Mathematics

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(Chapter 11)(Three Dimensional Geometry)

XII

Exercise 11.1

Question 1:

If a line makes angles 90°, 135°, 45° with x, y and z-axes respectively, find its direction cosines.

Answer

Let direction cosines of the line be l, m, and n.

$$l = \cos 90° = 0$$

$$m = \cos 135° = -\frac{1}{\sqrt{2}}$$

$$n = \cos 45° = \frac{1}{\sqrt{2}}$$

Therefore, the direction cosines of the line are $0, -\frac{1}{\sqrt{2}}$, and $\frac{1}{\sqrt{2}}$.

Question 2:

Find the direction cosines of a line which makes equal angles with the coordinate axes.

Answer

Let the direction cosines of the line make an angle α with each of the coordinate axes. ∴
l = cos α, m = cos α, n = cos α

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow 3\cos^2\alpha = 1$$

$$\Rightarrow \cos^2\alpha = \frac{1}{3}$$

$$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Thus, the direction cosines of the line, which is equally inclined to the coordinate axes, are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$, and $\pm\frac{1}{\sqrt{3}}$.

Question 3:

If a line has the direction ratios −18, 12, −4, then what are its direction cosines?

Answer

If a line has direction ratios of −18, 12, and −4, then its direction cosines are

$$\frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$$

i.e., $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$

$\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

Thus, the direction cosines are $-\frac{9}{11}, \frac{6}{11}$, and $\frac{-2}{11}$.

Question 4:

Show that the points (2, 3, 4), (−1, −2, 1), (5, 8, 7) are collinear.

Answer

The given points are A (2, 3, 4), B (− 1, − 2, 1), and C (5, 8, 7).

It is known that the direction ratios of line joining the points, (x_1, y_1, z_1) and (x_2, y_2, z_2), are given by, $x_2 - x_1$, $y_2 - y_1$, and $z_2 - z_1$.

The direction ratios of AB are (−1 − 2), (−2 − 3), and (1 − 4) i.e., −3, −5, and −3.

The direction ratios of BC are (5 − (− 1)), (8 − (− 2)), and (7 − 1) i.e., 6, 10, and 6.

It can be seen that the direction ratios of BC are −2 times that of AB i.e., they are proportional.

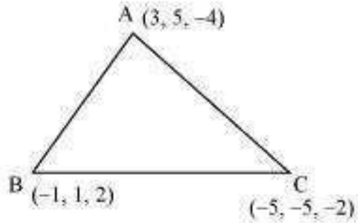
Therefore, AB is parallel to BC. Since point B is common to both AB and BC, points A, B, and C are collinear.

Question 5:

Find the direction cosines of the sides of the triangle whose vertices are (3, 5, − 4), (− 1, 1, 2) and (− 5, − 5, − 2)

Answer

The vertices of ΔABC are A (3, 5, −4), B (−1, 1, 2), and C (−5, −5, −2).



The direction ratios of side AB are (−1 − 3), (1 − 5), and (2 − (−4)) i.e., −4, −4, and 6.

$$\text{Then,}\sqrt{(-4)^2+(-4)^2+(6)^2}=\sqrt{16+16+36}$$
$$=\sqrt{68}$$
$$=2\sqrt{17}$$

Therefore, the direction cosines of AB are

$$\frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}},\frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}},\frac{6}{\sqrt{(-4)^2+(-4)^2+(6)^2}}$$

$$\frac{-4}{2\sqrt{17}},-\frac{4}{2\sqrt{17}},\frac{6}{2\sqrt{17}}$$

$$\frac{-2}{\sqrt{17}},\frac{-2}{\sqrt{17}},\frac{3}{\sqrt{17}}$$

The direction ratios of BC are (−5 − (−1)), (−5 − 1), and (−2 − 2) i.e., −4, −6, and −4. Therefore, the direction cosines of BC are

$$\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}},\frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}},\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

$$\frac{-4}{2\sqrt{17}},\frac{-6}{2\sqrt{17}},\frac{-4}{2\sqrt{17}}$$

The direction ratios of CA are (−5 − 3), (−5 − 5), and (−2 − (−4)) i.e., −8, −10, and 2.

Therefore, the direction cosines of AC are

$$\frac{-8}{\sqrt{(-8)^2+(10)^2+(2)^2}},\frac{-5}{\sqrt{(-8)^2+(10)^2+(2)^2}},\frac{2}{\sqrt{(-8)^2+(10)^2+(2)^2}}$$

$$\frac{-8}{2\sqrt{42}},\frac{-10}{2\sqrt{42}},\frac{2}{2\sqrt{42}}$$