

Mathematics

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(Chapter – 7) (Integrals)

(Class – XII)

Exercise 7.6

Integrate the functions in Exercises 1 to 22.

Question 1: $x \sin x$

Answer 1:

$$\text{Let } I = \int x \sin x \, dx$$

Taking x as first function and $\sin x$ as second function and integrating by parts, we obtain

$$\begin{aligned} I &= x \int \sin x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin x \, dx \right\} dx \\ &= x(-\cos x) - \int 1 \cdot (-\cos x) \, dx \\ &= -x \cos x + \sin x + C \end{aligned}$$

Question 2: $x \sin 3x$

Answer 2:

$$\text{Let } I = \int x \sin 3x \, dx$$

Taking x as first function and $\sin 3x$ as second function and integrating by parts, we obtain

$$\begin{aligned} I &= x \int \sin 3x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin 3x \, dx \right\} \\ &= x \left(\frac{-\cos 3x}{3} \right) - \int 1 \cdot \left(\frac{-\cos 3x}{3} \right) dx \\ &= \frac{-x \cos 3x}{3} + \frac{1}{3} \int \cos 3x \, dx \\ &= \frac{-x \cos 3x}{3} + \frac{1}{9} \sin 3x + C \end{aligned}$$



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Question 3: $x^2 e^x$

Answer 3:

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$\begin{aligned} I &= x^2 \int e^x dx - \int \left\{ \left(\frac{d}{dx} x^2 \right) \int e^x dx \right\} dx \\ &= x^2 e^x - \int 2x \cdot e^x dx \\ &= x^2 e^x - 2 \int x \cdot e^x dx \end{aligned}$$

Again integrating by parts, we obtain

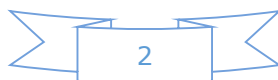
$$\begin{aligned} &= x^2 e^x - 2 \left[x \cdot \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \cdot \int e^x dx \right\} dx \right] \\ &= x^2 e^x - 2 \left[x e^x - \int e^x dx \right] \\ &= x^2 e^x - 2 \left[x e^x - e^x \right] \\ &= x^2 e^x - 2x e^x + 2e^x + C \\ &= e^x (x^2 - 2x + 2) + C \end{aligned}$$

Question 4: $x \log x$

Answer 4:

Let $I = \int x \log x dx$

Taking $\log x$ as first function and x as second function and integrating by parts, we obtain



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$$\begin{aligned} I &= \log x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x \, dx \right\} dx \\ &= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx \\ &= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C \end{aligned}$$

Question 5: $x \log 2x$

Answer 5:

$$\text{Let } I = \int x \log 2x \, dx$$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \log 2x \int x \, dx - \int \left\{ \left(\frac{d}{dx} 2 \log x \right) \int x \, dx \right\} dx \\ &= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx \\ &= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C \end{aligned}$$

Question 6: $x^2 \log x$

Answer 6:

$$\text{Let } I = \int x^2 \log x \, dx$$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain



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$$\begin{aligned} I &= \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx \\ &= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx \\ &= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx \\ &= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C \end{aligned}$$

Question 7: $x \sin^{-1} x$

Answer 7:

Let $I = \int x \sin^{-1} x dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \sin^{-1} x \int x dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x dx \right\} dx \\ &= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2} dx - \int \frac{1}{\sqrt{1-x^2}} dx \right\} \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C \\ &= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C \\ &= \frac{1}{4} (2x^2 - 1) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C \end{aligned}$$



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Question 8: $x \tan^{-1} x$

Answer 8:

Let $I = \int x \tan^{-1} x \, dx$

Taking $\tan^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \tan^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int x \, dx \right\} dx \\ &= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\ &= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right) dx \\ &= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx \\ &= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} (x - \tan^{-1} x) + C \\ &= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C \end{aligned}$$

Question 9: $x \cos^{-1}x$

Answer 9:

Let $I = \int x \cos^{-1} x \, dx$

Taking $\cos^{-1} x$ as first function and x as second function and integrating by parts, we obtain



