Exercise 7.11

Question 1:

$\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$

Answer 1:

$I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx \qquad ...(1)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad ...(2)$

Adding (1) and (2), we obtain

$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$

$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$

$\Rightarrow 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

Question 2:

$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Answer 2:

$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \qquad ...(1)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos} + \sqrt{\sin x}} dx \qquad ...(2)$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 3:

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$$

Answer 3:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 4:

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$$

Answer 4:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \quad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 5:

$$\int_{-5}^{5} |x+2| dx$$

Answer 5:

$$\text{Let } I = \int_{-5}^{5} |x+2| dx$$

It can be seen that $(x + 2) \leq 0$ on $[-5, -2]$ and $(x + 2) \geq 0$ on $[-2, 5]$.

$$\therefore I=\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{5}(x+2)dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$I=-\left[\frac{x^2}{2}+2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{5}$$

$$=-\left[\frac{(-2)^2}{2}+2(-2)-\frac{(-5)^2}{2}-2(-5)\right]+\left[\frac{(5)^2}{2}+2(5)-\frac{(-2)^2}{2}-2(-2)\right]$$

$$=-\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$$

$$=-2+4+\frac{25}{2}-10+\frac{25}{2}+10-2+4$$

$$=29$$

Question 6:

$$\int_2^8|x-5|dx$$

Answer 6:

$$\text{Let } I=\int_2^8|x-5|dx$$

It can be seen that $(x-5)\le 0$ on $[2, 5]$ and $(x-5)\ge 0$ on $[5, 8]$.

$$I=\int_2^5-(x-5)dx+\int_2^8(x-5)dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$=-\left[\frac{x^2}{2}-5x\right]_2^5+\left[\frac{x^2}{2}-5x\right]_5^8$$

$$=-\left[\frac{25}{2}-25-2+10\right]+\left[32-40-\frac{25}{2}+25\right]$$

$$=9$$

Question 7:

$$\int_0^1 x(1-x)^n\,dx$$

Answer 7:

$$\text{Let } I=\int_0^1 x(1-x)^n\,dx$$

$$\therefore I=\int_0^1 (1-x)\left(1-(1-x)\right)^n\,dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$=\int_0^1 (1-x)(x)^n\,dx$$

$$=\int_0^1 \left(x^n-x^{n+1}\right)dx$$

$$=\left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1$$

$$=\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$$

$$=\frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$=\frac{1}{(n+1)(n+2)}$$

Question 8:

$$\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$$

Answer 8:

$$\text{Let } I=\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx \qquad \ldots(1)$$

$$\therefore I=\int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log 2\,dx-\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log 2dx - I \qquad [\text{From } (1)]$$

$$\Rightarrow 2I=\left[x\log 2\right]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I=\frac{\pi}{4}\log 2$$

$$\Rightarrow I=\frac{\pi}{8}\log 2$$

Question 9:

$$\int_0^2 x\sqrt{2-x}\,dx$$

Answer 9:

Let $I = \int_0^2 x\sqrt{2-x}\,dx$

$I = \int_0^2 (2-x)\sqrt{x}\,dx$ $\quad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\}dx$

$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$

$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$

$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$

$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$

$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$

$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$

$= \frac{16\sqrt{2}}{15}$

Question 10:

$$\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx$$

Answer 10:

$$\text{Let } I=\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log(2\sin x\cos x)\}\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log\sin x-\log\cos x-\log 2\}\,dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\sin x-\log\cos x-\log 2\}\,dx \qquad ...(1)$$

$$\text{It is known that, }\left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\cos x-\log\sin x-\log 2\}\,dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}(-\log 2-\log 2)\,dx$$

$$\Rightarrow 2I=-2\log 2\int_0^{\frac{\pi}{2}}1\,dx$$

$$\Rightarrow I=-\log 2\left[\frac{\pi}{2}\right]$$

$$\Rightarrow I=\frac{\pi}{2}(-\log 2)$$

$$\Rightarrow I=\frac{\pi}{2}\left[\log\frac{1}{2}\right]$$

$$\Rightarrow I=\frac{\pi}{2}\log\frac{1}{2}$$

Question 11:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\,dx$$

Answer 11:

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\,dx$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if f(x) is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

$$I = 2\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx$$

$$= \int_0^{\frac{\pi}{2}} (1-\cos 2x)\,dx$$

$$= \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

Question 12:

$$\int_0^{\pi} \frac{x\,dx}{1+\sin x}$$

Answer 12:

$$\text{Let } I = \int_0^{\pi} \frac{x\,dx}{1+\sin x} \quad \ldots(1)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)}\,dx \quad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin x}\,dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^\pi \frac{\pi}{1+\sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{(1-\sin x)}{(1+\sin x)(1-\sin x)} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \frac{1-\sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi \int_0^\pi \left\{\sec^2 x - \tan x \sec x\right\} dx$$

$$\Rightarrow 2I = \pi\left[\tan x - \sec x\right]_0^\pi$$

$$\Rightarrow 2I = \pi[2]$$

$$\Rightarrow I = \pi$$

Question 13:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$$

Answer 13:

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx \qquad \ldots(1)$$

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if f(x) is an odd function, then $\int_{-a}^{a} f(x) dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx = 0$$

Question 14:

$$\int_0^{2\pi} \cos^5 x dx$$

Answer 14:

$$\text{Let } I = \int_0^{2\pi} \cos^5 x dx \qquad \ldots(1)$$

$$\cos^5(2\pi - x) = \cos^5 x$$

It is known that,

$$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x)$$
$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$\therefore I = 2\int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \qquad \left[\cos^5(\pi - x) = -\cos^5 x\right]$$

Question 15:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x\cos x} dx$$

Answer 15:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x\cos x} dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right) - \cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\sin x\cos x} dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1+\sin x\cos x} dx$$

$$\Rightarrow I = 0$$

Question 16:

$$\int_0^{\pi} \log(1+\cos x) dx$$

Answer 16:

$$\text{Let } I = \int_0^{\pi} \log(1+\cos x) dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^{\pi} \log(1+\cos(\pi - x)) dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \log(1-\cos x) dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi}\{\log(1+\cos x)+\log(1-\cos x)\}\,dx$$

$$\Rightarrow 2I = \int_0^{\pi}\log(1-\cos^2 x)\,dx$$

$$\Rightarrow 2I = \int_0^{\pi}\log\sin^2 x\,dx$$

$$\Rightarrow 2I = 2\int_0^{\pi}\log\sin x\,dx$$

$$\Rightarrow I = \int_0^{\pi}\log\sin x\,dx \qquad ...(3)$$

sin (π − x) = sin x

$$\therefore I = 2\int_0^{\frac{\pi}{2}}\log\sin x\,dx \qquad ...(4)$$

$$\Rightarrow I = 2\int_0^{\frac{\pi}{2}}\log\sin\left(\frac{\pi}{2}-x\right)dx = 2\int_0^{\frac{\pi}{2}}\log\cos x\,dx \qquad ...(5)$$

Adding (4) and (5), we obtain

$$2I = 2\int_0^{\frac{\pi}{2}}(\log\sin x+\log\cos x)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}}(\log\sin x+\log\cos x+\log 2-\log 2)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}}(\log 2\sin x\cos x-\log 2)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}}\log\sin 2x\,dx - \int_0^{\frac{\pi}{2}}\log 2\,dx$$

Let 2x = t ⇒ 2dx = dt

When x = 0, t = 0

$$\therefore I = \frac{1}{2}\int_0^{\pi}\log\sin t\,dt - \frac{\pi}{2}\log 2$$

$$\Rightarrow I = \frac{I}{2} - \frac{\pi}{2}\log 2$$

$$\Rightarrow \frac{I}{2} = -\frac{\pi}{2}\log 2$$

$$\Rightarrow I = -\pi\log 2$$

Question 17:

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

Answer 17:

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \qquad ...(1)$$

It is known that, $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\Rightarrow 2I = \int_0^a 1\, dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Question 18:

$$\int_0^4 |x-1|\,dx$$

Answer 18:

$$I = \int_0^4 |x-1|\,dx$$

It can be seen that, (x − 1) ≤ 0 when 0 ≤ x ≤ 1 and (x − 1) ≥ 0 when 1 ≤ x ≤ 4

$$I = \int_0^1 |x-1|\,dx + \int_1^4 |x-1|\,dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1)dx + \int_1^4 (x-1)dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Question 19:

Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx,$ if f and g are defined as $f(x) = f(a-x)$ and $g(x)+g(a-x)=4$

Answer 19:

$$\text{Let } I = \int_0^a f(x)g(x)dx \qquad ...(1)$$

$$\Rightarrow I = \int_0^a f(a-x)g(a-x)dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^a f(x)g(a-x)dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \{f(x)g(x)+f(x)g(a-x)\}dx$$

$$\Rightarrow 2I = \int_0^a f(x)\{g(x)+g(a-x)\}dx$$

$$\Rightarrow 2I = \int_0^a f(x)\times 4\,dx \qquad [g(x)+g(a-x)=4]$$

$$\Rightarrow I = 2\int_0^a f(x)dx$$

Question 20:

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$ is

A. 0

B. 2

C. π

D. 1

Answer 20:

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1\cdot dx$$

It is known that if f(x) is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

if f(x) is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$

and

$$I = 0+0+0+2\int_{0}^{\frac{\pi}{2}} 1\cdot dx$$

$$= 2[x]_{0}^{\frac{\pi}{2}}$$

$$= \frac{2\pi}{2}$$

$\pi =$

Hence, the correct Answer is C.

Question 21:

The value of $\int_{0}^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is

A. 2

B. $\frac{3}{4}$

C. 0

D. -2

Answer 21:

$$\text{Let } I = \int_{0}^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \qquad ...(1)$$

$$\Rightarrow I = \int_{0}^{\frac{\pi}{2}} \log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx \qquad \left(\int_{0}^{a} f(x)\,dx = \int_{0}^{a} f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_{0}^{\frac{\pi}{2}} \log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left\{ \log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right) \right\} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x}\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1 dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 0 dx$$

$$\Rightarrow I = 0$$

Hence, the correct Answer is C.