Exercise 13.4

Question 1:

State which of the following are **not** the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P (X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P (X)	0.1	0.5	0.2	− 0.1	0.3

(iii)

Y	−1	0	1
P (Y)	0.6	0.1	0.2

(iv)

Z	3	2	1	0	−1
P (Z)	0.3	0.2	0.4	0.1	0.05

Answer 1:

It is known that the sum of all the probabilities in a probability distribution is one.

(i) Sum of the probabilities $= 0.4 + 0.4 + 0.2 = 1$

Therefore, the given table is a probability distribution of random variables.

(ii) It can be seen that for $X = 3$, $P(X) = -0.1$

It is known that probability of any observation is not negative. Therefore, the given table is not a probability distribution of random variables.

(iii) Sum of the probabilities $= 0.6 + 0.1 + 0.2 = 0.9 \neq 1$

Therefore, the given table is not a probability distribution of random variables.

(iv) Sum of the probabilities $= 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$

Therefore, the given table is not a probability distribution of random variables.

Question 2:

An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X? Is X a random variable?

Answer 2:

The two balls selected can be represented as BB, BR, RB, RR, where B represents a black ball and R represents a red ball.

X represents the number of black balls.

$\therefore X(BB) = 2$

$X(BR) = 1$

$X(RB) = 1$

$X(RR) = 0$

Therefore, the possible values of X are 0, 1, and 2.

Yes, X is a random variable.

Question 3:

Let X represents the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Answer 3:

A coin is tossed six times and X represents the difference between the number of heads and the number of tails.

$\therefore X(6\text{ H}, 0\text{T}) = |6-0| = 6$

$X(5\text{ H}, 1\text{ T}) = |5-1| = 4$

$X(4\text{ H}, 2\text{ T}) = |4-2| = 2$

$X(3\text{ H}, 3\text{ T}) = |3-3| = 0$

$X(2\text{ H}, 4\text{ T}) = |2-4| = 2$

$X(1\text{ H}, 5\text{ T}) = |1-5| = 4$

$X(0\text{H}, 6\text{ T}) = |0-6| = 6$

Thus, the possible values of X are 6, 4, 2, and 0.

Question 4:

Find the probability distribution of

(i) number of heads in two tosses of a coin

(ii) number of tails in the simultaneous tosses of three coins

(iii) number of heads in four tosses of a coin

Answer 4:

(i) When one coin is tossed twice, the sample space is

{HH, HT, TH, TT}

Let X represent the number of heads.

$\therefore$ X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0

Therefore, X can take the value of 0, 1, or 2.

It is known that,

$$P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$$

$$P(X = 0) = P(TT) = \frac{1}{4}$$

$$P(X = 1) = P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(X = 2) = P(HH) = \frac{1}{4}$$

Thus, the required probability distribution is as follows.

X	0	1	2
P (X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) When three coins are tossed simultaneously, the sample space is $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Let X represent the number of tails.

It can be seen that X can take the value of 0, 1, 2, or 3.

$$P(X=0) = P(HHH) = \frac{1}{8}$$

$$P(X=1) = P(HHT) + P(HTH) + P(THH) = \frac{1}{8}+\frac{1}{8}+\frac{1}{8} = \frac{3}{8}$$

$$P(X=2) = P(HTT) + P(THT) + P(TTH) = \frac{1}{8}+\frac{1}{8}+\frac{1}{8} = \frac{3}{8}$$

$$P(X=3) = P(TTT) = \frac{1}{8}$$

Thus, the probability distribution is as follows.

X	0	1	2	3
P (X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) When a coin is tossed four times, the sample space is

$$S = \begin{Bmatrix} HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, HTTH, HTTT, \\ THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT \end{Bmatrix}$$

Let X be the random variable, which represents the number of heads.

It can be seen that X can take the value of 0, 1, 2, 3, or 4.

$$P(X=0) = P(TTTT) = \frac{1}{16}$$

$$P(X=1) = P(TTTH) + P(TTHT) + P(THTT) + P(HTTT)$$

$$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16} = \frac{4}{16} = \frac{1}{4}$$

$$P(X=2) = P(HHTT) + P(THHT) + P(TTHH) + P(HTTH) + P(HTHT)$$

$$+ P(THTH)$$

$$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16} = \frac{6}{16} = \frac{3}{8}$$

$$P(X=3) = P(HHHT) + P(HHTH) + P(HTHH)\ P(THHH)$$

$$= \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16} = \frac{4}{16} = \frac{1}{4}$$

$$P(X=4) = P(HHHH) = \frac{1}{16}$$

Thus, the probability distribution is as follows.