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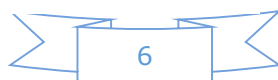
$$\begin{aligned} I &= \cos^{-1} x \int x dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1} x \right) \int x dx \right\} dx \\ &= \cos^{-1} x \frac{x^2}{2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx \\ &= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx \\ &= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \left\{ \sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}} \right) \right\} dx \\ &= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \left(\frac{-1}{\sqrt{1-x^2}} \right) dx \\ &= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2} I_1 - \frac{1}{2} \cos^{-1} x \quad \dots(1) \end{aligned}$$

where, $I_1 = \int \sqrt{1-x^2} dx$

$$\begin{aligned} \Rightarrow I_1 &= x\sqrt{1-x^2} - \int \frac{d}{dx} \sqrt{1-x^2} \int x dx \\ \Rightarrow I_1 &= x\sqrt{1-x^2} - \int \frac{-2x}{2\sqrt{1-x^2}} \cdot x dx \\ \Rightarrow I_1 &= x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}} dx \\ \Rightarrow I_1 &= x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx \\ \Rightarrow I_1 &= x\sqrt{1-x^2} - \left\{ \int \sqrt{1-x^2} dx + \int \frac{-dx}{\sqrt{1-x^2}} \right\} \\ \Rightarrow I_1 &= x\sqrt{1-x^2} - \{ I_1 + \cos^{-1} x \} \\ \Rightarrow 2I_1 &= x\sqrt{1-x^2} - \cos^{-1} x \\ \therefore I_1 &= \frac{x}{2} \sqrt{1-x^2} - \frac{1}{2} \cos^{-1} x \end{aligned}$$

Substituting in (1), we obtain

$$\begin{aligned} I &= \frac{x \cos^{-1} x}{2} - \frac{1}{2} \left(\frac{x}{2} \sqrt{1-x^2} - \frac{1}{2} \cos^{-1} x \right) - \frac{1}{2} \cos^{-1} x \\ &= \frac{(2x^2-1)}{4} \cos^{-1} x - \frac{x}{4} \sqrt{1-x^2} + C \end{aligned}$$



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Question 10: $(\sin^{-1}x)^2$

Answer 10:

$$\text{Let } I = \int (\sin^{-1}x)^2 \cdot 1 \, dx$$

Taking $(\sin^{-1}x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned} I &= (\sin^{-1}x) \int 1 \, dx - \int \left\{ \frac{d}{dx} (\sin^{-1}x)^2 \cdot \int 1 \cdot dx \right\} dx \\ &= (\sin^{-1}x)^2 \cdot x - \int \frac{2\sin^{-1}x}{\sqrt{1-x^2}} \cdot x \, dx \\ &= x(\sin^{-1}x)^2 + \int \sin^{-1}x \cdot \left(\frac{-2x}{\sqrt{1-x^2}} \right) dx \\ &= x(\sin^{-1}x)^2 + \left[\sin^{-1}x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1}x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right] \\ &= x(\sin^{-1}x)^2 + \left[\sin^{-1}x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right] \\ &= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2} \sin^{-1}x - \int 2 \, dx \\ &= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2} \sin^{-1}x - 2x + C \end{aligned}$$

Question 11: $\frac{x \cos^{-1}x}{\sqrt{1-x^2}}$

Answer 11:

$$\text{Let } I = \int \frac{x \cos^{-1}x}{\sqrt{1-x^2}} dx$$

$$I = \frac{-1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1}x \, dx$$



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Taking $\cos^{-1}x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$\begin{aligned} I &= \frac{-1}{2} \left[\cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right] \\ &= \frac{-1}{2} \left[\cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right] \\ &= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1} x + \int 2 dx \right] \\ &= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1} x + 2x \right] + C \\ &= - \left[\sqrt{1-x^2} \cos^{-1} x + x \right] + C \end{aligned}$$

Question 12: $x \sec^2 x$

Answer 12:

$$\text{Let } I = \int x \sec^2 x dx$$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$\begin{aligned} I &= x \int \sec^2 x dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sec^2 x dx \right\} dx \\ &= x \tan x - \int 1 \cdot \tan x dx \\ &= x \tan x + \log |\cos x| + C \end{aligned}$$

Question 13: $\tan^{-1}x$

Answer 13:

$$\text{Let } I = \int 1 \cdot \tan^{-1} x dx$$

Taking $\tan^{-1}x$ as first function and 1 as second function and integrating by parts,



