Exercise 7.7

Question 1:

$\sqrt{4-x^2}$

Answer 1:

$$\text{Let } I = \int \sqrt{4-x^2}\,dx = \int \sqrt{(2)^2-(x)^2}\,dx$$

$$\text{It is known that, } \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}\ \frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$

$$\therefore I = \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}+C$$

$$= \frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$$

Question 2:

$\sqrt{1-4x^2}$

Answer 2:

$$\text{Let } I = \int \sqrt{1-4x^2}\,dx = \int \sqrt{(1)^2-(2x)^2}\,dx$$

$$\text{Let } 2x = t \Rightarrow 2\,dx = dt$$

$$\therefore I = \frac{1}{2}\int \sqrt{(1)^2-(t)^2}\,dt$$

$$\text{It is known that, } \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$

$$\Rightarrow I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t\right]+C$$

$$= \frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t+C$$

$$= \frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$$

$$= \frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$$

Question 3:

$\sqrt{x^2+4x+6}$

Answer 3:

$$\text{Let } I = \int \sqrt{x^2+4x+6}\ dx$$
$$= \int \sqrt{x^2+4x+4+2}\ dx$$
$$= \int \sqrt{\left(x^2+4x+4\right)+2}\ dx$$
$$= \int \sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\ dx$$

It is known that, $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+6} + \frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right| + C$$
$$= \frac{(x+2)}{2}\sqrt{x^2+4x+6} + \log\left|(x+2)+\sqrt{x^2+4x+6}\right| + C$$

Question 4:

$\sqrt{x^2+4x+1}$

Answer 4:

$$\text{Let } I = \int \sqrt{x^2+4x+1}\,dx$$
$$= \int \sqrt{\left(x^2+4x+4\right)-3}\,dx$$
$$= \int \sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\ dx$$

It is known that, $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right| + C$$

Question 5:

$\sqrt{1-4x-x^2}$

Answer 5:

$$\text{Let } I = \int\sqrt{1-4x-x^2}\,dx$$
$$= \int\sqrt{1-\left(x^2+4x+4-4\right)}\,dx$$
$$= \int\sqrt{1+4-(x+2)^2}\,dx$$
$$= \int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

Question 6:

$\sqrt{x^2+4x-5}$

Answer 6:

$$\text{Let } I = \int\sqrt{x^2+4x-5}\,dx$$
$$= \int\sqrt{\left(x^2+4x+4\right)-9}\,dx$$
$$= \int\sqrt{(x+2)^2-(3)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C$$

Question 7:

$\sqrt{1+3x-x^2}$

Answer 7:

$$\text{Let } I = \int\sqrt{1+3x-x^2}\,dx$$

$$= \int\sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}\,dx$$

$$= \int\sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}\,dx$$

$$= \int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I = \frac{x-\frac{3}{2}}{2}\sqrt{1+3x-x^2}+\frac{13}{4\times 2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+C$$

$$= \frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$$

Question 8:

$\sqrt{x^2+3x}$

Answer 8:

$$\text{Let } I = \int\sqrt{x^2+3x}\,dx$$

$$= \int\sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\,dx$$

$$= \int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$
$$=\frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

Question 9:

$\sqrt{1+\frac{x^2}{9}}$

Answer 9:

Let $I=\int\sqrt{1+\frac{x^2}{9}}\,dx=\frac{1}{3}\int\sqrt{9+x^2}\,dx=\frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$

It is known that, $\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$
$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

Question 10:

$\int\sqrt{1+x^2}\,dx$ is equal to

A. $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

B. $\frac{2}{3}\left(1+x^2\right)^{\frac{2}{3}}+C$

C. $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+C$

D. $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+C$

Answer 10:

It is known that, $\int\sqrt{a^2+x^2}\,dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$\therefore \int\sqrt{1+x^2}\,dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

Hence, the correct Answer is A.

Question 11:

$\int\sqrt{x^2-8x+7}\,dx$ is equal to

A. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

B. $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

C. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

D. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Answer 11:

$$\begin{aligned}\text{Let } I &= \int\sqrt{x^2-8x+7}\,dx\\ &= \int\sqrt{(x^2-8x+16)-9}\,dx\\ &= \int\sqrt{(x-4)^2-(3)^2}\,dx\end{aligned}$$

It is known that, $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$\therefore I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$

Hence, the correct Answer is D.