

Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Exercise 7.1

Question 1:

Find an anti-derivative (or integral) of the following functions by the method of inspection. $\sin 2x$

Answer 1:

The anti-derivative of $\sin 2x$ is a function of x whose derivative is $\sin 2x$. It is known that,

$$\frac{d}{dx}(\cos 2x) = -2 \sin 2x$$

$$\Rightarrow \sin 2x = -\frac{1}{2} \frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2} \cos 2x\right)$$

Therefore, the anti-derivative of $\sin 2x$ is $-\frac{1}{2} \cos 2x$

Question 2:

Find an anti-derivative (or integral) of the following functions by the method of inspection. $\cos 3x$

Answer 2:

The anti-derivative of $\cos 3x$ is a function of x whose derivative is $\cos 3x$.

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3 \cos 3x$$

$$\Rightarrow \cos 3x = \frac{1}{3} \frac{d}{dx}(\sin 3x)$$

$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3} \sin 3x\right)$$

Therefore, the anti-derivative of $\cos 3x$ is $\frac{1}{3} \sin 3x$



Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 3:

Find an anti-derivative (or integral) of the following functions by the method of inspection. e^{2x}

Answer 3:

The anti-derivative of e^{2x} is the function of x whose derivative is e^{2x} .

It is known that,

$$\frac{d}{dx}(e^{2x}) = 2e^{2x}$$

$$\Rightarrow e^{2x} = \frac{1}{2} \frac{d}{dx}(e^{2x})$$

$$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti-derivative of e^{2x} is $\frac{1}{2}e^{2x}$

Question 4:

Find an anti-derivative (or integral) of the following functions by the method of inspection. $(ax + b)^2$

Answer 4:

The anti-derivative of $(ax + b)^2$ is the function of x whose derivative is $(ax + b)^2$.

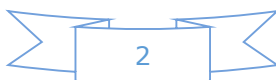
It is known that,

$$\frac{d}{dx}(ax + b)^3 = 3a(ax + b)^2$$

$$\Rightarrow (ax + b)^2 = \frac{1}{3a} \frac{d}{dx}(ax + b)^3$$

$$\therefore (ax + b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax + b)^3\right)$$

Therefore, the anti derivative of $(ax + b)^2$ is $\frac{1}{3a}(ax + b)^3$



Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 5:

Find an anti-derivative (or integral) of the following functions by the method of inspection. $\sin 2x - 4e^{3x}$

Answer 5:

The anti-derivative of $\sin 2x - 4e^{3x}$ is the function of x whose derivative is $\sin 2x - 4e^{3x}$

It is known that,

$$\frac{d}{dx} \left(-\frac{1}{2} \cos 2x - \frac{4}{3} e^{3x} \right) = \sin 2x - 4e^{3x}$$

Therefore, the anti derivative of $(\sin 2x - 4e^{3x})$ is $\left(-\frac{1}{2} \cos 2x - \frac{4}{3} e^{3x} \right)$.

Question 6: $\int (4e^{3x} + 1) dx$

Answer 6:

$$\begin{aligned} & \int (4e^{3x} + 1) dx \\ &= 4 \int e^{3x} dx + \int 1 dx \\ &= 4 \left(\frac{e^{3x}}{3} \right) + x + C \\ &= \frac{4}{3} e^{3x} + x + C \end{aligned}$$

Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 7: $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$

Answer 7:

$$\begin{aligned} & \int x^2 \left(1 - \frac{1}{x^2}\right) dx \\ &= \int (x^2 - 1) dx \\ &= \int x^2 dx - \int 1 dx \\ &= \frac{x^3}{3} - x + C \end{aligned}$$

Question 8: $\int (ax^2 + bx + c) dx$

Answer 8:

$$\begin{aligned} & \int (ax^2 + bx + c) dx \\ &= a \int x^2 dx + b \int x dx + c \int 1 dx \\ &= a \left(\frac{x^3}{3}\right) + b \left(\frac{x^2}{2}\right) + cx + C \\ &= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C \end{aligned}$$

Question 9: $\int (2x^2 + e^x) dx$

Answer 9:

$$\begin{aligned} & \int (2x^2 + e^x) dx \\ &= 2 \int x^2 dx + \int e^x dx \\ &= 2 \left(\frac{x^3}{3}\right) + e^x + C \\ &= \frac{2}{3} x^3 + e^x + C \end{aligned}$$



Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 10: $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx$

Answer 10:

$$\begin{aligned} & \int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx \\ &= \int \left(x + \frac{1}{x} - 2 \right) dx \\ &= \int x dx + \int \frac{1}{x} dx - 2 \int 1 dx \\ &= \frac{x^2}{2} + \log|x| - 2x + C \end{aligned}$$

Question 11: $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

Answer 11:

$$\begin{aligned} & \int \frac{x^3 + 5x^2 - 4}{x^2} dx \\ &= \int (x + 5 - 4x^{-2}) dx \\ &= \int x dx + 5 \int 1 dx - 4 \int x^{-2} dx \\ &= \frac{x^2}{2} + 5x - 4 \left(\frac{x^{-1}}{-1} \right) + C \\ &= \frac{x^2}{2} + 5x + \frac{4}{x} + C \end{aligned}$$

Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 12: $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