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we obtain

$$\begin{aligned} I &= \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int 1 \cdot dx \right\} dx \\ &= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x dx \\ &= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx \\ &= x \tan^{-1} x - \frac{1}{2} \log |1+x^2| + C \\ &= x \tan^{-1} x - \frac{1}{2} \log(1+x^2) + C \end{aligned}$$

Question 14: $x (\log x)^2$

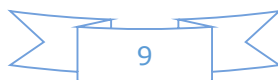
Answer 14:

$$I = \int x (\log x)^2 dx$$

Taking $(\log x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned} I &= (\log x)^2 \int x dx - \int \left\{ \left(\frac{d}{dx} (\log x)^2 \right) \int x dx \right\} dx \\ &= \frac{x^2}{2} (\log x)^2 - \left[\int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx \right] \\ &= \frac{x^2}{2} (\log x)^2 - \int x \log x dx \end{aligned}$$

Again integrating by parts, we obtain



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$$\begin{aligned} I &= \frac{x^2}{2}(\log x)^2 - \left[\log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx \right] \\ &= \frac{x^2}{2}(\log x)^2 - \left[\frac{x^2}{2} - \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right] \\ &= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx \\ &= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + C \end{aligned}$$

Question 15: $(x^2 + 1) \log x$

Answer 15:

$$\text{Let } I = \int (x^2 + 1) \log x dx = \int x^2 \log x dx + \int \log x dx$$

$$\text{Let } I = I_1 + I_2 \dots (1)$$

$$\text{Where, } I_1 = \int x^2 \log x dx \quad \text{and} \quad I_2 = \int \log x dx$$

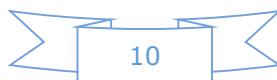
$$I_1 = \int x^2 \log x dx$$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$\begin{aligned} I_1 &= \log x - \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx \\ &= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx \\ &= \frac{x^3}{3} \log x - \frac{1}{3} \left(\int x^2 dx \right) \\ &= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 \quad \dots (2) \end{aligned}$$

$$I_2 = \int \log x dx$$

Taking $\log x$ as first function and 1 as second function and integrating by parts, we obtain



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$$\begin{aligned} I_2 &= \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\} \\ &= \log x \cdot x - \int \frac{1}{x} \cdot x dx \\ &= x \log x - \int 1 dx \\ &= x \log x - x + C_2 \quad \dots (3) \end{aligned}$$

Using equations (2) and (3) in (1), we obtain

$$\begin{aligned} I &= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2 \\ &= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2) \\ &= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C \end{aligned}$$

Question 16: $e^x (\sin x + \cos x)$

Answer 16:

Let $I = \int e^x (\sin x + \cos x) dx$

Let $f(x) = \sin x$

$$f'(x) = \cos x$$

$$I = \int e^x \{f(x) + f'(x)\} dx$$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$



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$$\therefore I = e^x \sin x + C$$

Question 17: $\int \frac{xe^x}{(1+x)^2} dx$

Answer 17:

Let
$$I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$$

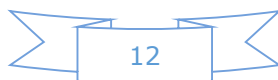
$$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$$
$$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$$

Let $f(x) = \frac{1}{1+x}$ $f'(x) = \frac{-1}{(1+x)^2}$

$$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{ f(x) + f'(x) \} dx$$

It is known that, $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + C$

$$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$



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Question 18: $e^x \left(\frac{1 + \sin x}{1 + \cos x} \right)$

Answer 18:

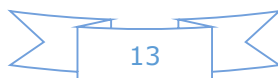
$$\begin{aligned} & e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) \\ &= e^x \left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) \\ &= \frac{e^x \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}{2 \cos^2 \frac{x}{2}} \\ &= \frac{1}{2} e^x \cdot \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2 \\ &= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2 \\ &= \frac{1}{2} e^2 \left(1 + \tan \frac{x}{2} \right)^2 \\ &= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\ &= \frac{1}{2} e^x \left[\sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\ & \frac{e^x (1 + \sin x) dx}{(1 + \cos x)} = e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \quad \dots(1) \end{aligned}$$

Let $\tan \frac{x}{2} = f(x)$ So $f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

