

# Mathematics

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(Chapter 8)(Application of Integrals)

XII

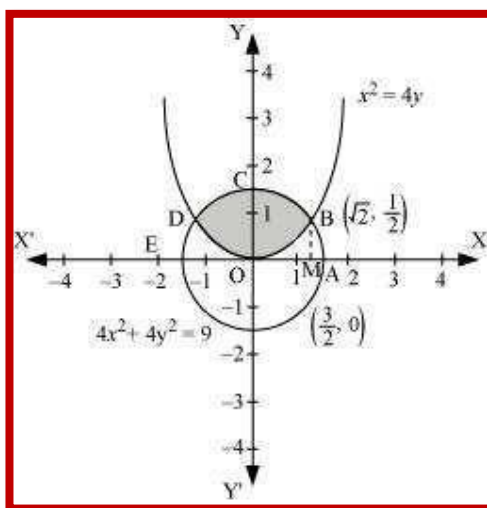
## Exercise 8.2

### Question 1:

Find the area of the circle  $4x^2 + 4y^2 = 9$  which is interior to the parabola  $x^2 = 4y$

### Answer 1:

The required area is represented by the shaded area OBCDO.



Solving the given equation of circle,  $4x^2 + 4y^2 = 9$ , and parabola,  $x^2 = 4y$ , we obtain the

point of intersection as  $B \left( \sqrt{2}, \frac{1}{2} \right)$  and  $D \left( -\sqrt{2}, \frac{1}{2} \right)$ .

It can be observed that the required area is symmetrical about y-axis.

$$\therefore \text{Area OBCDO} = 2 \times \text{Area OBCO}$$

We draw BM perpendicular to OA.

Therefore, the coordinates of M are  $\left( \frac{\sqrt{2}}{2}, 0 \right)$ .

Therefore, Area OBCO = Area OMBCO - Area OMBO



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$$\begin{aligned} &= \int_0^{\sqrt{2}} \sqrt{\frac{9-4x^2}{4}} dx - \int_0^{\sqrt{2}} \sqrt{\frac{x^2}{4}} dx \\ &= \frac{1}{2} \int_0^{\sqrt{2}} \sqrt{9-4x^2} dx - \frac{1}{4} \int_0^{\sqrt{2}} x^2 dx \\ &= \frac{1}{4} \left[ x\sqrt{9-4x^2} + \frac{9}{2} \sin^{-1} \frac{2x}{3} \right]_0^{\sqrt{2}} - \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^{\sqrt{2}} \\ &= \frac{1}{4} \left[ \sqrt{2}\sqrt{9-8} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] - \frac{1}{12} (\sqrt{2})^3 \\ &= \frac{\sqrt{2}}{4} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6} \\ &= \frac{\sqrt{2}}{12} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} \\ &= \frac{1}{2} \left( \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right) \end{aligned}$$

Therefore, the required area OBCDO is

$$\left( 2 \times \frac{1}{2} \left[ \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \right) = \left[ \frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right]$$

units

## Question 2:

Find the area bounded by curves  $(x-1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$

## Answer 2:

The area bounded by the curves,  $(x-1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$ , is represented by the shaded area as

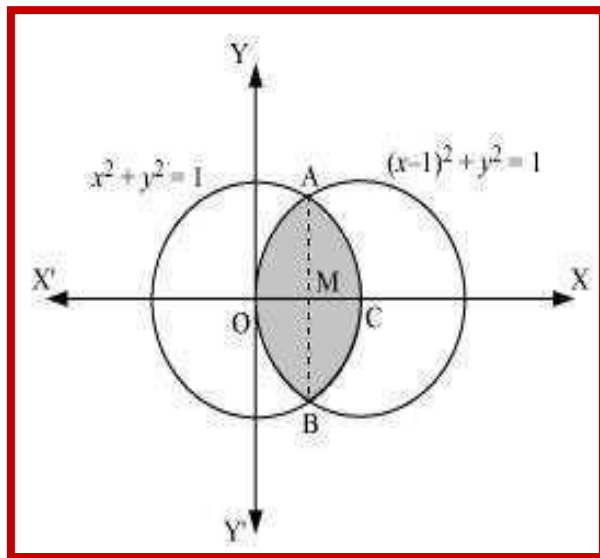


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On solving the equations,  $(x - 1)^2 + y^2 = 1$  and  $x^2 + y^2 = 1$ , we obtain the point of

intersection as A  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$  and B  $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ .

It can be observed that the required area is symmetrical about x-axis.

$\therefore$  Area OBCAO = 2  $\times$  Area OCAO

We join AB, which intersects OC at M, such that AM is perpendicular to OC.

The coordinates of M are  $\left(\frac{1}{2}, 0\right)$ .