X	0	1	2	3	4
P (X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Question 5:

Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(i) number greater than 4

(ii) six appears on at least one die

Answer 5:

When a die is tossed two times, we obtain $(6 \times 6) = 36$ number of observations.

Let X be the random variable, which represents the number of successes.

i. Here, success refers to the number greater than 4.

P (X = 0) = P (number less than or equal to 4 on both the tosses) $= \frac{4}{6} \times \frac{4}{6} = \frac{4}{9}$

P (X = 1) = P (number less than or equal to 4 on first toss and greater than 4 on second toss) + P (number greater than 4 on first toss and less than or equal to 4 on second toss)

$= \frac{4}{6} \times \frac{2}{6} + \frac{4}{6} \times \frac{2}{6} = \frac{4}{9}$

P (X = 2) = P (number greater than 4 on both the tosses)

$= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$

Thus, the probability distribution is as follows.

X	1	1	2
P (X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Here, success means six appears on at least one die.

P (Y = 0) = P (six does not appear on any of the dice) $= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

P (Y = 1) = P (six appears on at least one of the dice) $= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{5}{36} + \frac{5}{36} = \frac{10}{36}$

Thus, the required probability distribution is as follows.

Y	0	1
P (Y)	$\frac{25}{36}$	$\frac{10}{36}$

Question 6:

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Answer 6:

It is given that out of 30 bulbs, 6 are defective.

$\Rightarrow$ Number of non-defective bulbs = 30 − 6 = 24

4 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

$\therefore$ P (X = 0) = P (4 non-defective and 0 defective) $= {}^4C_0 \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} \frac{4}{5} = \frac{256}{625}$

P (X = 1) = P (3 non-defective and 1 defective) $= {}^4C_1 \cdot \left(\frac{1}{5}\right) \cdot \left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P (X = 2) = P (2 non-defective and 2 defective) $= {}^4C_2 \cdot \left(\frac{1}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P (X = 3) = P (1 non-defective and 3 defective) $= {}^4C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right) = \frac{16}{625}$

P (X = 4) = P (0 non-defective and 4 defective) $= {}^4C_4 \cdot \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^0 = \frac{1}{625}$

Therefore, the required probability distribution is as follows.

X	0	1	2	3	4
P (X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Question 7:

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Answer 7:

Let the probability of getting a tail in the biased coin be x.

$\therefore$ P (T) = x

$\Rightarrow$ P (H) = $3x$

For a biased coin, P (T) + P (H) = 1

$\Rightarrow x + 3x = 1$

$\Rightarrow 4x = 1$

$\Rightarrow x = \frac{1}{4}$

$\therefore P(T) = \frac{1}{4}$ and $P(H) = \frac{3}{4}$

When the coin is tossed twice, the sample space is {HH, TT, HT, TH}.

Let X be the random variable representing the number of tails.

$\therefore$ P (X = 0) = P (no tail) = P (H) × P (H) $= \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

P (X = 1) = P (one tail) = P (HT) + P (TH)

$$= \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4}$$
$$= \frac{3}{16} + \frac{3}{16}$$
$$= \frac{3}{8}$$

P (X = 2) = P (two tails) = P (TT) $= \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P (X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

Question 8:

A random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7
P (X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine

(i) k

(ii) P (X < 3)

(iii) P (X > 6)

(iv) P (0 < X < 3)

Answer 8:

(i) It is known that the sum of probabilities of a probability distribution of random variables is one.

$$\therefore 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + \left(7k^2 + k\right) = 1$$
$$\Rightarrow 10k^2 + 9k - 1 = 0$$
$$\Rightarrow (10k - 1)(k + 1) = 0$$
$$\Rightarrow k = -1, \frac{1}{10}$$

$k = -1$ is not possible as the probability of an event is never negative.