From equation (1), we obtain

$$\int \frac{e^x (1 + \sin x)}{(1 + \cos x)} dx = e^x \tan \frac{x}{2} + C$$



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Question 19: $e^x \left(\frac{1}{x} - \frac{1}{x^2} \right)$

Answer 19:

$$\text{Let } I = \int e^x \left[\frac{1}{x} - \frac{1}{x^2} \right] dx$$

Also, let $\frac{1}{x} = f(x)$ $f'(x) = \frac{-1}{x^2}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$\therefore I = \frac{e^x}{x} + C$$

Question 20: $\frac{(x-3)e^x}{(x-1)^3}$

Answer 20:

$$\begin{aligned} \int e^x \left\{ \frac{x-3}{(x-1)^3} \right\} dx &= \int e^x \left\{ \frac{x-1-2}{(x-1)^3} \right\} dx \\ &= \int e^x \left\{ \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right\} dx \end{aligned}$$

Let $f(x) = \frac{1}{(x-1)^2}$ $f'(x) = \frac{-2}{(x-1)^3}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$\therefore \int e^x \left\{ \frac{(x-3)}{(x-1)^2} \right\} dx = \frac{e^x}{(x-1)^2} + C$$

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Question 21: $e^{2x} \sin x$

Answer 21:

Let $I = \int e^{2x} \sin x \, dx$... (1)

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} \, dx - \int \left\{ \left(\frac{d}{dx} \sin x \right) \int e^{2x} \, dx \right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} \, dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} \, dx - \int \left\{ \left(\frac{d}{dx} \cos x \right) \int e^{2x} \, dx \right\} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} \, dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x \, dx \right]$$

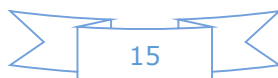
$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \quad \text{[From (1)]}$$

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5} [2 \sin x - \cos x] + C$$



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(Chapter – 7) (Integrals)

(Class – XII)

Question 22: $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Answer 22:

Let $x = \tan \theta$ $dx = \sec^2 \theta d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = \int 2\theta \cdot \sec^2 \theta d\theta = 2 \int \theta \cdot \sec^2 \theta d\theta$$

Integrating by parts, we obtain

$$2 \left[\theta \cdot \int \sec^2 \theta d\theta - \int \left\{ \left(\frac{d}{d\theta} \theta \right) \int \sec^2 \theta d\theta \right\} d\theta \right]$$

$$= 2 \left[\theta \cdot \tan \theta - \int \tan \theta d\theta \right]$$

$$= 2 \left[\theta \tan \theta + \log |\cos \theta| \right] + C$$

$$= 2 \left[x \tan^{-1} x + \log \left| \frac{1}{\sqrt{1+x^2}} \right| \right] + C$$

$$= 2x \tan^{-1} x + 2 \log (1+x^2)^{-\frac{1}{2}} + C$$

$$= 2x \tan^{-1} x + 2 \left[-\frac{1}{2} \log (1+x^2) \right] + C$$

$$= 2x \tan^{-1} x - \log (1+x^2) + C$$

Question 23:

$\int x^2 e^{x^3} dx$ equals

(A) $\frac{1}{3} e^{x^3} + C$

(B) $\frac{1}{3} e^{x^2} + C$

(C) $\frac{1}{2} e^{x^3} + C$

(D) $\frac{1}{3} e^{x^2} + C$

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Answer 23:

Let $I = \int x^2 e^{x^3} dx$

Also, let $x^3 = t$ so $3x^2 dx = dt$

$$\begin{aligned}\Rightarrow I &= \frac{1}{3} \int e^t dt \\ &= \frac{1}{3} (e^t) + C \\ &= \frac{1}{3} e^{x^3} + C\end{aligned}$$

Hence, the correct Answer is A.

Question 24:

$\int e^x \sec x (1 + \tan x) dx$ equals

- (A) $e^x \cos x + C$ (B) $e^x \sec x + C$
(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

Answer 24:

$$\int e^x \sec x (1 + \tan x) dx$$

Let $I = \int e^x \sec x (1 + \tan x) dx = \int e^x (\sec x + \sec x \tan x) dx$

Also, let $\sec x = f(x)$ $\sec x \tan x = f'(x)$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

Hence, the correct Answer is B.