Exercise 7.1

Question 1:

Find an anti-derivative (or integral) of the following functions by the method of inspection. sin 2x

Answer 1:

The anti-derivative of sin 2x is a function of x whose derivative is sin 2x.It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$

$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti-derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

Question 2:

Find an anti-derivative (or integral) of the following functions by the method of inspection. cos 3x

Answer 2:

The anti-derivative of cos 3x is a function of x whose derivative is cos 3x.

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$

$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$

$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, the anti-derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x$

Question 3:

Find an anti-derivative (or integral) of the following functions by the method of inspection. e^{2x}

Answer 3:

The anti-derivative of e^{2x} is the function of x whose derivative is e^{2x}.

It is known that,

$$\frac{d}{dx}\left(e^{2x}\right) = 2e^{2x}$$

$$\Rightarrow e^{2x} = \frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$$

$$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti-derivative of e^{2x} is $\frac{1}{2}e^{2x}$

Question 4:

Find an anti-derivative (or integral) of the following functions by the method of inspection. $(ax + b)^2$

Answer 4:

The anti-derivative of $(ax + b)^2$ is the function of x whose derivative is $(ax + b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$$

$$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$

$$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$

Question 5:

Find an anti-derivative (or integral) of the following functions by the method of inspection. $\sin 2x - 4e^{3x}$

Answer 5:

The anti-derivative of $\sin 2x - 4e^{3x}$ is the function of x whose derivative is $\sin 2x - 4e^{3x}$

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

Therefore, the anti derivative of $\left(\sin 2x - 4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$.

Question 6: $\int\left(4e^{3x}+1\right)dx$

Answer 6:

$$\int\left(4e^{3x}+1\right)dx$$

$$= 4\int e^{3x}dx + \int 1dx$$

$$= 4\left(\frac{e^{3x}}{3}\right) + x + C$$

$$= \frac{4}{3}e^{3x} + x + C$$

Question 7: $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Answer 7:

$$\int x^2\left(1-\frac{1}{x^2}\right)dx$$
$$=\int\left(x^2-1\right)dx$$
$$=\int x^2dx-\int 1dx$$
$$=\frac{x^3}{3}-x+C$$

Question 8: $\int\left(ax^2+bx+c\right)dx$

Answer 8:

$$\int\left(ax^2+bx+c\right)dx$$
$$=a\int x^2dx+b\int xdx+c\int 1.dx$$
$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$
$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

Question 9: $\int\left(2x^2+e^x\right)dx$

Answer 9:

$$\int\left(2x^2+e^x\right)dx$$
$$=2\int x^2dx+\int e^xdx$$
$$=2\left(\frac{x^3}{3}\right)+e^x+C$$
$$=\frac{2}{3}x^3+e^x+C$$

Question 10: $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$

Answer 10:

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

$$=\int\left(x+\frac{1}{x}-2\right)dx$$

$$=\int x dx+\int\frac{1}{x}dx-2\int 1.dx$$

$$=\frac{x^2}{2}+\log|x|-2x+C$$

Question 11: $\int\frac{x^3+5x^2-4}{x^2}dx$

Answer 11:

$$\int\frac{x^3+5x^2-4}{x^2}dx$$

$$=\int\left(x+5-4x^{-2}\right)dx$$

$$=\int x dx+5\int 1.dx-4\int x^{-2}dx$$

$$=\frac{x^2}{2}+5x-4\left(\frac{x^{-1}}{-1}\right)+C$$

$$=\frac{x^2}{2}+5x+\frac{4}{x}+C$$

Question 12: $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

Answer 12:

$$\int \frac{x^3+3x+4}{\sqrt{x}}dx$$

$$= \int\left(x^{\frac{5}{2}}+3x^{\frac{1}{2}}+4x^{-\frac{1}{2}}\right)dx$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+\frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}}+C$$