$$\therefore\ k = \frac{1}{10}$$

(ii) P (X < 3) = P (X = 0) + P (X = 1) + P (X = 2)

$$= 0 + k + 2k$$
$$= 3k$$
$$= 3 \times \frac{1}{10}$$
$$= \frac{3}{10}$$

(iii) P (X > 6) = P (X = 7)

$$= 7k^2 + k$$
$$= 7\times\left(\frac{1}{10}\right)^2 + \frac{1}{10}$$
$$= \frac{7}{100} + \frac{1}{10}$$
$$= \frac{17}{100}$$

(iv) P (0 < X < 3) = P (X = 1) + P (X = 2)

$$= k + 2k$$
$$= 3k$$
$$= 3\times\frac{1}{10}$$
$$= \frac{3}{10}$$

Question 9:

The random variable X has probability distribution P(X) of the following form, where k is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the value of k.

(b) Find $P(X < 2)$, $P(X \geq 2)$, $P(X \geq 2)$.

Answer 9:

(a) It is known that the sum of probabilities of a probability distribution of random variables is one.

$$\therefore k + 2k + 3k + 0 = 1$$

$$\Rightarrow 6k = 1$$

$$\Rightarrow k = \frac{1}{6}$$

(b) $P(X < 2) = P(X = 0) + P(X = 1)$

$$= k + 2k$$
$$= 3k$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

$$P(X \le 2) = P(X=0)+P(X=1)+P(X=2)$$
$$= k+2k+3k$$
$$= 6k$$
$$= \frac{6}{6}$$
$$= 1$$

$$P(X \ge 2) = P(X=2)+P(X>2)$$
$$= 3k+0$$
$$= 3k$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

Question 10:

Find the mean number of heads in three tosses of a fair coin.

Answer 10:

Let X denote the success of getting heads.

Therefore, the sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that X can take the value of 0, 1, 2, or 3.

$$\therefore P(X=0) = P(TTT)$$
$$= P(T)\cdot P(T)\cdot P(T)$$
$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{1}{8}$$

$$\therefore P(X=1) = P(HHT) + P(HTH) + P(THH)$$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{3}{8}$$

$$\therefore P(X=2) = P(HHT) + P(HTH) + P(THH)$$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$
$$= \frac{3}{8}$$

$$\therefore P(X=3) = P(HHH)$$
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{8}$$

Therefore, the required probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean of X E(X), $\mu = \sum X_i P(X_i)$

$$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$
$$= \frac{3}{8} + \frac{3}{4} + \frac{3}{8}$$
$$= \frac{3}{2}$$
$$= 1.5$$

Question 11:

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Answer 11:

Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Therefore, X can take the value of 0, 1, or 2.

$\therefore$ P (X = 0) = P (not getting six on any of the dice) = $\frac{25}{36}$

P (X = 1) = P (six on first die and no six on second die) + P (no six on first die and six on second die)

$$= 2\left(\frac{1}{6} \times \frac{5}{6}\right) = \frac{10}{36}$$

P (X = 2) = P (six on both the dice) = $\frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of X = E(X) = $\sum X_i P(X_i)$

$$= 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36}$$
$$= \frac{1}{3}$$

Question 12:

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find E(X).

Answer 12:

The two positive integers can be selected from the first six positive integers without replacement in $6 \times 5 = 30$ ways

X represents the larger of the two numbers obtained. Therefore, X can take the value of 2, 3, 4, 5, or 6.

For $X = 2$, the possible observations are (1, 2) and (2, 1).

$$\therefore P(X=2)=\frac{2}{30}=\frac{1}{15}$$

For $X = 3$, the possible observations are (1, 3), (2, 3), (3, 1), and (3, 2).

$$\therefore P(X=3)=\frac{4}{30}=\frac{2}{15}$$

For $X = 4$, the possible observations are (1, 4), (2, 4), (3, 4), (4, 3), (4, 2), and (4, 1).

$$\therefore P(X=4)=\frac{6}{30}=\frac{1}{5}$$

For $X = 5$, the possible observations are (1, 5), (2, 5), (3, 5), (4, 5), (5, 4), (5, 3), (5, 2), and (5, 1).

$$\therefore P(X=5)=\frac{8}{30}=\frac{4}{15}$$

For $X = 6$, the possible observations are (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 4), (6, 3), (6, 2), and (6, 1).