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$\Rightarrow$  Area OCAO = Area OMAO + Area MCAM

$$\begin{aligned} &= \left[ \int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2} dx + \int_{\frac{1}{2}}^1 \sqrt{1-x^2} dx \right] \\ &= \left[ \frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{\frac{1}{2}} + \left[ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{\frac{1}{2}}^1 \\ &= \left[ -\frac{1}{4} \sqrt{1-\left(-\frac{1}{2}\right)^2} + \frac{1}{2} \sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2} \sin^{-1}(-1) \right] + \\ &\quad \left[ \frac{1}{2} \sin^{-1}(1) - \frac{1}{4} \sqrt{1-\left(\frac{1}{2}\right)^2} - \frac{1}{2} \sin^{-1}\left(\frac{1}{2}\right) \right] \\ &= \left[ -\frac{\sqrt{3}}{8} + \frac{1}{2} \left(-\frac{\pi}{6}\right) - \frac{1}{2} \left(-\frac{\pi}{2}\right) \right] + \left[ \frac{1}{2} \left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2} \left(\frac{\pi}{6}\right) \right] \\ &= \left[ -\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12} \right] \\ &= \left[ -\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} \right] \\ &= \left[ \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right] \end{aligned}$$

Therefore, required area OBCAO =  $2 \times \left( \frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right) = \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)$  units

### Question 3:

Find the area of the region bounded by the curves  $y = x^2 + 2$ ,  $y = x$ ,  $x = 0$  and  $x = 3$

### Answer 3:

The area bounded by the curves,  $y = x^2 + 2$ ,  $y = x$ ,  $x = 0$ , and  $x = 3$ , is represented by the shaded area OCBAO as

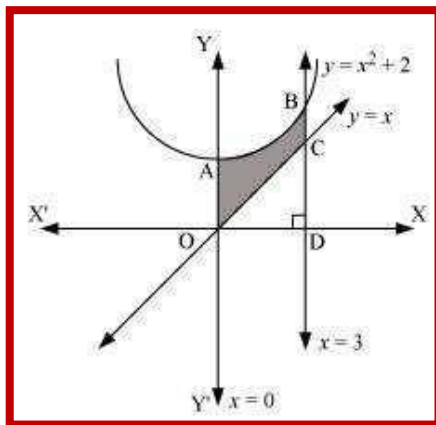


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Then, Area OCBAO = Area ODBAO - Area ODCO

$$\begin{aligned} &= \int_0^3 (x^2 + 2) dx - \int_0^3 x dx \\ &= \left[ \frac{x^3}{3} + 2x \right]_0^3 - \left[ \frac{x^2}{2} \right]_0^3 \\ &= [9 + 6] - \left[ \frac{9}{2} \right] \\ &= 15 - \frac{9}{2} \\ &= \frac{21}{2} \text{ units} \end{aligned}$$

#### Question 4:

Using integration finds the area of the region bounded by the triangle whose vertices are  $(-1, 0)$ ,  $(1, 3)$  and  $(3, 2)$ .

#### Answer 4:

BL and CM are drawn perpendicular to x-axis.

It can be observed in the following figure that,

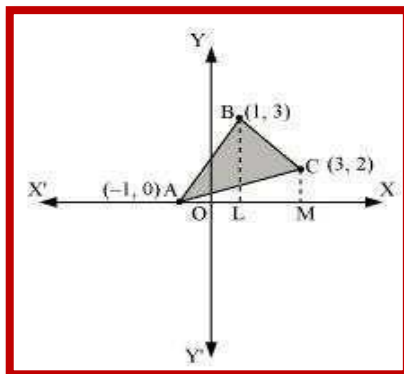
$$\text{Area } (\triangle ACB) = \text{Area } (ALBA) + \text{Area } (BLMCB) - \text{Area } (AMCA) \dots (1)$$

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Equation of line segment AB is

$$y - 0 = \frac{3 - 0}{1 - (-1)}(x + 1)$$

$$y = \frac{3}{2}(x + 1)$$

$$\therefore \text{Area(ALBA)} = \int_{-1}^1 \frac{3}{2}(x + 1) dx = \frac{3}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^1 = \frac{3}{2} \left[ \frac{1}{2} + 1 - \frac{1}{2} + 1 \right] = 3 \text{ units}$$

Equation of line segment BC is

$$y - 3 = \frac{2 - 3}{3 - 1}(x - 1)$$

$$y = \frac{1}{2}(-x + 7)$$

$$\therefore \text{Area(BLMCB)} = \int_1^3 \frac{1}{2}(-x + 7) dx = \frac{1}{2} \left[ -\frac{x^2}{2} + 7x \right]_1^3 = \frac{1}{2} \left[ -\frac{9}{2} + 21 + \frac{1}{2} - 7 \right] = 5 \text{ units}$$

Equation of line segment AC is

$$y - 0 = \frac{2 - 0}{3 - (-1)}(x + 1)$$

$$y = \frac{1}{2}(x + 1)$$

$$\therefore \text{Area(AMCA)} = \frac{1}{2} \int_{-1}^3 (x + 1) dx = \frac{1}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^3 = \frac{1}{2} \left[ \frac{9}{2} + 3 - \frac{1}{2} + 1 \right] = 4 \text{ units}$$

