given points A, B, and C are collinear.

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Question 17:

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

Answer 17:

Let vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ be position vectors of points A, B, and C respectively.

$$\text{i.e., } \overline{OA} = 2\hat{i} - \hat{j} + \hat{k}, \overline{OB} = \hat{i} - 3\hat{j} - 5\hat{k} \text{ and } \overline{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

Now, vectors \overline{AB} , \overline{BC} , and \overline{AC} represent the sides of ΔABC .

$$\text{i.e., } \overline{OA} = 2\hat{i} - \hat{j} + \hat{k}, \overline{OB} = \hat{i} - 3\hat{j} - 5\hat{k}, \text{ and } \overline{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$$

$$\therefore \overline{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overline{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\overline{AC} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$|\overline{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$$

$$|\overline{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$|\overline{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1+9+25} = \sqrt{35}$$

$$\therefore |\overline{BC}|^2 + |\overline{AC}|^2 = 6 + 35 = 41 = |\overline{AB}|^2$$

Hence, ΔABC is a right-angled triangle.

Question 18:

If \vec{a} is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then $\lambda \vec{a}$ is unit vector if

- (A) $\lambda = 1$ (B) $\lambda = -1$ (C) $a = |\lambda|$ (D) $a = \frac{1}{|\lambda|}$

Answer 18:

Vector $\lambda \vec{a}$ is a unit vector if $|\lambda \vec{a}| = 1$.

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Now,

$$|\lambda \vec{a}| = 1$$

$$\Rightarrow |\lambda| |\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} \quad [\lambda \neq 0]$$

$$\Rightarrow a = \frac{1}{|\lambda|} \quad [|\vec{a}| = a]$$

Hence, vector $\lambda \vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$

The correct answer is D.