$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8x^{\frac{1}{2}}+C$$

$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8\sqrt{x}+C$$

Question 13: $\int \frac{x^3-x^2+x-1}{x-1}dx$

Answer 13:

$$\int \frac{x^3-x^2+x-1}{x-1}dx$$

On dividing, we obtain

$$= \int\left(x^2+1\right)dx$$

$$= \int x^2dx+\int 1dx$$

$$= \frac{x^3}{3}+x+C$$

Question 14: $\int (1-x)\sqrt{x}\,dx$

Answer 14:

$$\int (1-x)\sqrt{x}\,dx$$
$$= \int\left(\sqrt{x} - x^{\frac{3}{2}}\right)dx$$
$$= \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx$$
$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$
$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$$

Question 15: $\int \sqrt{x}\left(3x^2+2x+3\right)dx$

Answer 15:

$$\int \sqrt{x}\left(3x^2+2x+3\right)dx$$
$$= \int\left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right)dx$$
$$= 3\int x^{\frac{5}{2}}dx + 2\int x^{\frac{3}{2}}dx + 3\int x^{\frac{1}{2}}dx$$
$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + 3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + C$$
$$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

Question 16: $\int\left(2x-3\cos x+e^{x}\right)dx$

Answer 16:

$$\int\left(2x-3\cos x+e^{x}\right)dx$$
$$=2\int xdx-3\int\cos xdx+\int e^{x}dx$$
$$=\frac{2x^{2}}{2}-3(\sin x)+e^{x}+C$$
$$=x^{2}-3\sin x+e^{x}+C$$

Question 17: $\int\left(2x^{2}-3\sin x+5\sqrt{x}\right)dx$

Answer 17:

$$\int\left(2x^{2}-3\sin x+5\sqrt{x}\right)dx$$
$$=2\int x^{2}dx-3\int\sin xdx+5\int x^{\frac{1}{2}}dx$$
$$=\frac{2x^{3}}{3}-3(-\cos x)+5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$
$$=\frac{2}{3}x^{3}+3\cos x+\frac{10}{3}x^{\frac{3}{2}}+C$$

Question 18: $\int\sec x(\sec x+\tan x)dx$

Answer 18:

$$\int\sec x(\sec x+\tan x)dx$$
$$=\int\left(\sec^{2}x+\sec x\tan x\right)dx$$
$$=\int\sec^{2}xdx+\int\sec x\tan xdx$$
$$=\tan x+\sec x+C$$

Question 19: $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

Answer 19:

$$\int \frac{\sec^2 x}{\cos ec^2 x} dx$$
$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$$
$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$
$$= \int \tan^2 x dx$$
$$= \int (\sec^2 x - 1) dx$$
$$= \int \sec^2 x dx - \int 1 dx$$
$$= \tan x - x + C$$

Question 20: $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

Answer 20:

$$\int \frac{2 - 3\sin x}{\cos^2 x} dx$$
$$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right) dx$$
$$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$$
$$= 2\tan x - 3\sec x + C$$

Question 21:

The anti-derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}+C$ (B) $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^{2}+C$

(C) $\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$ (D) $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+C$

Answer 21:

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$$

$$=\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$$

$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$

$$=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$$

Hence, the correct Answer is C.

Question 22: If $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$ such that f(2) = 0, then f(x) is

(A) $x^4+\frac{1}{x^3}-\frac{129}{8}$ (B) $x^3+\frac{1}{x^4}+\frac{129}{8}$

(C) $x^4+\frac{1}{x^3}+\frac{129}{8}$ (D) $x^3+\frac{1}{x^4}-\frac{129}{8}$

Answer 22:

It is given that, $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$

Anti-derivative of $4x^3-\frac{3}{x^4}=f(x)$

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4}\,dx$$

$$f(x) = 4\int x^3 dx - 3\int (x^{-4})dx$$

$$f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also,

$$f(2) = 0$$

$$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = \frac{-129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

Hence, the correct Answer is A.