Therefore, from equation (1), we obtain

$$\text{Area}(\Delta ABC) = (3 + 5 - 4) = 4 \text{ units}$$



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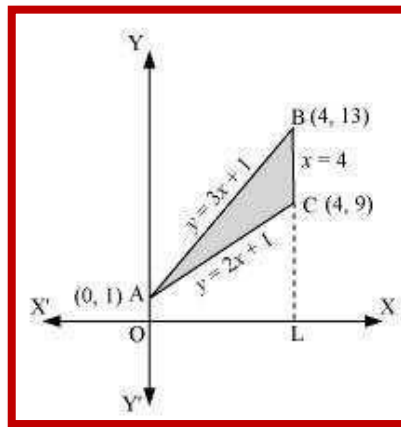
**Question 5:**

Using integration find the area of the triangular region whose sides have the equations  $y = 2x + 1$ ,  $y = 3x + 1$  and  $x = 4$ .

**Answer 5:**

The equations of sides of the triangle are  $y = 2x + 1$ ,  $y = 3x + 1$ , and  $x = 4$ .

On solving these equations, we obtain the vertices of triangle as  $A(0, 1)$ ,  $B(4, 13)$ , and  $C(4, 9)$ .



It can be observed that,

Area ( $\Delta ACB$ ) = Area (OLBAO) - Area (OLCAO)

$$= \int_0^4 (3x+1) dx - \int_0^4 (2x+1) dx$$

$$= \left[ \frac{3x^2}{2} + x \right]_0^4 - \left[ \frac{2x^2}{2} + x \right]_0^4$$

$$= (24+4) - (16+4)$$

$$= 28 - 20$$

$$= 8 \text{ units}$$

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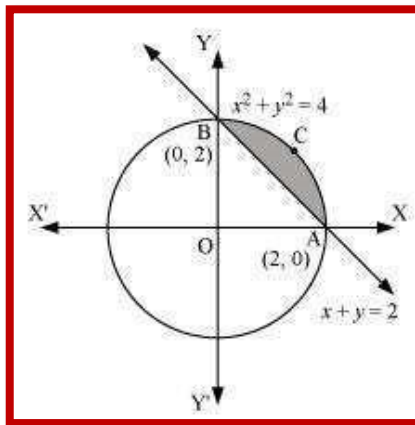
## Question 6:

Smaller area enclosed by the circle  $x^2 + y^2 = 4$  and the line  $x + y = 2$  is

- A.  $2(\pi - 2)$
- B.  $\pi - 2$
- C.  $2\pi - 1$
- D.  $2(\pi + 2)$

## Answer 6:

The smaller area enclosed by the circle,  $x^2 + y^2 = 4$ , and the line,  $x + y = 2$ , is represented by the shaded area ACBA as



It can be observed that,

Area ACBA = Area OACBO - Area ( $\Delta$ OAB)

$$\begin{aligned} &= \int_0^2 \sqrt{4-x^2} dx - \int_0^2 (2-x) dx \\ &= \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[ 2x - \frac{x^2}{2} \right]_0^2 \\ &= \left[ 2 \cdot \frac{\pi}{2} \right] - [4 - 2] \\ &= (\pi - 2) \text{ units} \end{aligned}$$

Thus, the correct answer is B.





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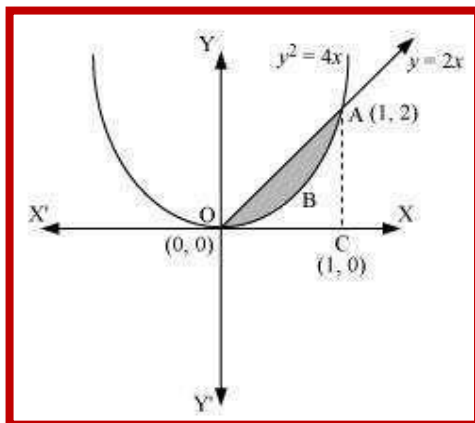
**Question 7:**

Area lying between the curve  $y^2 = 4x$  and  $y = 2x$  is

- A.  $\frac{2}{3}$
- B.  $\frac{1}{3}$
- C.  $\frac{1}{4}$
- D.  $\frac{3}{4}$

**Answer 7:**

The area lying between the curve,  $y^2 = 4x$  and  $y = 2x$ , is represented by the shaded area OBAO as



The points of intersection of these curves are O (0, 0) and A (1, 2).

We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

∴ Area OBAO = Area (ΔOCA) – Area (OCABO)

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$$\begin{aligned} &= \int_0^1 2x \, dx - \int_0^1 2\sqrt{x} \, dx \\ &= 2 \left[ \frac{x^2}{2} \right]_0^1 - 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 \\ &= \left| 1 - \frac{4}{3} \right| \\ &= \left| -\frac{1}{3} \right| \\ &= \frac{1}{3} \text{ units} \end{aligned}$$

Thus, the correct answer is B.