$$\therefore P(X=6)=\frac{10}{30}=\frac{1}{3}$$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

$$\begin{aligned}\text{Then, } E(X) &= \Sigma X_i P(X_i)\\ &= 2\cdot\frac{1}{15}+3\cdot\frac{2}{15}+4\cdot\frac{1}{5}+5\cdot\frac{4}{15}+6\cdot\frac{1}{3}\\ &= \frac{2}{15}+\frac{2}{5}+\frac{4}{5}+\frac{4}{3}+2\\ &= \frac{70}{15}\\ &= \frac{14}{3}\end{aligned}$$

Question 13:

Let X denotes the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

Answer 13:

When two fair dice are rolled, $6 \times 6 = 36$ observations are obtained.

$P(X=2)=P(1,1)=\frac{1}{36}$

$P(X=3)=P(1,2)+P(2,1)=\frac{2}{36}=\frac{1}{18}$

$P(X=4)=P(1,3)+P(2,2)+P(3,1)=\frac{3}{36}=\frac{1}{12}$

$P(X=5)=P(1,4)+P(2,3)+P(3,2)+P(4,1)=\frac{4}{36}=\frac{1}{9}$

$P(X=6)=P(1,5)+P(2,4)+P(3,3)+P(4,2)+P(5,1)=\frac{5}{36}$

$P(X=7)=P(1,6)+P(2,5)+P(3,4)+P(4,3)+P(5,2)+P(6,1)=\frac{6}{36}=\frac{1}{6}$

$P(X=8)=P(2,6)+P(3,5)+P(4,4)+P(5,3)+P(6,2)=\frac{5}{36}$

$P(X=9)=P(3,6)+P(4,5)+P(5,4)+P(6,3)=\frac{4}{36}=\frac{1}{9}$

$P(X=10)=P(4,6)+P(5,5)+P(6,4)=\frac{3}{36}=\frac{1}{12}$

$P(X=11)=P(5,6)+P(6,5)=\frac{2}{36}=\frac{1}{18}$

$P(X=12)=P(6,6)=\frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Then, $E(X)=\sum X_i \cdot P(X_i)$

$$=2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9}+6\times\frac{5}{36}+7\times\frac{1}{6}$$
$$+8\times\frac{5}{36}+9\times\frac{1}{9}+10\times\frac{1}{12}+11\times\frac{1}{18}+12\times\frac{1}{36}$$
$$=\frac{1}{18}+\frac{1}{6}+\frac{1}{3}+\frac{5}{9}+\frac{5}{6}+\frac{7}{6}+\frac{10}{9}+1+\frac{5}{6}+\frac{11}{18}+\frac{1}{3}$$
$$=7$$

$$E(X^2)=\sum X_i^2 \cdot P(X_i)$$

$$=4\times\frac{1}{36}+9\times\frac{1}{18}+16\times\frac{1}{12}+25\times\frac{1}{9}+36\times\frac{5}{36}+49\times\frac{1}{6}$$

$$+64\times\frac{5}{36}+81\times\frac{1}{9}+100\times\frac{1}{12}+121\times\frac{1}{18}+144\times\frac{1}{36}$$

$$=\frac{1}{9}+\frac{1}{2}+\frac{4}{3}+\frac{25}{9}+5+\frac{49}{6}+\frac{80}{9}+9+\frac{25}{3}+\frac{121}{18}+4$$

$$=\frac{987}{18}=\frac{329}{6}=54.833$$

$$\text{Then, } \mathrm{Var}(X)=E(X^2)-[E(X)]^2$$

$$=54.833-(7)^2$$

$$=54.833-49$$

$$=5.833$$

$$\therefore \text{Standard deviation}=\sqrt{\mathrm{Var}(X)}$$

$$=\sqrt{5.833}$$

$$=2.415$$

Question 14:

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

Answer 14:

There are 15 students in the class. Each student has the same chance to be chosen. Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$P(X=14)=\frac{2}{15}$, $P(X=15)=\frac{1}{15}$, $P(X=16)=\frac{2}{15}$, $P(X=16)=\frac{3}{15}$,

