Exercise 7.9

Question 1:

$$\int_{-1}^{1}(x+1)dx$$

Answer 1:

Let $I = \int_{-1}^{1}(x+1)dx$

$$\int(x+1)\,dx = \frac{x^2}{2}+x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(-1)$$

$$= \left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)$$

$$= \frac{1}{2}+1-\frac{1}{2}+1$$

$$= 2$$

Question 2:

$$\int_{2}^{3}\frac{1}{x}dx$$

Answer 2:

Let $I = \int_{2}^{3}\frac{1}{x}\,dx$

$$\int\frac{1}{x}dx = \log|x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \log|3| - \log|2| = \log\frac{3}{2}$$

Question 3:

$$\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right)dx$$

Answer 3:

Let $I = \int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right)dx$

$$\int \left(4x^3 - 5x^2 + 6x + 9\right)dx = 4\left(\frac{x^4}{4}\right) - 5\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + 9(x)$$

$$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(2) - F(1)$$

$$I = \left\{2^4 - \frac{5 \cdot (2)^3}{3} + 3(2)^2 + 9(2)\right\} - \left\{(1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1)\right\}$$

$$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$$

$$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$$

$$= 33 - \frac{35}{3}$$

$$= \frac{99 - 35}{3}$$

$$= \frac{64}{3}$$

Question 4:

$$\int_0^{\frac{\pi}{4}} \sin 2x\, dx$$

Answer 4:

Let $I = \int_0^{\frac{\pi}{4}} \sin 2x\, dx$

$$\int \sin 2x\, dx = \left(\frac{-\cos 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right) - \cos 0\right]$$

$$= -\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right]$$

$$= -\frac{1}{2}[0 - 1]$$

$$= \frac{1}{2}$$

Question 5:

$$\int_0^{\frac{\pi}{2}} \cos 2x\, dx$$

Answer 5:

Let $I = \int_0^{\frac{\pi}{2}} \cos 2x\, dx$

$$\int \cos 2x\, dx = \left(\frac{\sin 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{2}\right) - F(0)$$

$$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$$

$$= \frac{1}{2}[\sin \pi - \sin 0]$$

$$= \frac{1}{2}[0 - 0] = 0$$

Question 6:

$$\int_4^5 e^x \, dx$$

Answer 6:

Let $I = \int_4^5 e^x \, dx$

$\int e^x \, dx = e^x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(5) - F(4)$

$= e^5 - e^4$

$= e^4(e-1)$

Question 7:

$$\int_0^{\frac{\pi}{4}} \tan x \, dx$$

Answer 7:

Let $I = \int_0^{\frac{\pi}{4}} \tan x \, dx$

$\int \tan x \, dx = -\log|\cos x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{4}\right) - F(0)$

$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$

$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$

$= -\log(2)^{-\frac{1}{2}}$

$= \frac{1}{2}\log 2$

Question 8:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos ec\, x\, dx$$

Answer 8:

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos ec\, x\, dx$

$\int \cos ec\, x\, dx = \log|\cos ec\, x - \cot x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\dfrac{\pi}{4}\right) - F\left(\dfrac{\pi}{6}\right)$

$= \log\left|\cos ec\dfrac{\pi}{4} - \cot\dfrac{\pi}{4}\right| - \log\left|\cos ec\dfrac{\pi}{6} - \cot\dfrac{\pi}{6}\right|$

$= \log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$

$= \log\left(\dfrac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$

Question 9:

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

Answer 9:

Let $I = \int_0^1 \dfrac{dx}{\sqrt{1-x^2}}$

$\int \dfrac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(0)$

$= \sin^{-1}(1) - \sin^{-1}(0)$

$= \dfrac{\pi}{2} - 0$

$= \dfrac{\pi}{2}$

Question 10:

$$\int_0^1 \frac{dx}{1+x^2}$$

Answer 10:

Let $I = \int_0^1 \frac{dx}{1+x^2}$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4}$$

Question 11:

$$\int_2^3 \frac{dx}{x^2-1}$$

Answer 11:

Let $I = \int_2^3 \frac{dx}{x^2-1}$

$$\int \frac{dx}{x^2-1} = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \frac{1}{2} \left[\log \left| \frac{3-1}{3+1} \right| - \log \left| \frac{2-1}{2+1} \right| \right]$$

$$= \frac{1}{2} \left[\log \left| \frac{2}{4} \right| - \log \left| \frac{1}{3} \right| \right]$$

$$= \frac{1}{2} \left[\log \frac{1}{2} - \log \frac{1}{3} \right]$$

$$= \frac{1}{2} \left[\log \frac{3}{2} \right]$$

Question 12:

$$\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

Answer 12:

Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

$$\int \cos^2 x \, dx = \int \left(\frac{1 + \cos 2x}{2} \right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[F\left(\frac{\pi}{2} \right) - F(0) \right]$$

$$= \frac{1}{2} \left[\left(\frac{\pi}{2} - \frac{\sin \pi}{2} \right) - \left(0 + \frac{\sin 0}{2} \right) \right]$$

$$= \frac{1}{2} \left[\frac{\pi}{2} + 0 - 0 - 0 \right]$$

$$= \frac{\pi}{4}$$

Question 13:

$$\int_2^3 \frac{x \, dx}{x^2 + 1}$$

Answer 13:

Let $I = \int_2^3 \frac{x}{x^2 + 1} dx$

$$\int \frac{x}{x^2 + 1} dx = \frac{1}{2} \int \frac{2x}{x^2 + 1} dx = \frac{1}{2} \log \left(1 + x^2 \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \frac{1}{2} \left[\log \left(1 + (3)^2 \right) - \log \left(1 + (2)^2 \right) \right]$$

$$= \frac{1}{2} \left[\log(10) - \log(5) \right]$$

$$= \frac{1}{2} \log \left(\frac{10}{5} \right) = \frac{1}{2} \log 2$$

Question 14:

$$\int_0^1 \frac{2x+3}{5x^2+1}dx$$

Answer 14:

Let $I = \int_0^1 \frac{2x+3}{5x^2+1}dx$

$$\int \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\cdot \frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right)$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \left\{\frac{1}{5}\log(5+1) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)\right\} - \left\{\frac{1}{5}\log(1) + \frac{3}{\sqrt{5}}\tan^{-1}(0)\right\}$$

$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$$

Question 15:

$$\int_0^1 x e^{x^2} dx$$

Answer 15:

Let $I = \int_0^1 x e^{x^2} dx$

Put $x^2 = t \Rightarrow 2x \, dx = dt$

As $x \to 0, t \to 0$ and as $x \to 1, t \to 1$,

$\therefore I = \dfrac{1}{2} \int_0^1 e^t dt$

$\dfrac{1}{2} \int e^t dt = \dfrac{1}{2} e^t = F(t)$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(0)$

$= \dfrac{1}{2} e - \dfrac{1}{2} e^0$

$= \dfrac{1}{2} (e - 1)$

Question 16:

$$\int_0^1 \frac{5x^2}{x^2 + 4x + 3}$$

Answer 16:

Let $\quad I = \displaystyle\int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$

Dividing $5x^2$ by $x^2 + 4x + 3$, we obtain

$I = \displaystyle\int_1^2 \left\{ 5 - \frac{20x + 15}{x^2 + 4x + 3} \right\} dx$

$= \displaystyle\int_1^2 5 dx - \int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx$

$= \left[5x \right]_1^2 - \displaystyle\int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx$

$I = 5 - I_1$, where $I = \displaystyle\int_1^2 \frac{20x + 15}{x^2 + 4x + 3} dx \quad$ ...(1)

Consider $I_1 = \int_1^2 \dfrac{20x+15}{x^2+4x+8}\,dx$

Let $20x+15 = A\dfrac{d}{dx}\left(x^2+4x+3\right)+B$

$\qquad\qquad\quad = 2Ax+(4A+B)$

Equating the coefficients of x and constant term, we obtain

A = 10 and B = −25

$\Rightarrow I_1 = 10\int_1^2 \dfrac{2x+4}{x^2+4x+3}\,dx - 25\int_1^2 \dfrac{dx}{x^2+4x+3}$

Let $x^2+4x+3 = t$

$\Rightarrow (2x+4)dx = dt$

$\Rightarrow I_1 = 10\int \dfrac{dt}{t} - 25\int \dfrac{dx}{(x+2)^2 - 1^2}$

$\qquad = 10\log t - 25\left[\dfrac{1}{2}\log\left(\dfrac{x+2-1}{x+2+1}\right)\right]$

$\qquad = \left[10\log\left(x^2+4x+3\right)\right]_1^2 - 25\left[\dfrac{1}{2}\log\left(\dfrac{x+1}{x+3}\right)\right]_1^2$

$\qquad = \left[10\log 15 - 10\log 8\right] - 25\left[\dfrac{1}{2}\log\dfrac{3}{5} - \dfrac{1}{2}\log\dfrac{2}{4}\right]$

$\qquad = \left[10\log(5\times 3) - 10\log(4\times 2)\right] - \dfrac{25}{2}\left[\log 3 - \log 5 - \log 2 + \log 4\right]$

$\qquad = \left[10\log 5 + 10\log 3 - 10\log 4 - 10\log 2\right] - \dfrac{25}{2}\left[\log 3 - \log 5 - \log 2 + \log 4\right]$

$\qquad = \left[10 + \dfrac{25}{2}\right]\log 5 + \left[-10 - \dfrac{25}{2}\right]\log 4 + \left[10 - \dfrac{25}{2}\right]\log 3 + \left[-10 + \dfrac{25}{2}\right]\log 2$

$\qquad = \dfrac{45}{2}\log 5 - \dfrac{45}{2}\log 4 - \dfrac{5}{2}\log 3 + \dfrac{5}{2}\log 2$

$\qquad = \dfrac{45}{2}\log\dfrac{5}{4} - \dfrac{5}{2}\log\dfrac{3}{2}$

Substituting the value of I_1 in (1), we obtain

$I = 5 - \left[\dfrac{45}{2}\log\dfrac{5}{4} - \dfrac{5}{2}\log\dfrac{3}{2}\right]$

