Exercise 9.2

Question 1:

$y = e^x + 1 \quad : \quad y'' - y' = 0$

Answer

$y = e^x + 1$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + 1\right)$$

$$\Rightarrow y' = e^x \qquad ...(1)$$

Now, differentiating equation (1) with respect to x, we get:

$$\frac{d}{dx}(y') = \frac{d}{dx}\left(e^x\right)$$

$$\Rightarrow y'' = e^x$$

Substituting the values of y' and y'' in the given differential equation, we get the L.H.S. as:

$$y'' - y' = e^x - e^x = 0 = \text{R.H.S.}$$

Thus, the given function is the solution of the corresponding differential equation.

Question 2:

$y = x^2 + 2x + C \quad : \quad y' - 2x - 2 = 0$

Answer

$y = x^2 + 2x + C$

Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}\left(x^2 + 2x + C\right)$$

$$\Rightarrow y' = 2x + 2$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. = $y' - 2x - 2 = 2x + 2 - 2x - 2 = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 3:

$y = \cos x + C \quad : \quad y' + \sin x = 0$

Answer

$y = \cos x + C$

Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}(\cos x + C)$$
$$\Rightarrow y' = -\sin x$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. = $y' + \sin x = -\sin x + \sin x = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 4: $y = \sqrt{1+x^2} \quad : \quad y' = \frac{xy}{1+x^2}$

Answer

$y = \sqrt{1+x^2}$

Differentiating both sides of the equation with respect to x, we get:

$$y' = \frac{d}{dx}\left(\sqrt{1+x^2}\right)$$
$$y' = \frac{1}{2\sqrt{1+x^2}} \cdot \frac{d}{dx}\left(1+x^2\right)$$
$$y' = \frac{2x}{2\sqrt{1+x^2}}$$
$$y' = \frac{x}{\sqrt{1+x^2}}$$
$$\Rightarrow y' = \frac{x}{1+x^2} \times \sqrt{1+x^2}$$
$$\Rightarrow y' = \frac{x}{1+x^2} \cdot y$$
$$\Rightarrow y' = \frac{xy}{1+x^2}$$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 5:

$y = Ax \quad : \quad xy' = y \ (x \neq 0)$

Answer

$y = Ax$

Differentiating both sides with respect to x, we get:

$$y' = \frac{d}{dx}(Ax)$$
$$\Rightarrow y' = A$$

Substituting the value of y' in the given differential equation, we get:

$$\text{L.H.S.} = xy' = x \cdot A = Ax = y = \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 6:

$y = x\sin x \quad : \quad xy' = y + x\sqrt{x^2 - y^2} \ (x \neq 0 \text{ and } x > y \text{ or } x < -y)$

Answer

$y = x\sin x$

Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}(x\sin x)$$
$$\Rightarrow y' = \sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)$$
$$\Rightarrow y' = \sin x + x\cos x$$

Substituting the value of y' in the given differential equation, we get:

$$\begin{aligned}\text{L.H.S.} = xy' &= x(\sin x + x\cos x) \\ &= x\sin x + x^2\cos x \\ &= y + x^2 \cdot \sqrt{1 - \sin^2 x} \\ &= y + x^2\sqrt{1 - \left(\frac{y}{x}\right)^2} \\ &= y + x\sqrt{y^2 - x^2} \\ &= \text{R.H.S.}\end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 7:

$xy = \log y + C \qquad : \quad y' = \frac{y^2}{1-xy} \left(xy \neq 1\right)$

Answer

$xy = \log y + C$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$

$$\Rightarrow y \cdot \frac{d}{dx}(x) + x \cdot \frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow y + xy' = \frac{1}{y} y'$$

$$\Rightarrow y^2 + xy\, y' = y'$$

$$\Rightarrow (xy-1)\, y' = -y^2$$

$$\Rightarrow y' = \frac{y^2}{1-xy}$$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 8:

$y - \cos y = x \qquad : \quad (y \sin y + \cos y + x)\, y' = y$

Answer

$y - \cos y = x \qquad \ldots(1)$

Differentiating both sides of the equation with respect to x, we get:

$$\frac{dy}{dx} - \frac{d}{dx}(\cos y) = \frac{d}{dx}(x)$$
$$\Rightarrow y' + \sin y \cdot y' = 1$$
$$\Rightarrow y'(1+\sin y) = 1$$
$$\Rightarrow y' = \frac{1}{1+\sin y}$$

Substituting the value of y' in equation (1), we get:

$$\text{L.H.S.} = (y\sin y + \cos y + x)\, y'$$
$$= (y\sin y + \cos y + y - \cos y) \times \frac{1}{1+\sin y}$$
$$= y(1+\sin y) \cdot \frac{1}{1+\sin y}$$
$$= y$$
$$= \text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 9:

$x + y = \tan^{-1} y \quad : \quad y^2 y' + y^2 + 1 = 0$

Answer

$x + y = \tan^{-1} y$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{d}{dx}(x+y)=\frac{d}{dx}\left(\tan^{-1} y\right)$$

$$\Rightarrow 1+y'=\left[\frac{1}{1+y^2}\right]y'$$

$$\Rightarrow y'\left[\frac{1}{1+y^2}-1\right]=1$$

$$\Rightarrow y'\left[\frac{1-\left(1+y^2\right)}{1+y^2}\right]=1$$

$$\Rightarrow y'\left[\frac{-y^2}{1+y^2}\right]=1$$

$$\Rightarrow y'=\frac{-\left(1+y^2\right)}{y^2}$$

Substituting the value of y' in the given differential equation, we get:

$$\begin{aligned}\text{L.H.S.} = y^2y'+y^2+1 &= y^2\left[\frac{-\left(1+y^2\right)}{y^2}\right]+y^2+1\\ &=-1-y^2+y^2+1\\ &=0\\ &=\text{R.H.S.}\end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 10:

$y=\sqrt{a^2-x^2}\, x\in(-a,a)$: $x+y\frac{dy}{dx}=0\,(y\neq 0)$

Answer

$y=\sqrt{a^2-x^2}$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{a^2-x^2}\right)$$

$$\Rightarrow\frac{dy}{dx}=\frac{1}{2\sqrt{a^2-x^2}}\cdot\frac{d}{dx}\left(a^2-x^2\right)$$

$$=\frac{1}{2\sqrt{a^2-x^2}}(-2x)$$

$$=\frac{-x}{\sqrt{a^2-x^2}}$$

Substituting the value of $\frac{dy}{dx}$

$$\text{L.H.S.}=x+y\frac{dy}{dx}=x+\sqrt{a^2-x^2}\times\frac{-x}{\sqrt{a^2-x^2}}$$

$$=x-x$$

$$=0$$

$$=\text{R.H.S.}$$

Hence, the given function is the solution of the corresponding differential equation.

Question 11:

The numbers of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4

Answer

We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth order differential equation is four.

Hence, the correct answer is D.

Question 12:

The numbers of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3 (B) 2 (C) 1 (D) 0

Answer

In a particular solution of a differential equation, there are no arbitrary constants.

Hence, the correct answer is D.