$P(X=18)=\frac{1}{15}$, $P(X=19)=\frac{2}{15}$, $P(X=20)=\frac{3}{15}$, $P(X=21)=\frac{1}{15}$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Then, mean of X = E(X)

$$= \sum X_i P(X_i)$$
$$= 14\times\frac{2}{15}+15\times\frac{1}{15}+16\times\frac{2}{15}+17\times\frac{3}{15}+18\times\frac{1}{15}+19\times\frac{2}{15}+20\times\frac{3}{15}+21\times\frac{1}{15}$$
$$= \frac{1}{15}(28+15+32+51+18+38+60+21)$$
$$= \frac{263}{15}$$
$$= 17.53$$

$$E(X^2) = \sum X_i^2 P(X_i)$$

$$= (14)^2\cdot\frac{2}{15}+(15)^2\cdot\frac{1}{15}+(16)^2\cdot\frac{2}{15}+(17)^2\cdot\frac{3}{15}+$$
$$(18)^2\cdot\frac{1}{15}+(19)^2\cdot\frac{2}{15}+(20)^2\cdot\frac{3}{15}+(21)^2\cdot\frac{1}{15}$$
$$= \frac{1}{15}\cdot(392+225+512+867+324+722+1200+441)$$
$$= \frac{4683}{15}$$
$$= 312.2$$

$$\therefore \text{Variance}(X) = E(X^2)-[E(X)]^2$$
$$= 312.2-\left(\frac{263}{15}\right)^2$$
$$= 312.2-307.4177$$
$$= 4.7823$$
$$\approx 4.78$$

$$\text{Standard derivation} = \sqrt{\text{Variance}(X)}$$
$$= \sqrt{4.78}$$
$$= 2.186 \approx 2.19$$

Question 15:

In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var(X).

Answer 15:

It is given that $P(X = 0) = 30\% = \frac{30}{100} = 0.3$

$P(X = 1) = 70\% = \frac{70}{100} = 0.7$

Therefore, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

Then, $E(X) = \sum X_i P(X_i)$

$$= 0 \times 0.3 + 1 \times 0.7$$
$$= 0.7$$

$$E(X^2) = \sum X_i^2 P(X_i)$$
$$= 0^2 \times 0.3 + (1)^2 \times 0.7$$
$$= 0.7$$

It is known that, $\text{Var}(X) = E(X^2) - [E(X)]^2$

$= 0.7 - (0.7)^2$

$= 0.7 - 0.49$

$= 0.21$

Question 16:

The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 (B) 2 (C) 5 (D) $\frac{8}{3}$

Answer 16:

Let X be the random variable representing a number on the die.

The total number of observations is six.

$$\therefore P(X = 1) = \frac{3}{6} = \frac{1}{2}$$
$$P(X = 2) = \frac{2}{6} = \frac{1}{3}$$
$$P(X = 5) = \frac{1}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$$\text{Mean} = E(X) = \sum p_i x_i$$

$$= \frac{1}{2}\times 1 + \frac{1}{3}\times 2 + \frac{1}{6}\cdot 5$$
$$= \frac{1}{2} + \frac{2}{3} + \frac{5}{6}$$
$$= \frac{3+4+5}{6}$$
$$= \frac{12}{6}$$
$$= 2$$

The correct answer is B.

Question 17:

Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

(A) $\frac{37}{221}$ (B) $\frac{5}{13}$ (C) $\frac{1}{13}$ (D) $\frac{2}{13}$

Answer 17::

Let X denote the number of aces obtained. Therefore, X can take any of the values of 0, 1, or 2.

In a deck of 52 cards, 4 cards are aces. Therefore, there are 48 non-ace cards.

$\therefore$ P (X = 0) = P (0 ace and 2 non-ace cards) $= \frac{{}^{4}C_0 \times {}^{48}C_2}{{}^{52}C_2} = \frac{1128}{1326}$

P (X = 1) = P (1 ace and 1 non-ace cards) $= \frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{192}{1326}$

P (X = 2) = P (2 ace and 0 non- ace cards) $= \frac{{}^{4}C_2 \times {}^{48}C_0}{{}^{52}C_2} = \frac{6}{1326}$

Thus, the probability distribution is as follows.

X	0	1	2
P(X)	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

Then, $E(X) = \sum p_i x_i$

$= 0 \times \frac{1128}{1326} + 1 \times \frac{192}{1326} + 2 \times \frac{6}{1326}$

$= \frac{204}{1326}$

$= \frac{2}{13}$

Therefore, the correct answer is D.