$\quad = 5 - \dfrac{5}{2}\left[9\log\dfrac{5}{4} - \log\dfrac{3}{2}\right]$

Question 17:

$$\int_0^{\frac{\pi}{4}} \left(2\sec^2 x + x^3 + 2\right) dx$$

Answer 17:

Let $I = \int_0^{\frac{\pi}{4}} \left(2\sec^2 x + x^3 + 2\right) dx$

$\int \left(2\sec^2 x + x^3 + 2\right) dx = 2\tan x + \dfrac{x^4}{4} + 2x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\dfrac{\pi}{4}\right) - F(0)$

$= \left\{ \left(2\tan\dfrac{\pi}{4} + \dfrac{1}{4}\left(\dfrac{\pi}{4}\right)^4 + 2\left(\dfrac{\pi}{4}\right)\right) - \left(2\tan 0 + 0 + 0\right)\right\}$

$= 2\tan\dfrac{\pi}{4} + \dfrac{\pi^4}{4^5} + \dfrac{\pi}{2}$

$= 2 + \dfrac{\pi}{2} + \dfrac{\pi^4}{1024}$

Question 18:

$$\int_0^{\pi} \left(\sin^2\dfrac{x}{2} - \cos^2\dfrac{x}{2}\right) dx$$

Answer 18:

Let $I = \int_0^{\pi} \left(\sin^2\dfrac{x}{2} - \cos^2\dfrac{x}{2}\right) dx$

$= -\int_0^{\pi} \left(\cos^2\dfrac{x}{2} - \sin^2\dfrac{x}{2}\right) dx$

$= -\int_0^{\pi} \cos x \, dx$

$\int \cos x \, dx = \sin x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(\pi) - F(0)$

$= \sin\pi - \sin 0$

$= 0$

Question 19:

$$\int_0^2 \frac{6x+3}{x^2+4}dx$$

Answer 19:

Let $I = \int_0^2 \frac{6x+3}{x^2+4}dx$

$$\int \frac{6x+3}{x^2+4}dx = 3\int \frac{2x+1}{x^2+4}dx$$

$$= 3\int \frac{2x}{x^2+4}dx + 3\int \frac{1}{x^2+4}dx$$

$$= 3\log(x^2+4) + \frac{3}{2}\tan^{-1}\frac{x}{2} = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(2) - F(0)$$

$$= \left\{3\log(2^2+4) + \frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\} - \left\{3\log(0+4) + \frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$$

$$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$$

$$= 3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$$

$$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$$

$$= 3\log 2 + \frac{3\pi}{8}$$

Question 20:

$$\int_0^1 \left(xe^x + \sin\frac{\pi x}{4} \right) dx$$

Answer 20:

Let $I = \int_0^1 \left(xe^x + \sin\frac{\pi x}{4} \right) dx$

$$\int \left(xe^x + \sin\frac{\pi x}{4} \right) dx = x\int e^x dx - \int\left\{ \left(\frac{d}{dx}x\right)\int e^x dx \right\} dx + \left\{ \frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}} \right\}$$

$$= xe^x - \int e^x dx - \frac{4\pi}{\pi}\cos\frac{x}{4}$$

$$= xe^x - e^x - \frac{4\pi}{\pi}\cos\frac{x}{4}$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \left(1.e^1 - e^1 - \frac{4}{\pi}\cos\frac{\pi}{4} \right) - \left(0.e^0 - e^0 - \frac{4}{\pi}\cos 0 \right)$$

$$= e - e - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}}\right) + 1 + \frac{4}{\pi}$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

Question 21:

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$$

A. $\dfrac{\pi}{3}$

B. $\dfrac{2\pi}{3}$

C. $\dfrac{\pi}{6}$

D. $\dfrac{\pi}{12}$ equals

Answer 21:

$$\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = F(\sqrt{3}) - F(1)$$

$$= \tan^{-1} \sqrt{3} - \tan^{-1} 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

Hence, the correct Answer is D.

Question 22:

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$$

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{12}$

C. $\dfrac{\pi}{24}$

D. $\dfrac{\pi}{4}$ equals

Answer 22:

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2 + (3x)^2}$$

Put $3x = t \Rightarrow 3dx = dt$

$$\therefore \int \frac{dx}{(2)^2 + (3x)^2} = \frac{1}{3} \int \frac{dt}{(2)^2 + t^2}$$

$$= \frac{1}{3} \left[\frac{1}{2} \tan^{-1} \frac{t}{2} \right]$$

$$= \frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right)$$

$$= F(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = F\left(\frac{2}{3}\right) - F(0)$$

$$= \frac{1}{6} \tan^{-1}\left(\frac{3}{2} \cdot \frac{2}{3}\right) - \frac{1}{6} \tan^{-1} 0$$

$$= \frac{1}{6} \tan^{-1} 1 - 0$$

$$= \frac{1}{6} \times \frac{\pi}{4}$$

$$= \frac{\pi}{24}$$

Hence, the correct Answer is C.