Answer 12:

$$\begin{aligned} & \int \frac{x^3 + 3x + 4}{\sqrt{x}} dx \\ &= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} \right) dx \\ &= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3 \left(x^{\frac{3}{2}} \right)}{\frac{3}{2}} + \frac{4 \left(x^{\frac{1}{2}} \right)}{\frac{1}{2}} + C \\ &= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C \\ &= \frac{2}{7} x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C \end{aligned}$$

Question 13: $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

Answer 13:

$$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$$

On dividing, we obtain

$$\begin{aligned} &= \int (x^2 + 1) dx \\ &= \int x^2 dx + \int 1 dx \\ &= \frac{x^3}{3} + x + C \end{aligned}$$



Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 14: $\int (1-x)\sqrt{x} dx$

Answer 14:

$$\begin{aligned} & \int (1-x)\sqrt{x} dx \\ &= \int \left(\sqrt{x} - x^{\frac{3}{2}} \right) dx \\ &= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx \\ &= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C \\ &= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C \end{aligned}$$

Question 15: $\int \sqrt{x} (3x^2 + 2x + 3) dx$

Answer 15:

$$\begin{aligned} & \int \sqrt{x} (3x^2 + 2x + 3) dx \\ &= \int \left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) dx \\ &= 3 \int x^{\frac{5}{2}} dx + 2 \int x^{\frac{3}{2}} dx + 3 \int x^{\frac{1}{2}} dx \\ &= 3 \left(\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} \right) + 2 \left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right) + 3 \left(\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right) + C \\ &= \frac{6}{7} x^{\frac{7}{2}} + \frac{4}{5} x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C \end{aligned}$$



Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 16: $\int (2x - 3 \cos x + e^x) dx$

Answer 16:

$$\begin{aligned} & \int (2x - 3 \cos x + e^x) dx \\ &= 2 \int x dx - 3 \int \cos x dx + \int e^x dx \\ &= \frac{2x^2}{2} - 3(\sin x) + e^x + C \\ &= x^2 - 3 \sin x + e^x + C \end{aligned}$$

Question 17: $\int (2x^2 - 3 \sin x + 5\sqrt{x}) dx$

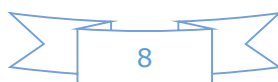
Answer 17:

$$\begin{aligned} & \int (2x^2 - 3 \sin x + 5\sqrt{x}) dx \\ &= 2 \int x^2 dx - 3 \int \sin x dx + 5 \int x^{\frac{1}{2}} dx \\ &= \frac{2x^3}{3} - 3(-\cos x) + 5 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) + C \\ &= \frac{2}{3} x^3 + 3 \cos x + \frac{10}{3} x^{\frac{3}{2}} + C \end{aligned}$$

Question 18: $\int \sec x (\sec x + \tan x) dx$

Answer 18:

$$\begin{aligned} & \int \sec x (\sec x + \tan x) dx \\ &= \int (\sec^2 x + \sec x \tan x) dx \\ &= \int \sec^2 x dx + \int \sec x \tan x dx \\ &= \tan x + \sec x + C \end{aligned}$$



Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 19:

$$\int \frac{\sec^2 x}{\cos \sec^2 x} dx$$

Answer 19:

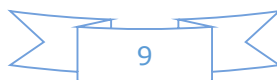
$$\begin{aligned} & \int \frac{\sec^2 x}{\cos \sec^2 x} dx \\ &= \int \frac{1}{\frac{\cos^2 x}{1}} dx \\ &= \int \frac{\sin^2 x}{\cos^2 x} dx \\ &= \int \tan^2 x dx \\ &= \int (\sec^2 x - 1) dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \tan x - x + C \end{aligned}$$

Question 20:

$$\int \frac{2 - 3 \sin x}{\cos^2 x} dx$$

Answer 20:

$$\begin{aligned} & \int \frac{2 - 3 \sin x}{\cos^2 x} dx \\ &= \int \left(\frac{2}{\cos^2 x} - \frac{3 \sin x}{\cos^2 x} \right) dx \\ &= \int 2 \sec^2 x dx - 3 \int \tan x \sec x dx \\ &= 2 \tan x - 3 \sec x + C \end{aligned}$$



Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

Question 21:

The anti-derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

Answer 21:

$$\begin{aligned} & \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx \\ &= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx \\ &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C \end{aligned}$$

Hence, the correct Answer is C.

Question 22: If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$, then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Answer 22:

It is given that, $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$

Anti-derivative of $4x^3 - \frac{3}{x^4} = f(x)$

Mathematics

(www.tiwariacademy.com: Focus on free education)

(Chapter – 7) (Integrals)

(Class – XII)

$$\therefore f(x) = \int 4x^3 - \frac{3}{x^4} dx$$

$$f(x) = 4 \int x^3 dx - 3 \int (x^{-4}) dx$$

$$f(x) = 4 \left(\frac{x^4}{4} \right) - 3 \left(\frac{x^{-3}}{-3} \right) + C$$

$$f(x) = x^4 + \frac{1}{x^3} + C$$

Also,

$$f(2) = 0$$

$$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$$

$$\Rightarrow 16 + \frac{1}{8} + C = 0$$

$$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$$

$$\Rightarrow C = \frac{-129}{8}$$

$$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$$

Hence, the correct Answer is A.